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ON n-CYCLIC MAPS

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1. Introduction

The concept of cyclic maps was first introduced and studied by D. H. Gottlieb [3] and K. Varadarajan [12]. Many properties of cyclic maps can be found in [6], [9] and [14]. In this paper, we shall define and study an n-cyclic map which is a generalization of a cyclic map. In Section 3, existence of n-cyclic maps is shown. Also, we study some properties of n-cyclic maps. In Section 4, we study the mapping spaces $L(\Sigma A, X)$ and n-cyclic maps. In general, the components of $L(\Sigma A, X)$ almost never have the same homotopy type [13]. In this section, we study some sufficient conditions for homotopy equivalence between the components of $L(\Sigma A, X)$. Also, we show that if $f: \Sigma A \to X$ is n-cyclic, then $[\Sigma^r B, L(\Sigma A, X; nf)]$ is isomorphic to $[\Sigma(\Sigma^r B \wedge A), X] \oplus [\Sigma^r B, X]$ where A and B are suspensions.

2. Prelimlnaries

Unless otherwise stated, we shall work in the category of spaces with base points and having the homotopy type of connected locally finite CW complexes. All maps shall mean continuous functions. As usual, all maps and homotopies are to preserve base points with the exception of Section 4. The base point as well as the constant map will be denoted by*. The identity map of space will be denoted by 1 when it is clear from the context. For simplicity, we some times use the same symbol for a map and its homotopy class. Also, we denote by [X, Y] the set of homotopy classes of pointed maps $X \to Y$. The holding map $\nabla : X \wedge X \to X$ is given by

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$\nabla(x,*)=\nabla(*,x)=x$ for each $x\in X$. ΣX denote the reduced suspension of X. Let $L(\Sigma A, X)$ be the space of free maps from ΣA to X with the compact open topology. For a pointed map $f:\Sigma A\to X$, $L(\Sigma A, X;f)$ denotes the path component of $L(\Sigma A, X)$ which contains f.

$L_0(\Sigma A, X)$ and $L_0(\Sigma A, X;f)$ will denote the spaces of base points preserving maps in $L(\Sigma A, X)$ and $L(\Sigma A, X;f)$ respectively. The evaluation map $\omega: L(\Sigma A, X)\to X$ is defined to be $\omega(k)=k(*)$ for each $k\in L(\Sigma A, X)$. Varadarajan [12] called a map $f:A\to X$ is cyclic if $\nabla(1\vee f): X\vee A\to X$ extends to a map $F: X\times A\to X$. Also, Lim [9] called a map $f:A\to X$ is cyclic if there exists a map $F: X\times A\to X$ such that $Fj: X\vee A\to X$ is homotophic to $\nabla(1\vee f): X\vee A\to X$, where $j: X\vee A\to X\times A$ is the inclusion. However, in our category the Lim's definition is equivalent to the Varadarajan's.

3. Definition and existence of n-cyclic maps

Let f and g be (pointed) maps from A to X where A is a co-H-space with μ a comultiplication. Define $f+g: A\to X$ to be the composition

$$A\xrightarrow{\mu} A\vee A\xrightarrow{f\vee g} X\vee X\xrightarrow{\nabla} X$$

It is known [9] that if $f, g: A\to X$ are cyclic maps, then $f+g: A\to X$ is cyclic. When n is a positive integer, define nf by;

$$1f=f,\quad nf=(n-1)f+f \text{ for } n>1.$$

Definition 3.1. Let A be a co-H-space. A map $f: A\to X$ is said to be *n-cyclic* if $nf: A\to X$ is cyclic.

Proposition 3.2. *If k is odd, then the identity map of S^k is 2-cyclic.*

Proof. It is known [11] that there is a map $H: S^k\times S^k\to S^k$ of type $(1,2)$, that is, $H|_{S^k\times *}$ has degree 1 and $H|_{*\times S^k}$ has degree 2. Thus $1_{S^k}: S^k\to S^k$ is 2-cyclic.

Remark 3.3. Let A be a co-H-space. According to [9], it follows immediately that if $f: A\to X$ is cyclic, then $f: A\to X$ is n-cyclic for any $n \in N$. But the converse does not hold. From Proposition 3.2, the identity map of S^5 is 2-cyclic. However, it is known [9] that X is an H-space if and only if 1_X is cyclic. Thus the identity map of S^5 is not cyclic.

THEOREM 3.4. *Any map $f: S^{2k+1} \to S^{2k+1}$ is 2-cyclic.*

Proof. If $k=0$, then $f: S^1 \to S^1$ is cyclic. Thus $f: S^1 \to S^1$ is 2-cyclic. If $k>0$, then we have, from the Freudental suspension theorem, that there is a map $g: S^{2k} \to S^{2k}$ such that Σg is homotopic to f. Since $\sum g$ is co-H-map and the identity map of S^{2k+1} is 2-cyclic, $f: S^{2k+1} \to S^{2k+1}$ is 2-cyclic.

THEOREM 3.5. *Let A be a co-H-space. If $f: A \to X$ is n-cyclic and $g: X \to Y$ has a right homotopy inverse, then $gf: A \to Y$ is n-cyclic.*

Proof. Since $f: A \to X$ is n-cyclic, there is a map $F: X \times A \to X$ such that $Fj = \nabla(1 \vee nf)$. Let $h: Y \to X$ be a right homotopy inverse of g. We define a map $G: Y \times A \to Y$ by letting $G(y, a) = gF(h(y), a)$. Then $Gj = gF(h \times 1)j = gFj(h \vee 1) = g\nabla(1 \vee nf)(h \vee 1) = \nabla(gh \vee g(nf)) \sim \nabla(1 \vee n(gf))$. This proves the theorem.

THEOREM 3.6. *Let A be a co-H-space. If $g: X \to Y$ has a left homotopy inverse and $f: A \to X$ is a map such that $gf: A \to Y$ is n-cyclic, then $f: A \to X$ is n-cyclic.*

Proof. Since $gf: A \to Y$ is n-cyclic, there is a map $G: Y \times A \to Y$ such that $Gj = \nabla(1 \vee n(gf))$. Let $h: Y \to X$ be a left homotopy inverse of g. We define a map $F: X \times A \to X$ by letting $F(x, a) = hG(g(x), a)$. Then $Fj = hG(g \times 1)j = hGj(g \vee 1) = h\nabla(g \vee n(gf)) = h\nabla(g \vee g(nf)) = hg\nabla(1 \vee nf) \sim \nabla(1 \vee nf)$. This proves the theorem.

COROLLARY 3.7. *Let $f: S^{2k+1} \to S^{2k+1}$ be any map. If $g: S^{2k+1} \to X$ has a right homotopy inverse, then $gf: S^{2k+1} \to X$ is 2-cyclic.*

4. Applications in mapping spaces

Let $L(\Sigma A, X)$ be the space of maps from ΣA to X with the compact open topology, let $f: \Sigma A \to X$ and $L(\Sigma A, X; f)$ the path component of $L(\Sigma A, X)$ containing f. $L_0(\Sigma A, X)$ and $L_0(\Sigma A, X; f)$ will denote the spaces of base point preserving maps in $L(\Sigma A, X)$ and $L(\Sigma A, X; f)$ respectively. In general, the components of $L(\Sigma A, X)$ almost never have the same homotopy type. For example, $L(S^2, S^2; *)$ and $L(S^2, S^2; 1)$ have different homotopy type [13]. However, it is known by Koh [7] that if X is an H-space, then $L(S^p, X; f)$ and $L(S^p, X; g)$ have the same homotopy type for arbitrary f and g in $\pi_p(X)$. In this section, we study some sufficient conditions for homotopy equivalence of $L(\Sigma A, X; f)$ and $L(\Sigma A, X; g)$ for $f, g \in [\Sigma A, X]$. Thus we obtain the above Koh's result as a corollary. Also, we study homotopy groups of free mapping space $L(\Sigma A, X)$ with an n-cyclic map as base point.

DEFINITION 4.1. Clearly the evaluation map $\omega: L(\Sigma A, X) \to X$ is a fibration. Let $f: \Sigma A \to X$ be a pointed map. Since X is path connected, the restriction $\omega_f = \omega|_{L(\Sigma A, X; f)}: L(\Sigma A, X; f) \to X$ is a fibration with fiber $L_0(\Sigma A, X; f)$. We call this fibration $(L(\Sigma A, X; f), \omega_f, X)$ *the evaluation fibration defined by* f.

The evaluation fibration $(L(\Sigma A, X; *), \omega_*, X)$ defined by the neutral $* \epsilon [\Sigma A, X]$ is called *the neutral evaluation fibration.* It has a canonical section $\rho_*: X \to L(\Sigma A, X; *)$ defined by $\rho_*(x)(y) = x$ for every $x \in X$ and every $y \in \Sigma A$. The existence of a section is a very special property for an evaluation fibration.

LEMMA 4.2. *The evaluation fibration* $(L(\Sigma A, X; nf),\ \omega_{nf}, X)$ *has a section if and only if* $f: \Sigma A \to X$ *is* n*-cyclic.*

Proof. Let $\rho: X \to L(\Sigma A, X; nf)$ be a map such that $\omega_{nf}\rho = 1_X$. Define a map $H: X \times \Sigma A \to X$ by letting $H(x, \langle a, t\rangle) = \rho(x)\langle a, t\rangle$. Then $H: X \times \Sigma A \to X$ is a continuous map and $H(x, *) = \rho(x)(*) = \omega_{nf}\rho(x) = x$ and $H(*, \langle a, t\rangle) = \rho(*)\langle a, t\rangle$. Since $\rho(*)$ belongs to $L_0(\Sigma A, X; nf)$, $\rho(*)$ is homotophic to nf. Thus $f: \Sigma A \to X$ is n-cyclic. On the other hand, suppose that $f: \Sigma A \to X$ is n-cyclic. Then there is a map $F: X \times \Sigma A \to X$ such that $Fj = \nabla(1 \vee nf)$. Define a map $\rho: X \to L(\Sigma A, X)$ by letting $\rho(x)\langle a, t\rangle = F(x, \langle a, t\rangle)$. Since X is a path connected, $\rho: X \to L(\Sigma A, X; nf)$ is a map such that $\omega_{nf}\rho = 1_X$.

Two fiber spaces E_1 and E_2 over the same space B with projections $p_i(E_i)=B(i=1,2)$ are *equivalent over* B [13] if there exist mappings $\phi_1 : E_1 \to E_2$, $\phi_2 : E_2 \to E_1$ such that i) $\phi_1\phi_2$ and $\phi_2\phi_1$ are homotopic to the identity maps of E_2 and E_1, respectively. ii) $p_2\phi_1$ is homotopic to p_1 and $p_1\phi_2$ is homotopic to p_2.

REMARK 4.3. However, it is known [2. Theorem 6.1] that two fiber spaces E_1 and E_2 are equivalent over B if and only if E_1 and E_2 are fiber homotopy equivalent.

PROPOSITION 4.4 (G.W. Whitehead). *$L(S^p, X ; f)$ is equivalent to $L(S^p, X ; *)$ over X if and only if the evaluation fibration $(L(S^p, X ; f), \omega_f, X)$ has a section.* (*p.* 464, *Theorem* 2.8 *in* [13]).

One can generalize Proposition 4.4, without essential changes in its proof, to the case of an arbitrary sphere S^p into a reduced suspension ΣA. Thus we have the following theorem.

THEOREM 4.5. *The following statements are equivalent ;*

i) *$f : \Sigma A \to X$ is n-cyclic*

ii) *$(L(\Sigma A, X ; nf), \omega_{nf}, X)$ has a section*

iii) *$L(\Sigma A, X ; nf)$ is fiber homotopic equivalent to $L(\Sigma A, X ; *)$.*

Proof. The assertions i)$\Longleftrightarrow$ii) follows from Lemma 4.2. ii) implies iii). Let ρ be a section of $\omega_{nf} : L(\Sigma A, X ; nf) \to X$. We can define a map $\phi : L(\Sigma A, X ; *) \to L(\Sigma A, X ; nf)$ by $\phi(h)=h+\rho(\omega_*(h))$, where $h+\rho(\omega_*(h)) : \Sigma A \to X$ is a map given by starting out with the co-H-structure $\mu : \Sigma A \to \Sigma A \vee \Sigma A$ by applying h to the first factor and $\rho(\omega_*(h))$ to the second factor in the wedge product $\Sigma A \vee \Sigma A$. For $h \in L(\Sigma A, X ; *)$, there is a path $p_1 : I \to L(\Sigma A, X)$ from $*$ to h. Let $p_2 : I \to L(\Sigma A, X)$ be defined by $p_2(t)=\rho\omega_* p_1(t)$. Then p_2 is a path from nf to $\rho\omega_*(h)$ and $\omega_f p_2(t)=\omega_* p_1(t)$. Thus we define a map $p : I \to L(\Sigma A, X)$ by $p(t)=p_1(t)+p_2(t)$. Then p is a path from nf to $h+\rho\omega_*(h)$. Thus $\phi : L(\Sigma A, X ; *) \to L(\Sigma A, X ; nf)$ is well defined. Similarly, define a map $\psi : L(\Sigma A, X ; nf) \to L(\Sigma A, X ; *)$ by $\psi(k)=k+(-(\rho(\omega_{nf}(k))))$. Then ψ is also well defined. It is easy to see that ϕ and ψ are fiber preserving maps and that $\psi\phi(h)=(h+\rho(\omega_*(h)))+(-(\rho(\omega_*(h))))$ and $\phi\psi(k)=(k+(-(\rho(\omega_{nf}(k)))))+\rho(\omega_{nf}(k))$. To show that $\psi\phi$ is homotopic to

1 and $\phi\psi$ is homotophic to 1, define maps $H:(\Sigma A, X;*)\times I\to L(\Sigma A, X;*)$ by letting

$$H(h,t)\langle a,s\rangle=\begin{cases} h\langle a, 4s/(1+3t)\rangle & ,0\leq s\leq (1+3t)/4 \\ \rho\omega_*(h)\langle a, 4s-(1+3t)\rangle & ,(1+3t)/4\leq s\leq (1+t)/2 \\ \rho\omega_*(h)\langle a, 2-2s\rangle & ,(1+t)/2\leq s\leq 1 \end{cases}$$

and $K:L(\Sigma A, X;nf)\times I\to L(\Sigma A, X;nf)$ by letting

$$K(k,t)\langle a,s\rangle=\begin{cases} k< a, 4s/1+3t)\rangle & ,0\leq s\leq (1+3t)/4 \\ \rho\omega_{nf}(k)\langle a, 2-4s+3t\rangle & ,(1+3t)/4\leq s\leq (1+t)/2 \\ \rho\omega_{nf}(k)\langle a, 2s-1\rangle & ,(1+t)/2\leq s\leq 1. \end{cases}$$

Then H is a homotopy from $\psi\phi$ to 1 and K is a homotopy from $\phi\psi$ to 1. Thus, by Remark 4.3, $L(\Sigma A, X;nf)$ is fiber homotopy equivalent to $L(\Sigma A, X;*)$.

iii) implies ii). There is a canonical section $\rho_*:X\to L(\Sigma A, X;*)$ such that $\omega_*\rho_*=1$. From the hypotheses, there is a map $\phi:L(\Sigma A, X;*)\to L(\Sigma A, X;nf)$ such that $\omega_{nf}\phi$ is homotopic to ω_*. Let $\rho=\phi\rho_*$. Then $\omega_{nf}\rho=\omega_{nf}\phi\rho_*\sim\omega_*\rho_*=1$. By the covering homotopy property, there is a map $\bar{\rho}:X\to L(\Sigma A, X;nf)$ such that ρ is homotophic to $\bar{\rho}$ and $\omega_{nf}\bar{\rho}=1$. Thus $\omega_{nf}:L(\Sigma A, X;nf)\to X$ has a section $\bar{\rho}$.

Corollary 4.6. *If $f:\Sigma A\to X$ is m-cyclic and $g:\Sigma A\to X$ is n-cyclic, then $L(\Sigma A, X;mf)$ and $L(\Sigma A, X;ng)$ are homotopy equivalent.*

This corollary tells why all the components in $L(S^3, S^2)$ (or $L(S^7, S^4)$) have the same homotopy type.

Gottlieb [4] introduced the evaluation subgroups $G_m(X)$ of homotopy groups $\pi_m(X)$. $G_m(X)$ is defined to be the set of all $f\in\pi_m(X)$ for which $f:S_m\to X$ is cyclic. Also, a space X satisfying $G_m(X)=\pi_m(X)$ for all m is called a G-space.

Corollary 4.7. *If X is a G-space, then $L(S^p, X;f)$ and $L(S^p, X;g)$ have the same homotopy type for arbitrary f and g in $\pi_p(X)$.*

Cleary, any H-space is a G-space, but the converse is not true [10]. Thus the above corollary generalize Koh's result to the case of H-spaces into G-spaces.

Proposition 4.8. [5]. *For f and g in $\pi_m(S^n)$, if the Whitehead* *product* $[f, 1_n]=[g, 1_n]$ (*or* $[f, 1_n]=-[g, 1_n]$), *then the evaluation fibrations* $(L(S^m, S^n ; f),\ \omega_f, S^n)$ *and* $(L(S^m, S^n ; g),\ \omega_g, S^n)$ *are fiber homotopy equivalent.*

However, it is known [6] that $f : \Sigma A \to \Sigma X$ is cyclic if and only if $[1_{\Sigma X}, f]=0$, where $[\,,\,]$ is the generalized Whitehead product. Thus we may can interpret the hypotheses of Proposition 4.8. in terms of cyclic maps. Next theorem gives another sufficient condition for homotopy equivalence of $L(\Sigma A, X ; f)$ and $L(\Sigma A, X ; g)$.

THEOREM 4.9. *For pointed maps* $f, g : \Sigma A \to X$, *if* $f+g$ (*or* $f-g$) $: \Sigma A \to X$ *is cyclic, then the evaluation fibrations* $(L(\Sigma A, X ; f),\ \omega_f, X)$ *and* $(L(\Sigma A, X ; g), \omega_g, X)$ *are fiber homotopy equivalent.*

Proof. For proving this case the technique is identically the same as that used by Hansen [5] for proving Proposition 4.8. We carry out the proof for the case $f+g : \Sigma A \to X$ is cyclic and the other case is similar. By Lemma 4.2, there is a section $\rho : X \to L(\Sigma A, X ; f+g)$ for the evaluation fibration defined by $f+g$. Let us consider maps $\phi : L(\Sigma A, X ; f) \to L(\Sigma A, X ; g)$ is given by $\phi(h)=(-h)+\rho(\omega_f(h))$ and $\psi : L(\Sigma A, X ; g) \to L(\Sigma A, X ; f)$ is given by $\psi(k)=\rho(\omega_g(k))+(-k)$. Then it follows that ϕ and ψ are fiber preserving maps and that $\psi\phi$ and $\phi\psi$ are fiber homotopic to the respectively identity maps. This prove the theorem.

COROLLARY 4.10. *If* $f : \Sigma A \to X$ *is* n-*cyclic, then* $L(\Sigma A, X ; (n-r)f)$ *is homotopy equivalent to* $L(\Sigma A, X ; rf)$, *where* $0 \le r \le n$ *and* $0f$ *is the constant map**.

PROPOSITION 4.11 (G. E. Lang, Jr.). *Let* $f \in [\Sigma A, X]$. *For any* $r \ge 1$, *we have then a commutative diagram*

$$\begin{array}{ccccccc}
\to [\Sigma^{r+1}B, X] & \xrightarrow{\partial_f} & [\Sigma^r B, L_0(\Sigma A, X ; f)] & \xrightarrow{i_*} & [\Sigma^r B, L(\Sigma A, X ; f)] & \xrightarrow{\omega_*} & \\
\Big| & & \Big\uparrow \hat{f}_* & & & & \\
\Big| p_f & & [\Sigma^r B, L_0(\Sigma A, X ; *)] & & & & \\
\Big| & & \Big\uparrow \theta & & & & \\
\big\lfloor & \longrightarrow & [\Sigma(\Sigma^r B \wedge A), X] & & & &
\end{array}$$

where ∂_f *is the boundary operator in the exact homotopy sequence* *for the evaluation fibration defined by f, θ is the isomorphism given by $\theta(k)(r)$ is the map taking $\langle a,t\rangle$ to $k\ \langle\langle a,r\rangle,t\rangle$ in X, $\hat{f}_*$ is the isomorphism induced by the homotopy equivalence $\hat{f}: L_0(\Sigma A, X; *)\to L_0(\Sigma A, X; f)$ given by $\hat{f}(h)=h+f$, and p_f is the f-Whitehead homorphism($p_f(g)=[g,f]$ for any g in $[\Sigma^{r+1}B, X]$).*

LEMMA 4.12. *If A and B are suspensions and $f:\Sigma A\to X$ is n-cyclic, then the nf-Whitehead homomorphism $P_{nf}:[\Sigma^r B, X]\to[\Sigma(\Sigma^{r-1}B\wedge A, X]$ is zero map.*

Proof. It follows from Theorem 3.2 in [12] and Proposition 3.4 in [1].

THEOREM 4.13. *If A and B are suspensions and $f:\Sigma A\to X$ is n-cyclic, then $[\Sigma^r B, L(\Sigma A, X; nf)]$ is isomorphic to $[\Sigma(\Sigma^r B\wedge A), X]\oplus[\Sigma^r B, X]$ for any integer $r\geq 1$.*

Proof. For $nf\in[\Sigma A, X]$, the evaluation map $\omega: L(\Sigma A, X; nf)\to X$ is a fibration with fiber $L_0(\Sigma A, X; nf)$. Then, by the Proposition 4.11, there is a long exact nf-component EHP-sequence

$$\xrightarrow{p_{nf}}[\Sigma(\Sigma^r B\wedge A), X]\xrightarrow{i''_*}[\Sigma^r B, L[\Sigma A, X; nf)]\xrightarrow{\omega_*}[\Sigma^r B, X]\xrightarrow{p_{nf}}[\Sigma(\Sigma^{r-1}B\wedge A), X]$$

where $i''_*=i_*\hat{f}_*\theta$. According to the above lemma, we obtain a short exact sequence

$$0\longrightarrow[\Sigma(\Sigma^r B\wedge A), X]\xrightarrow{i''_*}[\Sigma^r B, L(\Sigma A, X; nf)]\xrightarrow{\omega_*}[\Sigma^r B, X]\longrightarrow 0$$

From Lemma 4.2, there is a section $\rho: X\to L(\Sigma A, X; nf)$ for the evaluation fibration defined by nf. Thus $\rho_*:[\Sigma^r B, X]\to[\Sigma^r B, L(\Sigma A, X; nf)]$ is a homorphism such that $\omega_*\rho_*=1_{[\Sigma^r B, X]}$. Hence this sequence is split. This completes the proof.

COROLLARY 4.14 [14]. *If $f: S^p\to X$ is cyclic, then $\pi_q(L(S^p, X; f))$ is isomorphic to $\pi_{p+q}(X)\oplus\pi_q(X)$ for all $p,q>1$.*

References

1. M. Arkowitz, *The generalized Whitehead product*, Pacific J. Math., **12** (1962), 7-23.

2. A. Dold, *Partitions of unity in the theory of fibrations*, Ann. of Math., **78** (1963), 223-255.
3. D.H. Gottlieb, *A certain subgroup of the fundamental group*, Amer. J. Math., **87**(1965), 840-856.
4. D.H. Gottlieb, *Evaluation subgroups of homotopy groups*, Amer. J. Math., **91**(1969), 729-756.
5. V.L. Hansen, *The homotopy problem for the components in the space of maps on the n-sphere*, Quart. J. Math. Oxford (3), **25**(1974), 313-321.
6. C.S. Hoo, *Cyclic maps from suspensions to suspensions*, Can. J. Math., **24** (5)(1972), 789-791.
7. S.S. Koh, *Note on the properties of the components of the mapping spaces* X^{S^q}, Proc. Amer. Math. Soc., **11**(1960), 896-904.
8. G.E. Lang, Jr., *The evaluation map and EHP sequences*, Pacific J. Math., **44**(1)(1973), 201-210.
9. K.L. Lim, *On cyclic maps*, J. Austral. Math. Soc., (series A) **32**(1982), 349-357.
10. J. Siegel, *G-spaces, W-spaces and H-spaces*, Pacific J. Math., **31**(1969), 209-214.
11. N.E. Steenrod, *Cohomology operations*, Princeton, New Jersey, Princeton Univ. Press, 1962.
12. K. Varadarajan, *Generalized Gottlieb groups*, J. Indian Math. Soc., **33** (1969), 141-164.
13. G.W. Whitehead, *On products in homotopy groups*, Ann. Math., **47**(1946), 460-475.
14. M.H. Woo and Y.S. Yoon, *On some properties of G-spaces*, Comm. Korean Math. Soc., **2**(1987), 117-121.

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