H-Polarized Scattering by an Inversely Tapered Resistive Half Plane

(반비례적으로 변하는 저항율을 갖는 반평면에 의한 H 분극산란)

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要 約

가장자리로 부터 역비례로 저항율이 변하는 반평면에 H분극 평면파가 입사되는 경우, 산란파를 Kontorovich-Lebedev 변환을 이용해서 정확한 적분식으로 얻었으며, 이로부터 모든 각도에서 쓸 수 있는 균일 근사식 및 계산된 산란파를 광선적(光線的)으로 해석할 수 있도록 해주는 비균일 근사식도 구했다. 저항율의 상수 여러 값에 대하여 가장자리 회절패턴을 보였다. 결과들은 물리적으로 타당함을 알 수 있었다.

Abstract

For H-polarized incident plane wave, an exact integral expression for the scattered field by an inversely tapered resistive half plane is obtained by using Kontorovich-Lebedev transform. Uniform asymptotic results available for all angles are obtained, and non-uniform asymptotic results which provide the ray-optical interpretation of the calculated scattered field are also obtained. The edge diffraction patterns for several values of inverse proportionality of resistivity are shown. We find out that the results are in agreement with physical reasoning.

I. Introduction

Resistive materials are of interest for radar cross section reduction. The uniform resistive half plane has been studied [1] and the uniform or tapered resistive strips are also studied [2,3]. For the backscattering from the tapered resistive strips, numerical results show that the smooth variation of the resistivity reduces the near edge-on backscattering below that of the uniform resistive strip. This leads us to seek the sqlution for an electromagnetic plane wave incident on a varying resistive half plane.

Wedge of constant and varying impedance walls

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are studied[4,5], but do not provide any information for the scattered electromagnetic wave from the resistive half plane of varying resistivity.

As a particular case when the resistivity varies linearly with distance from the edge of the resistive half plane, an exact intergral expression for the scattered field for the plane wave incidence of electric polarization was obtained by using Kontorovich-Lebedev transform [6].

In this paper for H-polarization case, an exact integral expression for the scattered field by a half plane with inversely varying resistivity is also obtained in the same way for E-polarization case. The integral converges in a limited angular region and could be analytically continued to be valid in the whole angular region. Uniform and non-uniform asymptotic results for the scattered field are obtained, which provide the ray-optical interpretation. Our results are in agreement with physical reasoning. As the inverse proportionality of resistivity decreases, the edge diffraction coefficient is shown to approach that of a conducting half plane.

II. Kontorovich-Lebedev Transform

An electromagnetic plane wave H-polarized in z-direction is incident upon a resistive half plane with incident angle ϕ_0, $0<\phi_0<\pi$, as shown in Fig. 1. The scattered field H_z^s satisfies the Helmholtz equation

$$(\nabla^2+k^2)H_z^s=0 \tag{1}$$

subject to the boundary conditions on the resistive sheet for the total field as

$$[\partial H_z/\partial\phi]_{\phi=2\pi}^{\phi=0}=0 \tag{2a}$$

$$\partial H_z/\partial\phi\,|_{\phi=0}=jk\rho R\,[H_z]_{\phi=2\pi}^{\phi=0}=ja\,[H_z]_{\phi=2\pi}^{\phi=0} \tag{2b}$$

where [ ] denotes the discontinuity across the half plane, k is the wave number of free space, and $R=a/k\rho$ is the resisitivity normalized to the free space intrinsic impedance and inversely proportional to ρ.

In order to apply the Kontorovich-Lebedev (K-L) transform [6]

$$f(k\rho)=\frac{1}{4j}\int_{-j\infty}^{j\infty}\nu F(\nu)\ \exp(-j\nu\pi)\ \sin\nu\pi\, H_\nu^{(2)}(k\rho)\,d\nu \tag{3a}$$

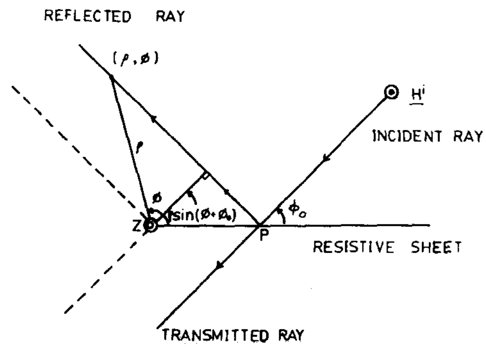


Fig.1. Resistive half plane and field points.

$$F(\nu)=\int_0^\infty \rho^{-1}f(k\rho)\,H_\nu^{(2)}(k\rho)\,d\rho,$$
$$k=|k|\exp(-j\alpha),\quad 0<\alpha<\pi \tag{3b}$$

one may introduce f to assume the convergence of the transform in (3b) as

$$f(\rho,\phi)=H_z^s(\rho,\phi)-f_o\ \exp(-jk\rho) \tag{4a}$$

where

$$f_o=H_z^s(\rho=0,\phi) \tag{4b}$$

so that f becomes zero as ρ approaches zero or

$$\lim_{\rho\to 0} f(\rho,\phi)=0\ . \tag{5a}$$

The edge condition for the ϕ component of electric field yields

$$\lim_{\rho\to 0}\ \rho\ \partial f/\partial\rho=0\,. \tag{5b}$$

Assuming a slightly lossy medium in free space, the far field satisfies the boundary conditions

$$\lim_{\rho\to\infty} f(\rho,\phi)=0 \tag{5c}$$

$$\lim_{\rho\to\infty}\partial f/\partial\rho=0\ . \tag{5d}$$

Substituting H_z^s in (4a) into (1), one obtains a differential equation for f,

$$(\nabla^2+k^2)\,f=\frac{jk}{\rho}f_o\ \exp(-jk\rho) \tag{6}$$

and one obtains a differential equation for $F(\nu,\phi)$ by K-L transform[6] as

$$(\partial^2/\partial\phi^2+\upsilon^2)F(\upsilon,\phi)=j2f_o\frac{\upsilon}{\sin\upsilon\pi}\exp(j\upsilon\pi/2), \quad (7)$$

the solution of which has been easily obtained as

$$F(\upsilon,\phi)=A(\upsilon)\cos(\upsilon\phi)+B(\upsilon)\sin(\upsilon\phi)+\frac{j2f_o\exp(j\upsilon\pi/2)}{\upsilon\sin(\upsilon\pi)} \quad (8)$$

where A(υ) and B(υ) are arbitrary constant to be determied from the boundary conditions defined in (2).

The total field is expressed as

$$H_z=\exp\{jk\rho\cos(\phi-\phi_o)\}+H_z^s \quad (9)$$

Substituting (4a) and (9) into (2), one obtains the corresponding boundary conditions for f along the resistive sheet. The K-L transform then yields the boundary conditions for F as

$$\left[\frac{\partial F(\upsilon,\phi)}{\partial\phi}\right]_{\phi=2\pi}^{\phi=0}=0 \quad (10a)$$

$$\left.\frac{\partial F(\upsilon,\phi)}{\partial\phi}\right|_{\phi=0}-ja\,[F]_{\phi=2\pi}^{\phi=0}=\frac{-j2\exp(j\upsilon\pi/2)\sin\upsilon(\pi-\phi_o)}{\sin\upsilon\pi} \quad (10b)$$

Subsituting (8) into (10), one obtains A and B constants and then F in (8) becomes

$$F(\upsilon,\phi)=\frac{2\exp(j\upsilon\pi/2)}{j\upsilon\sin(\upsilon\pi)}\,[-f_o-W(\upsilon,\phi)] \quad (11a)$$

where

$$W(\upsilon,\phi)=\frac{\upsilon\sin\upsilon(\pi-\phi_o)\sin\upsilon(\pi-\phi)}{\upsilon\cos\upsilon\pi+j2a\sin\upsilon\pi} \quad (11b)$$

Since the inverse K-L transform of the first term in the right hand side of (11a) equals the additional term $-f_o\exp(-jk\rho)$, the scattered field becomes the inverse K-L transform of the second term in (11a), i.e.

$$H_z^s(\rho,\phi)=\int_0^{j\infty}W(\upsilon,\phi)\exp(-j\upsilon\pi/2)H_\upsilon^{(2)}(k\rho)d\upsilon. \quad (12)$$

From the convergence requirement, it is noted that the restriction $|\phi-\pi|<\phi_o$ must be imposed, i.e. the representation is valid only in the region where H_z^s is an outgoing cylindrical wave. For the other angular regions, the analytical continuation of the integral (12) may be used.

III. Analytic Continuation and Asymptotic Approximation

We may decompose W(ν, ϕ) as

$$W(\upsilon,\phi)=[W-d]+d \quad (13)$$

where

$$d=\frac{\upsilon\exp\{j\upsilon(\phi_o-\psi)\}}{2(\upsilon-2a)},\quad \psi=|\pi-\phi| \quad (14)$$

is that part of W which diverges in either reflection region ($0\leqslant\phi<\pi-\phi_o$) or shadow region ($\pi+\phi_o<\phi\leqslant2\pi$), as shown in Fig.1.

Let

$$I_k=\int_0^{j\infty}W_k(\upsilon,\phi)\exp(-j\upsilon\pi/2)H_\upsilon^{(2)}(k\rho)d\upsilon,\quad (k=1,2) \quad (15)$$

where $W_1=d$ and $W_2=W-d$. Then the integral I_2 converges for all values of ϕ. The integral I_1, on the other hand, may be obtained by using the integral representation[7]

$$H_\upsilon^{(2)}(k\rho)=\frac{j\exp(j\upsilon\pi/2)}{\pi}\int_{-\infty}^{\infty}\exp(-jk\rho\cosh t-\upsilon t)dt,\quad -1<\mathrm{Re}\,\upsilon<1 \quad (16)$$

and after reversal of the order of the integraton permissible in $|\phi-\pi|<\phi_o$ as

$$I_1=(ja/\pi)\int_{-\infty}^{\infty}\exp(-jk\rho\cosh t+w)\{E_1(w)-\exp(-w)/w\}dt \quad (17)$$

where

$$w=2a\{-t+j(\phi_o-\psi)\} \quad (18)$$

and

$$E_1(w)=\int_w^{\infty}\frac{\exp(-\mu)}{\mu}d\mu,\quad |\arg w|<\pi \quad (19)$$

is the exponential intergal [8].

The exponential integral of integrand in (17) has a logarithmic branch point at w=0 or $t=t_b=j(\phi_o-\psi)$ and the second term in (17) has a pole at $t=t_p=j(\phi_o-\psi)$. The integrand in (17) decays as $t=\pm\infty$ for the slightly lossy medium (Im k < 0), and the integration path is along the real t axis. As the branch point t_b or the pole t_p crosses the integration path, the integral may be analytically continued into other angular regions by adding the integral around this branch cut and the pole contribution. The asymptotic branch cut contribution I_{1b} is

$$I_{1b}=(\pi/ja)\int_{C_b}\exp(-jk\rho\cosh t+w)\ E_1(w)\ dt$$

$$\cong\frac{-2a\ \exp\{-jk\rho\cos(\phi_o-\psi)\}}{k\rho\sin(\phi_o-\psi)-2a},$$

$$\psi=|\pi-\phi|>\phi_o \qquad (20)$$

where C_b is the integration path around the branch cut in Fig. 2a. And the pole contribution I_{1p} is

$$I_{1P}=(j/2\pi)\int_{C_P}\frac{\exp(-jk\rho\cosh t)}{t-j(\phi_o-\psi)}dt$$

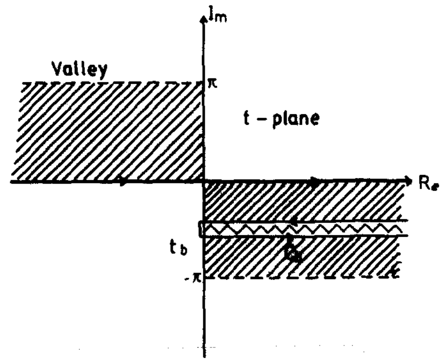
$$\cong-\exp\{jk\rho\cos(\phi-\phi_o)\}, \qquad (21)$$

where C_p is the intergation path around the pole in Fig. 2b. The saddle point contribution I_{1s} may be obtained by expanding the integrand near the saddle point t=0 as[4]

$$I_{1S}\cong(2/\pi k\rho)^{1/2}\exp\{-j(k\rho-\pi/4)\}$$

$$\left\{a\ \exp\{j2a(\phi_o-\psi)\}\ E_1\{j2a(\phi_o-\psi)\}+\frac{j}{2(\phi_o-\psi)}\right\} \qquad (22)$$

Similarly the integral I_2 may be evaluated asymptotically since W-d does not have any singularities in the υ-plane. Then the scattered field may be obtained by adding I_{1s}, I_{1b} and I_{1p} to the saddle point contribution of I_2 as



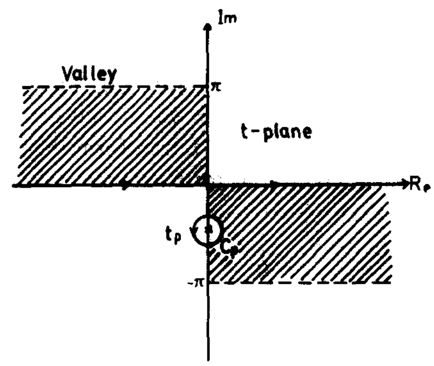


Fig.2. (a) Integration path C_b in the t-plane.
(b) Integration path C_p in the t-plane.

$$H_z^s(\rho,\phi)\cong(2/\pi k\rho)^{1/2}\exp\{-j(k\rho-\pi/4)\}D_S(\phi,\phi_o)$$

$$+I_{1S}+(I_{1b}+I_{1P})\ u(\psi-\phi_o), \qquad (23)$$

where $u(\psi-\phi_o)$ is the unit step function which yields one for $\psi>\phi_o$ and zero for $\psi<\phi_o$ and

$$D_S(\phi,\phi_o)=\int_0^{j\infty}\left\{W(\upsilon,\phi)-\frac{\upsilon\ \exp\{j\upsilon(\phi_o-\psi)\}}{2(2a-\upsilon)}\right\}d\upsilon. \qquad (24)$$

The asymptotic approximation for the scattered field in (23) is valid now in all ranges of ϕ provided that the branch point ($t=t_b$) and the pole ($t=t_p$) are not near the first-order saddle point (t=0). As ϕ approaches $\psi=|\pi-\phi|$, the branch point and pole approach the saddle point and a uniform asymptotic evaluation of I_1 in (17) is needed for H_z^S in (12).

Defining

$$J_k=(a/j\pi)\int_{-\infty}^{\infty}\exp(-jk\rho\cosh t+W)Y_k(w)dt,\quad (k=1,2,3) \tag{25}$$

where $Y_1=-\ln(w/2a)$, $Y_2=E_1(w)+\ln(w/2a)$ and $Y_3=\exp(-w)/w$, then $I_1=J+J_2+J_3$. The uniform asymptotic approximation of the intergal J_2 may easily be obtained since its integrand has no singularities, and that of the integral J_3 may also be obtained[4]. But the integral J_1 needs more attention and a new variable $z=t+j(\phi o-\psi)$ may be introduced such that

$$J_1=\frac{ja}{\pi}\int_{-\infty+j(\phi_0-\psi)}^{\infty+j(\phi_0-\psi)}\exp\{-jk\rho\cosh[z-j(\phi_0-\psi)]+2az\}\ln z\,dz. \tag{26}$$

For the asymptotically large parameter $k\rho$, the uniform asymptotic approximation may be obtained from the following relationship [9].

$$\frac{-j\pi J_1}{a}=\lim_{r\to 0}\frac{\partial}{\partial r}\int_{-\infty+j(\phi_0-\psi)}^{\infty+j(\phi_0-\psi)}\exp\{-jk\rho\cosh[z-j(\phi_0-\psi)]+2az\}z^r\,dz \tag{27}$$

where the integral in (27) may be expressed in terms of the parabolic cylinder function D_r. Hence the uniform approximation for H_z^S may be obtained by adding the first term in (23) to I_1 as

$$H_z^S\cong(2/\pi k\rho)^{1/2}\exp\{-j(k\rho-\pi/4)\}\{D_s(\phi,\phi_0)-a\exp(j2a\psi')[E_1(j2a\psi')+\ln(j2a\psi')]\}+\frac{j\exp(-jk\rho\cos\psi')}{2\pi}\left\{W_{-1}^{(2)}(0)+\frac{1-G(\psi)}{r}W_0^{(2)}(0)\right.$$
$$+\frac{ja\exp(-jk\rho\cos\psi')}{\pi}\left[\ln\left(\frac{\gamma}{(k\rho)^{1/2}\sin\psi'}\right)\frac{G(\psi')}{(k\rho)^{1/2}}W_0^{(2)}(Q)+\frac{G(\psi')}{(k\rho)^{1/2}}\frac{\partial W_r^{(2)}(0)}{\partial r}\bigg|_{r=0}+\right.$$
$$\left\{G(\psi')\ln\left(\frac{\gamma}{k\rho\sin\psi'}\right)-S(\psi')\ln(-j\psi'/(Q)\right\}\frac{W_1^{(2)}(Q)}{\gamma k\rho}+\frac{G(\psi)-S(\psi')}{k\rho\gamma}\frac{\partial W_{r+1}^{(2)}(Q)}{\partial r}\bigg|_{r=0}\Bigg] \tag{28}$$

where

$$\psi'=\phi_0-\psi \tag{29}$$

$$\gamma=\pm\exp(-j\pi/4)\{2(1-\cos\psi')\}^{1/2},\ \psi'\gtrless 0 \tag{30}$$

$$Q=\gamma(k\rho)^{1/2} \tag{31}$$

$$G_P(\psi')=\pm\exp(j\pi/4)\gamma/\psi',\ \psi'\gtrless 0 \tag{32}$$

$$G(\psi')=\gamma/\sin\psi' \tag{33}$$

$$S(\psi')=\pm\exp(-j\pi/4+j2a\psi'),\ \psi'\gtrless 0 \tag{34}$$

$$W_r^{(2)}(z)=(2\pi)^{1/2}\exp(j\pi r/2+z^2/4)D_r(jz) \tag{35}$$

One may show that the uniform asymptotic result in (28) is reduced into the asymptotic result in (23) as $|\phi_0-\psi|$ increases.

IV. Ray Optical Interpretation

The first two terms in (23) represent the edge diffracted field whose diffraction coefficient is D_S plus $\exp(-j2a(\phi o-\psi))+[-j2\phi_0-\psi)]^{-1}$ in (22). Calculated values of the edge diffraction are shown in Fig. 3 for the backscattering for different values of the inverse proportionality constant of inversely varing resistivity, $\alpha'=a/2\pi$. Since the results are symmetrical about the resistive half plane, only the range $0<\phi<180$ degrees is included. Our results are in agreement with physical reasoning. As the inverse proportionality of resistivity α' decreases, the edge diffraction coefficient is shown to approach that of a conducting half plane.

The third term in (23) from the branch cut and pole contributions may be interpreted as the geometric optics term. It repersents the reflected wave in the reflection region, while the transmitted wave in the shadow region may be obtained by adding the incident wave to this term. The reflection and transmission coefficients Γ and T may be obtained as

$$\Gamma=\frac{k\rho\sin(\phi+\phi)}{2a+k\rho\sin(\phi+\phi_0)} \tag{36a}$$

$$T=\frac{2a}{2a-k\rho\sin(\phi+\phi_0)} \tag{36b}$$

The coefficients in (36) are constant along the rays, respectively, and are the same as those of

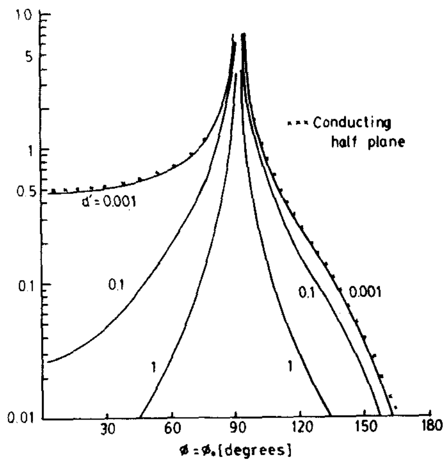


Fig.3. Magnitude of the edge diffraction coefficient for the backscattering ($\phi = \phi_0$).

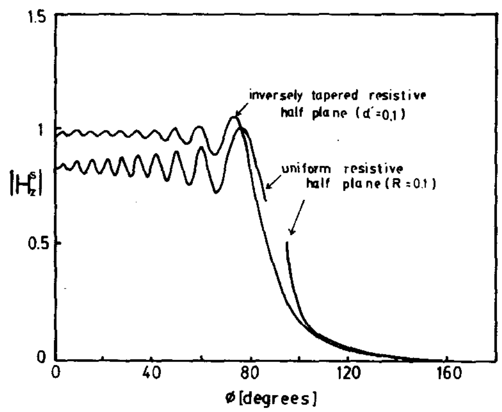


Fig.4. Magnitude of the scattered field H_z^s ($\phi_0 = 90°, \rho=10\ \lambda$).

the uniform resistive half plane if the local resistivity, where the ray originates, is equal to that of the uniform resistive plane.

Through the uniform asymptotic expression in (28), typical amplitudes of the scattered fields may be calculated as in Fig. 4 for the incident angle 90 degrees, $\alpha'=0.1$ and $\rho=10$ wavelengths. And non-uniform results for a uniform resistive half plane (R=0.1) are also shown[10]. As for the uniform resistive half plane, the scattered fields consist of the reflected (or transmitted) and edge diffracted fields. The amplitude of the interference fringes is smaller, which means that the diffracted field is also smaller, than that for the uniform resistive one. The peak of the interference fringe occurring last near $\phi=75°$ for the inversely tapered resistive half plane is shifted toward 90° in the uniform resistive half plane. For the angle larger than the reflection region (90° in Fig.4) only the edge diffraction contributes, and its amplitude decreases monotonically to zero as in Fig. 3.

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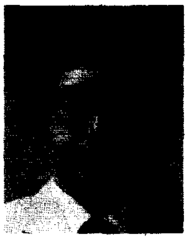

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