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Basic Study on the Reliability Analysis of Structural Systems

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시스템 신뢰성 해석에 과한 기초연구

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Key Words : Failure Mode(파괴모드), Incremental Load Method(하중증분법), Probability of Failure(파괴확률), Reliability Index(신뢰성 지수), Safety Margin(안전여유), Strength Modelling Parameter(강도공식 계수), System Reliability(시스템 신뢰성)

초 록

본 논문의 주목적은 불연속 또는 연속계 구조물의 시스템 신뢰성해석(system reliability analysis)을 위한 보다 일반적인 방법을 소개하는 데 있다. 본 논문에서는, 확대하중증분법(extended incremental load method)이라고 불리우는데, 지금까지의 신뢰성 해석법 중 종래의 하중증분법이 갖는 단점을 보완하고, 여러 형태의 하중이 작용하는 구조물에 대해, 부재의 파괴후 거동(post-ultimate behaviour)을 다른 방법보다 더 실제적으로 고려할 수 있는 장점을 갖도록 개발한 것이다. 본 방법의 또 하나의 장점은 구조설계시 사용하는 강도공식(strength formula)을 시스템 신뢰성 해석에서 직접 이용할 수 있다는 점이다. 이 방법은 부유식 해양구조물 같은 연속계 구조물의 시스템 신뢰성 해석을 위해 개발되었는데, 이 논문에서는 실제 구조물은 다루지 않고, 방법의 정당성과 아울러 수정된 안전여유식의 적용가능성을 보여주는 것에 중점을 두었다. 본 논문의 부유식 해양구조물들에 적용한 결과는 후일 발표할 예정이다.

Nomenclature

B_{ml} : loading coefficient of the lth loading for the mth failure mode

C_{mk} : resistance coefficient of component k (or r_k) for the mth failure mode

G : interaction equation under the combined loading condition (or failure equation)

j : number of failed components

L : number of loading types

P_{f_m} : probability of failure of the mth failure mode

Q_k : load effect of component k (or r_k)

R_k : resistance of component k (or r_k)

V_{X_M} : strength modelling uncertainty

X_M : stregnth modelling parameter

$\bar{XM}$: mean bias of strength modelling parameter

Z_m, Z_m' : safety margin for the mth failure

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mode and its modified form

β_m : reliability index of the mth failure mode

λ_j : load factor up to the jth failure stage

λ_T : total load factor at the failure stage

$\{P^{(1)}\}$: vector of load increment for the 1th loading

$\{R_e\}$: vector of component resistance

$[a^{(1)}]$: utilisation matrix for the 1th loading

$[A]$: total utilization matrix

under bar : mean of random variable

1. Introduction

1.1 General

Structural reliability theory is concerned with the rational treatment of uncertainties in structural engineering and with the methods for assessing the safety and serviceability of structures. Sutrctural reliability theory has grown rapidly during the last two decades. It has become a design decision tool in some cases with the object of achieving a more uniform and consistent reliability within the structural system.

For many years the reliability theory has been applied to various structures[1)~5)]. Most applications are, however, based on component reliability analysis. It has been recognised for many years that a more complete estimate of the reliability of a structure must include a structural system reliability analysis. During the last decade this need has been emphasised and many studies have been performed. Some are concerned with sensitivity studies[6)] and some with optimal structural design[7)]. Most of them, however, deal with discrete structures such as jacket platforms for which the possible failure modes (or paths) can be identified relatively easily. In the case of continuous structures, such as Tension Leg Platforms (TLPs) and semisubmersibles, it will not generally be so simple and easy as in the case of discrete structures to identify the possible failure modes and to define the re-distribution of load effects following the failure of a component.

An important task in structural system reliability analysis is to identify the failure modes. It will not usually be practical to include all possible failure modes in a complex structure. In such cases only those modes which are expected to contribute significantly to the system failure must be taken into account in estimating the system reliability. These are often referred to as the most likely[8)], or the most important or the most significant failure modes[9),10)], or the stochastically dominant failure modes[11)]. Hereafter these will be referred to as 'the most important failure modes'. And a major difficulty is to find an efficient algorithm to identify the most important failure modes in a complex structure.

1.2 Review of System Reliability Analysis Methods

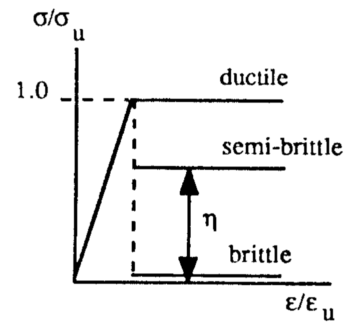
The system reliability analysis method generally can be divided into three categories which are : analytical methods, approximate methods and hybrid methods, which may combine analytical and approximate methods or combine different analytical methods. Theoretically analytical methods may give the exact probability of system failure, but they can only be applied to quite idealized problems and so provide no practical results for most real structures.

After Moses and Stahl[9)] suggested the incremental load method several approximate methods based on this have been developed[10),11)]. approximate methods can be categorized into three : failure path approach, survival set approach[12)] and plascticity-based approach[13)], among which the failure path approach is most popular. In this approach, since the less important paths are neglected, the result may lie on the unconservative side. In the survival set approach, survival paths are generated. Since the number of these is much larger than that of the failure paths, the failure path approach is more efficient from the computational point of view.

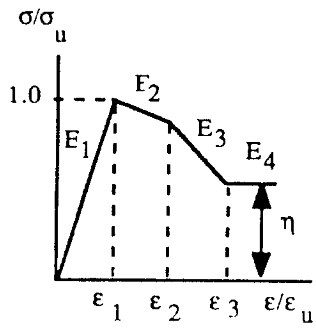
Additionally, this approach can be applied to the structure with ductile, brittle and semibrittle component behaviour, and is therefore more general than the plasticity-based approach. Because of this, most research on system reliability analysis has used the failure path approach. Table 1 shows the comparison of two typical failure path approaches : the incremental load method (ILM) and the element replacement method (ERM).

Lin and Corotis[14] suggested a possible hybrid analytical approxiamte method to take into account the non-linearity in the system analysis in which the incremental load method was used to obtain the system resistance, and non-linear structural analysis was included. This method seems to be a possible way of more realistically taking into account the effect of non-linearities. However, considering the fact that in system reliability analysis the structural analysis takes the main portion of computational time, even simple structures may require excessive computational time. For this this method would seem to be unsuitable for complex and large structures.

A major aim of the present study is to develop the system reliability algorithm called the "Extended Incremental Load Method" based on an extension of the conventional incremental load method[10] to include multiple loadings and to more



(a) Two-state model



(b) Multi-state model

Fig. 1 Typical piecewise model for non-linear behaviour

Table 1 Comparison between incremental load method and element replacement method

	ILM	ERM
Behaviour of the failed components	Deformation of failed components can be allowed to follow the post-ultimate behaviour.	Deformation of the failed components are restricted. Failed components are removed and their strength are replaced by the equivalent force acting on the structure.
Merit	Post-ultimate behaviour of failed components can be treated more realistically than the element replacement method.	Applicable to multiple loading case
Limitation	Applicable only to a single loading case	Post-ultimate behaviour of a faild component is idealised as a two-state model : ductile, semi-brittle and brittle.

realistically take into account the post ultimate behaviour of a failed component. The method has been developed for the system reliability analysis of continuous structures, such as TLPs and semi-submersibles. The present system reliability method is detailed together with the modified safety margin equation proposed herein, with which the strength models of principle components in a structure can be directly used and the strength modelling parameter can be incorporated in system analysis.

The method has been applied to discrete structures to show its validity. Its application to continuous structures will be presented in the near future.

2. Present Method : Extended Incremental Load Method

As seen in Table 1, one of the major limitations of the incremental load method is that its application is restricted to a single load pattern. Moses and Rashed[15] presented the results of multiple loading case for the ducitle system by incrementing one load and keeping the rest fixed at their final values. Their formulation, however, is neither clear nor consistent because all loads should probably be incremented proportionally until failure occurs.

Since in the incremental load method the load factor up to the particular failure stage can be obtained, it can be used to predicted the deformation of a failed component, in a general sense, based on the mean value of applied load. This method may have potential to account for the post-ultimate strength more realistically than other methods. In addition, the method can easily applied to evaluating and assessing the system reliability with pre-defined failure modes for sensitivity studies.

In the method the utilization ratio generally represents the relationship between component strength and load increment, and may be expressed as a stress in some cases or even a more complex expression, such as interaction formula under the combined loading. The existing strength formulae developed for the principle components in a structure, therefore, can be directly used for this purpose.

2.1 Derivation of the Safety Margin Equation

A general expression of the linear safety margin, Z_m for the mth failure mode is expressed as Eq. (1)[16] :

$$Z_m = \sum_{k=1}^{j} C_{mk} R_k - \sum_{l=1}^{L} B_{ml} P_l \quad \cdots\cdots\cdots \quad (1)$$

where R_k is the resistance of component k (or r_k), P_l, is the lth loading acting on a structure, C_{mk} is the resistance coefficient for R_k, B_{ml} is the loading coefficient for P_l, j and L refer to the number of failed components and the number of loading cases, respectively. The first part of Eq. (1) will be called the resistance term and the second part the loading term. When the failure of the jth component leads to the collapse of the structure, then $C_{mj} = 1.0$.

The procedure for deriving the safety margin equation for the multiple loading case is similar to that for the single loading case, except that the contribution factor defined below for each loading is included. Let L loadings act on a structure in which j components, r_1, r_2, $\cdots$, r_j have failed. The utilization equation for each loading may be expressed as Eq. (2) similar to the single loading case [9,10]. For the lth loading :

$$\begin{Bmatrix} R_1 \\ R_2 \\ \cdot \\ \cdot \\ R_j \end{Bmatrix} = \begin{bmatrix} a_{11}^{(l)} & & & & \\ a_{21}^{(l)} & a_{22}^{(l)} & & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & & \cdot & \\ a_{j1}^{(l)} & a_{j2}^{(l)} & \cdot & \cdot & a_{jj}^{(l)} \end{bmatrix} \begin{Bmatrix} P_1^{(l)} \\ P_2^{(l)} \\ \cdot \\ \cdot \\ P_j^{(l)} \end{Bmatrix} \quad \cdots\cdots \quad (2)$$

where the element $a_{ki}^{(l)}$ is called the utilization ration and is defined as the proportion of strength of component r_k utilized in the ith load increment, which represents the relationship between component strength and load increment, R_k is the strength of the failed component, r_k and $P_k^{(1)}$ is the load increment to fail the component r_k. In matrix form :

$$\{R_e\} = [a^{(1)}] \{P^{(1)}\} \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

where matrix $[a^{(1)}]$ is the utilization matrix and is a triangle matrix of which all elements in the upper triangle are zero, and $\{R_e\}$ is the resistance vector whose elements are the strength of failed components and $\{P^{(1)}\}$ is the load increment vector with elements for each of the j components. In this equation the superscript (1) represents the terms related to the 1th loading. The load increment vector is obtained by solving Eq. (3).

$$\{P^{(1)}\} = [a^{(1)}]^{-1} \{R_e\} \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

System resistance is expressed as the sum of each load increments :

$$R_{sys} = P_1 + P_2 + \cdots + P_j = \sum_{k=1}^{j} C_k^{(1)'} \cdots\cdots\cdots\cdots\cdots\cdots (5)$$

where $C_k^{(1)'}$ is the sum of the kth column of $[a^{(1)}]^{-1}$.

$$C_k^{(1)'} = \sum_{i=k}^{j} a_{ik}^{(1)-1} \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

Normalizing the coefficient by the final one gives the resistance coefficients for the 1th load. The resistance coefficient $C_k^{(1)}$ corresponding to resistance R_k for the 1th loading is :

$$C_k^{(1)} = \frac{C_k^{(1)'}}{C_j^{(1)'}}, \quad k = 1, 2, \cdots, j\text{-}1 \cdots\cdots\cdots (7)$$

and

$$C_j^{(1)} = 1.0 \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

When the contribution of resistance for each loading to the system resistance is represented by the contribution factor, $CF^{(1)}$, which is defined here as the relative proportion of utilization of the 1th loading. The resultant resistance coefficients for all loadings are obtained by summing up the resistance coefficients for each loading multiplied by their corresponding contribution factors, i.e. the resultant resistance term is expressed as a sum of contributions of resistance for each loading to the system resistance :

$$C_k = \sum_{1=1}^{L} C_k^{(1)} . CF^{(1)} \quad k = 1, 2, \cdots, j\text{-}1 \cdots (9)$$

Hence, the resistance term in the safety margin equation can be expressed as :

$$R_{sys} = C_1 R_1 + C_2 R_2 + \cdots + C_j R_j \quad \cdots \quad (10)$$

where C_j is unity. The loading term can be easily obtained as the sum of the product of the utilisation ratio $a_{jj}^{(1)}$ and load $p^{(1)}$.

$$Q = \{a_{jj}^{(1)} p^{(1)} + a_{jj}^{(2)} p^{(2)} + \cdots + a_{jj}^{(1)} p^{(1)}\} \cdots (11)$$

where Q simply denotes the loading term. With Eqs. (10) and (11), the safety margin equation for the mth failure mode becomes :

$$Z_m = R_{sys} - Q = \sum_{k=1}^{j} C_{mk} R_k - \sum_{1=1}^{L} B_{ml} P^{(1)} \cdots\cdots\cdots\cdots\cdots\cdots (12)$$

where C_{mk} and B_{ml} are resistance and loading coefficients for the mth failure mode, respectively and $B_{ml} = a_{jj}^{(1)}$. This equation is the same as Eq. (1), except that the superscript '(1)' appears in the loading term.

Since summing up all elements of the inverse matrix of the utilization matrix results in the load factor up to the particular failure stage, using this concept the total load factor for a multiple loading case, when structural collapse occurs, can be obtained. In Eq. (2) each element of the utilization matrix is the utilized proportion of a component strength in a particular incremental stage due to unit load, $p^{(1)} = 1.0$, and if the mean value of the load is substituted, the element of the utilization matrix represents the mean utilization for that loading. When L loadings are applied the total mean utilization is simply the sum of each utilization. Let $\lambda_1, \lambda_2, \cdots, \lambda_j$ be mean load factors, then the utilization equation for mean load may be expressed as :

$$\begin{Bmatrix} R_1 \\ R_2 \\ \cdot \\ \cdot \\ R_j \end{Bmatrix} = \begin{bmatrix} A_{11} & & & & \\ A_{21} & A_{22} & & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & & \cdot & \\ A_{j1} & A_{j2} & \cdot & \cdot & A_{jj} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \lambda_j \end{Bmatrix} \quad \cdots\cdots (13)$$

or simply

$$\{R_e\} = [A]\ \{\lambda\} \quad \cdots\cdots (14)$$

where A_{ki} is the total mean utilization obtained from Eq. (15)

$$A_{ki} = \sum_{l=1}^{L} a_{ki}^{(l)} \underline{P}^{(l)} \quad \cdots\cdots (15)$$

and $\underline{P}^{(l)}$ is the mean value of load $P^{(l)}$. By inverting Eq. (14), the mean load factors for load increments can be obtained :

$$\{\lambda\} = [A]^{-1}\ \{R_e\} \quad \cdots\cdots (16)$$

Hence, when failure of j components leads to structural collapse, the total load factor, λ_T is given by :

$$\lambda_T = \sum_{i=1}^{j} \lambda_i \quad \cdots\cdots (17)$$

2.2 Modified Safety Margin Equation

As described in the previous section the utilization ratio represents the utilized proportion of a particular components strength, and this can be obtained by using the appropriate strength formulae of the component. For a simple case when there are only two random variables, Q and R for component k (or r_k), say strength R_k and Q_k as a resistance and load effect variables, respectively, the safety margin or limit state equation is given by :

$$Z_k(X_M, R, Q) = X_{M_k} R_k - Q_k \quad \cdots\cdots (18.a)$$

where X_{M_k} is the modelling parameter (or modelling error) for the strength formula of component k and defined as :

$$X_M = \frac{\text{actual behaviour}}{\text{predicted behaviour}} \quad \cdots\cdots (19)$$

which represents the mainly subjective uncertainty of the strength model in the reliability analysis[17]. The mean of X_M is referred to as the bias and when there is sufficient data the random component of X_M is usually referred to as the modelling uncertainty specified by its coefficient of variation V_{X_M}. Dividing both sides of Eq. (18.a) by the component strength gives the same safety margin without loss of any physical meaning :

$$Z_k'(X_M, R, Q) \times X_{M_k} - \frac{Q_k}{R_k} \quad \cdots\cdots (18.b)$$

in which Q_k/R_k implies the utilized proportion of component k as a loading variable in the safety margin. When using the Rackwitz-Fiessler algorithm Eqs. (18.a) and (18.b) give the same reliability indices because they both have the same physical meaning.

In the case of multiple load effects the safety margin (failure surface equation) for component k can be generally and conceptually expressed in the non-dimensional form as :

$$Z_k(\{R\}) = 1 - G(\{R\}_k, \{Q\}_l) \quad \cdots\cdots (20.a)$$

where $\{R\}_k$ denotes the resistance variable vector associated with the strength for component k, such as geometric and material properties, and $\{Q\}_l$ the loading variable vector associated with the load effects, such as axial stress, etc. As before when the strength modelling parameter for component k, X_{M_k} is introduced as another random variable Eq. (20.a) can be rewritten as :

$$Z_k(X_M, \{R\}, \{Q\}) = X_{M_k} - G(\{R\}_k, \{Q\}_l) \quad \cdots\cdots (20.b)$$

The mean bias of X_{M_k} implies that the failure surface is to be shifted from the surface given by Eq. (20.a), and its COV represents the scatter or the perturbation of the failure surface around the shifted one (see Fig. 2). Guenard et al[18] also proposed the same idea with the mean of X_M being hopefully close to unity, representing the bias of the strength model and its probability density function representing the modelling uncertainty.

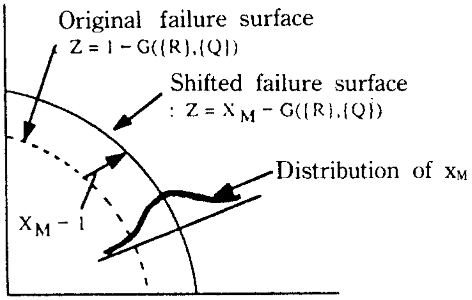


Fig. 2 Two-dimensional failure surface

In order to use the strength formulae for the present purpose the safety margin given by Eq. (12) or (1) can be modified as follows to consider the effect of strength modelling parameter in system reliability analysis. Separating the resistance term of component r_j in Eq. (12), which is the last failed component, and considering that jth resistance coefficient is unity :

$$Z_m = R_j + \sum_{k=1}^{j-1} C_{mk} R_k - \sum_{l=1}^{L} B_{ml} P^{(l)} = R_j - Q_j \quad \cdots\cdots (21)$$

where Q_j is the net load effect on component r_j due to the already failed components, $r_1, \cdots, r_{j-1}$ and due to the loading acting on a structure, namely :

$$Q_j = \sum_{l=1}^{L} B_{ml} P^{(l)} - \sum_{k=1}^{j-1} C_{mk} R_k \quad \cdots\cdots (22)$$

when introducing the strength modelling parameter of component r_j, X_{M_j} Eq. (21) becomes :

$$Z_m = X_{M_j} R_j - Q_j$$

Dividing both sides of the above equation by R_j and re-substituting Eq. (22) will give the safety margin which has the same physical meaning as Eq. (12) :

$$Z'_m = X_{M_j} - \frac{Q_j}{R_j} = X_{M_j} + \sum_{k=1}^{j-1} C_{mk} \frac{R_k}{R_j} + \sum_{l=1}^{L} B_{ml} \frac{P^{(l)}}{R_j} \quad \cdots\cdots (23)$$

Let $\{R\}_k$ and $\{Q\}_l$ be the resistance variable and loading variable vectors as before. Then the first summation term in Eq. (23) can be regarded as a function of $\{R\}_k$, $k=1, \cdots, j-1$, and the second term as a function of $\{Q\}_l$. Hence, with these the safety margin given by Eq. (12) can be conceptually modified in the non-dimensional form. That is :

$$Z'_m = X_{M_j} + \sum_{k=1}^{j-1} G_k(\{R\}_k, \{R\}_j) - \sum_{l=1}^{L} G_l(\{Q\}_l, \{R\}_j) \quad \cdots\cdots (24)$$

where the first summation term is the contribution of the strength of the already failed components to the system safety margin and the second term is the contribution of loadings. Function G_k and G_l represent those associated with the strength of component r_k and with loading 1, respectively.

Expressing the safety margin as Eq. (24) is a feasible way to use the strength formulae for the principle components in system analysis of a structure under multiple load effects and to take into account uncertainties of design variables in strength and load without loss of any mean and variance can be easily obtained using the concept of the First-Order Second Moment Method (FOSM). Doing this effectively represents the contributions of design variables to the safety margin. Let X be a vector of design variables, i.e. $\{X\} = [X_M, \{R\}^T, \{Q\}^T]^T$. Then, the mean and variance of function G_l (or G_k) are obtained from :

$$\bar{G}_l \cong G_l(\{\bar{X}\}) \quad \cdots\cdots (25.a)$$

$$\sigma^2_{G_l} \cong \sum_i \left\{\frac{\partial G_l}{\partial X_i}\right\}^2 \sigma^2_{X_i} + \sum_i \sum_j \left\{\frac{\partial G_l}{\partial X_i}\right\} \left\{\frac{\partial G_l}{\partial X_j}\right\} \sigma_{X_i} \sigma_{X_j} \rho_{ij} \quad \cdots (25.b)$$

where $\{\bar{X}\}$ is the mean value vector of random variables, σ_{X_j} is the standard deviation and ρ_{ij} is the correlation coefficient between variables X_i and X_j. The term in brackets of Eq. (25.b) represent the partial derivatives of function $G_l(\{X\})$ evaluated at the mean values of random variables.

2.3 Idenfication of the Important Failure Modes

Identification of the failure modes is one of the most important part in the failure path approach. In a complex structure the number of potential failure modes is usually very large, but as it is well recognized, only a few of them are important and dominant in evaluating the probability of system failure. At present there are several procedures to indetify the most important failure modes [19].

The present procedure of identifying the most important failure modes is aimed at reducing the computational time. The procedure consists of two parts : a Searching Procedure to discard the deterministically less important modes following a similar criteria as Moses[10]. The details were introduced by Lee and Faulkner[20] where a procedure is presented by which failure modes are obtained in decreasing order of failure probability or in increasing order of corresponding reliability indices, i.e. :

$$P_{f_1} > P_{f_2} > \cdots > P_{f_m} > \cdots \quad \cdots\cdots\cdots\cdots \quad (26.a)$$

$$\beta_1 < \beta_2 < \cdots < \beta_m < \cdots \quad \cdots\cdots\cdots\cdots \quad (26.b)$$

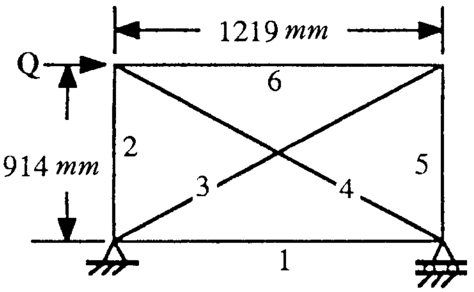
where P_f and β_m are the failure probability and the corresponding reliability index of the mth failure mode. In the procedure, the collapse of a structure is defined as the occurance of the singularity in the structural stiffness matrix, i.e. :

$$\text{Ket}[K] = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (27)$$

where [K] is the structural stiffness matrix. In evaluating the bounds of a failure mode (bounds of a parallel system) and the bounds of the system failure probability (bounds of a series system), Ditlevsen's bounds, called narrow bounds, are calculated[21].

3. Application to Discrete Structures

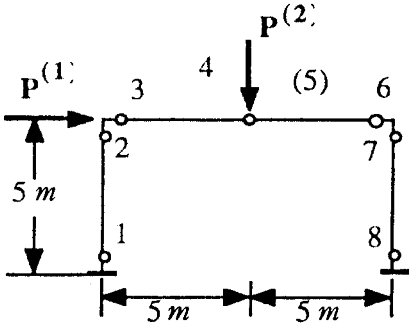
Two simple structures in Figs. 3 and 4 are selected to show the validity of the present method in deriving the safety margin of a structure under multiple loading and in identifying the most important failure modes to evaluate the structural system reliability.



Comp.	R_k	A_k
1, 2	36.708	133.0
3, 4	41.124	149.0
5, 6	36.708	133.0

A_k = Cross sectional area(mm^2)
R_k = Mean value of reference strength(= yielding axial force, kN)
E = 206000 MPa COV of R_k = 0.1 COV of Q = 0.2

Fig. 3 Truss model



Comp.	A_k	I_k	R_k
1, 2	4.0	3.58	0.075
3, 4	4.0	4.77	0.101
5, 6	4.0	4.77	0.101
7, 8	4.0	3.58	0.075

A_k = Cross sectional area(x $10^{-3}\,m^2$) I_k = Moment of inertia (x $10^{-5}\,m^4$)

Mean of yield stress = 276 *MPa* $P^{(1)} = 0.02\,MN$, $P^{(2)} = 0.04\,MN$
COV of R_k = 0.05 COV of $P^{(1)}$ and $P^{(2)}$ = 0.3

Fig. 4 Frame model

3.1 Plane Truss Model

The plane truss model[22] in Fig. 3, which is a sub-structure found in jacket platform, has six members as components and component failure is assumed when the axial stress in a member reaches the yield stress. The same strength for compression and tension is assumed. Table 2. shows all potential failure modes of the truss example. As can be seen in the table, failure modes are found in decreasing order of failure probability, and even this simple structure has lots of possible failure modes. Among them, when only the first four modes are taken, neglecting the remaining 13 modes overestimates the lower bound of the system reliability index by only 3.5%. This means that in practice only a few of important failure modes are likely needed to estimate the system reliability.

3.2 Plane Frame Model

The plane frame model in Fig. 4 has eight possible hinges as components. This model is espe-

Table 2 All potential failure modes of plane truss model

No	Failed comp.	β_{path}	$\beta_{s,u}$	$\beta_{s,l}$	no	Failed comp.	β_{path}	$\beta_{s,u}$	$\beta_{s,l}$
1	3, 4	2.25	2.25	2.25	10	4, 5	3.51	2.16	1.90
2	4, 3	2.25	2.16	2.06	11	6, 2	3.83	2.16	1.90
3	3, 1	2.52	2.16	2.01	12	1, 5	3.83	2.16	1.90
4	4, 6	2.52	2.16	1.97	13	2, 3	4.11	2.16	1.90
5	1, 3	2.71	2.16	1.94	14	5, 4	4.11	2.16	1.90
6	6, 4	2.71	2.16	1.92	15	5, 1	4.21	2.16	1.90
7	1, 6	2.93	2.16	1.91	16	2, 6	4.21	2.16	1.90
8	6, 1	2.93	2.16	1.90	17	5, 2	4.84	2.16	1.90
9	3, 2	3.51	2.16	1.90					

Note : $\beta_{s,u}$ = upper bound of β_{sys} $\beta_{s,l}$ = lower bound of β_{sys}

cially sensitive and has been frequently selected for evaluation of system reliability and sensitivity analysis[6), 11)]. Component failure is assumed when the bending moment at any particular hinge reaches the plastic bending moment. For this model the strength formula can be expressed simple as (see also Eqs. (18.b) and (20.b)) :

$$Z'(X_M, \{R\}, \{Q\}) = X_M - \frac{Q}{R} \qquad (28)$$

where Q is bending moment due to loading as the load effect and R the plastic bending moment as the strength. Here, modelling parameter, X_M is assumed to be deterministic having a mean of 1.0 and all resistance and loading terms in Eq. (24) are assumed to be normal.

Fig. 5 shows the failure states of important failure modes, and eight modes have been identified. Actually there are lots of possible failure modes. But as seen in the figure, the reliability index of the 8th mode is 3.29, and so the remaining neglected failure modes are expected to have greater reliability indices than this value and are likely to have small influence on the evaluated system reliability. In this paper when a failure mode has the same correlation as another mode already identified this new mode is not presented. For example the modes for path 7-4-8-2, 7-4-2, 7-8-4-2 and 4-7-2 have the close correlations as the first mode for path 4-7-8-2 and the safety margin of these five modes are same as given by Eq. (29) which is derived according to the proposed approach and expressed in the form of the safety margin equation (24) in this paper :

$$Z'_{4\text{-}7\text{-}8\text{-}2} = X_{M_2} + 1.0\,R'_7 + 2.693\,R'_4 - 0.795 \times 10^{-6}\,R'_8 - 0.16 \times 10^{-5}\,P^{(1)'} - 2.667\,P^{(2)'} \qquad (29)$$

where X_{M_2} is strength modelling parameter of component 2 with mean of unity and COV of zero as assumed. R'_7 and R'_4 effectively represent the strength of component 7 and 4 with mean of unity and COV of 0.071. $P^{(1)'}$ and $P^{(2)'}$ effectively represent the loading $P^{(1)}$ and $P^{(2)}$ with mean of unity and COV of 0.304. For this failure mode the horizontal load $P^{(1)}$ and component 8 theoretically do not move or participate in the failure when the associated mechanism forms. But due to computational truncation error very small coefficient do appear in Eq. (29). However their effects on the safety margin and the evaluated safety level can be seen to be negligible. From Eq. (29) component 8 does not contribute to the safety margin, i.e., without it a collapse mechanism can be formed which is path 4-7-2. This is one of the failure modes having the same correlation as the mode for path 4-7-8-2. This component 8 at col-

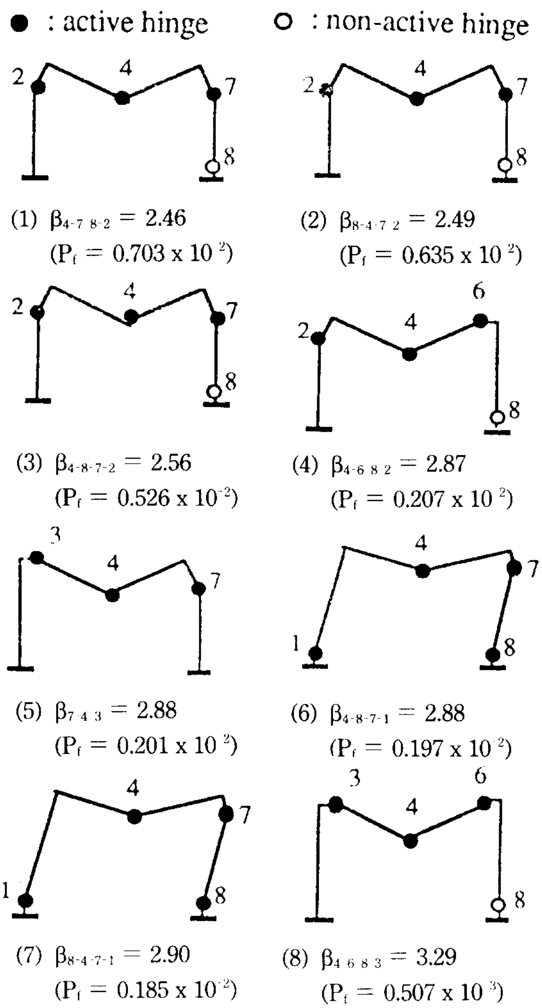


Fig. 5 Failure states of important failure modes for plane frame model

lapse is referred to as non-active hinge, but before collapse it was being considered in the particular failure path being examined.

As described in Section 2.1, when using the incremental load method the load factor λ_i, can be obtained, from which the mean load can be predicted up to the particular failure stage i. This can easily be extended to estimate the collapse load (in the mean sense) from Eq. (30):

$$[\text{Collapse Load}] = \lambda_T * [\text{Mean Applied Load}] \quad \cdots\cdots (30)$$

where the total load factor, λ_T can be obtained from Eq. (17) and is conceptually the ratio of collapse load to the mean applied load, and so it is related to the reserve strength index (RSI)[23]. λ_T of the modes in Fig. 5 are listed in Table 3. From the table we can see that the deterministically important failure mode is not identical with the probabilistically important one, that is, the mode having the smallest λ_T does not give the lowest β path.

Table 4 shows comparison of the results when using Eqs. (12) and (24) as safety margin. The proposed Equation (24) usually gives a lower reliability index than Eq. (12), but the difference is less than 1%, and can be neglected.

These results from two simple structural models justify the validity and the applicability of the present system reliability method to a structure under multiple loading to identify the most important failure modes and to evaluate the system reliability. From the results of the present study, in general, using Eq. (24) as a safety margin equation may give a different path reliability index of (or different path failure probability) and consequently, give a different system reliability index. But it can be said that using Eq. (24) is one possible way of using the strength formulae directly in system reliability analysis.

Table 3 Total load factors of the identified failure modes for frame model

Mode	β_{path}	λ_T	Mode	β_{path}	λ_T
4-7-8-2	2.45	1.76	7-4-3	2.88	1.89
8-4-7-2	2.49	1.76	4-8-7-1	2.88	1.67
4-8-7-2	2.56	1.76	8-4-7-1	2.90	1.67
4-6-8-2	2.87	1.89	4-6-8-3	3.29	2.02

Table 4 Comparison between two safety margin equations for the frame model

Failure Paths	7-4-2	7-4-8-2	7-4-3	7-4-8-3	7-4-8-1
Eq. (12)	2.48	2.48	2.88	2.88	2.91
Eq. (24)	2.46	2.46	2.88	2.88	2.90

4. Conclusions

In this study an "Extended Incremental Load Method" is introduced as another approximate method for system reliability analysis. It is an extension of the conventional incremental load method to allow for multiple loads. A modified form of the safety margin equation is developed to directly use the strength formulae in which mean values and COVs of resistance and loading coefficients are obtained using the FOSM.

The results for simple structure models justify the validity and the applicability of the present method to a structure under multiple loadings and to identifying the most important failure modes from which to evaluate the structural system reliability. Using the proposed modified safety margin expression makes it possible to directly use the existing strength models. From the results of the frame model, it is found that the deterministically most important failure mode is not identical with the probabilistically most important one.

The present paper has not shown the results for continuous structures and for the structure in which the post-ultimate behaviour may take the form of multi-stage unloading pattern (as in Fig. 1-b). Nevertheless, the extended incremental load method, together with the modified from of safety margin shown in Section 2.2, can now allow for the evaluation of system reliability level of continuous structures, of which component behaviour is ductile or not. The results of this extension will be presented later.

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★ News ★

EUROMS-90

EUROPEAN OFFSHORE MECHANICS SYMPOSIUM-1990

at NTH, Trondheim, Norway, 20~21 August 1990

Dear Colleagues :

On behalf of the organizing committee and the International Society of Offshore and Polar Engineers, (ISOPE), we would like to invite you to submit a paper abstract for the possible presentation at and for inclusion in the proceedings of a two-day seminar on selected offshore and polar topics which will be organized at the Norwegian Institute of Technology in Trondheim.

This international symposium is hoped to become an excellent forum for the European and Atlantic cooperation in the offshore and marine technology, and related emerging technologies. With this in mind we have formed the organizing committee, hopefully laying the ground work for the cooperation and initially representing the UK, the Netherlands, Norway, Canada and the USA. It is also intended to establish a link to the colleagues from other international regions.

The purposes of the symposium are to :

- premote technological progress and activities, the international technological transfer and cooperation, and opportunities for engineers to maintain and improve technical competence.
- to provide a timely international forum for the technical activities, cooperation, opportunity and fellowship among researchers and engineers.

Some baselines to maintain are :

- focused session topics with acceptance of high quality (in originality and significance) papers through rigorous review.
- establishment of high international reputation for publication and worldwide distribution.

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