EFFICIENT ANALYSIS OF PIPING SYSTEMS WITH JOINT DEFORMATION

접합부의 변형을 고려한 파이프 설비의 효율적인 해석

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요 약

파이프 설비는 다양한 두께 및 직경, 길이를 가진 파이프들로 이루어진 구조물로서 정확한 해석을 위해서는 3차원의 유한요소 모델이 필요하다. 그러나 유한요소 모델에 의한 해석 방법은 계산 시간이 길어 질뿐만 아니라 방대한 양의 컴퓨터 용량을 필요로 한다. 반면에, 파이프를 보요소로 가정한 단순한 해석 모델은 파이프 설비의 정확한 거동을 예측하기에 문제점이 많다.

따라서, 본 연구에서는 파이프 설비의 효율적인 해석 모델을 제안하고자 한다. 제안된 해석 모델은 보요소로 구성된 모델에 접합부 요소만을 추가함으로써 보요소만으로 구성된 모델만큼 단순하며, 또한 파이프 접합부의 국부적인 변형을 고려하기 때문에 파이프 설비의 거동을 정확히 예측할 수 있다. 예제 해석을 통하여 결과들을 비교함으로써 본 연구에서 제안한 모델의 효율성을 입증하였다.

Abstract

A piping system is a structure composed of pipes with various thickness, diameter and length. Accurate analysis of a piping system requires a complicated three dimensional finite element model and a computer system with large memory size, while a simplified model may result in syutem response predicition with deteriorated accuracy.

An efficient analysis model for piping systems is proposed in the present study. The proposed model is developed by introducing pipe joint elements which accounts for the behavior of a pipe joint. Pipes are represented by beam elements and the effect of local deformation of pipe joints is replaced by joint element deformations.

The proposed model which is as simple and efficient as a beam model can be used to obtain piping system response with accuracy close to that of a finite element model.

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1. INTRODUCTION

A piping system is generally considered to include the complete interconnection of pipes, including in-line components such as pipe fittings and flanges. The contributions of piping systems are essential in an industrialized society - they provide drinking water to cities, irrigation water to farms, and cooling water to buildings and nuclear power plants. In the design of piping systems and their supports, the factors that need to be taken into consideration depend on the type of plant in which the piping systems will be installed. For nuclear power piping systems, the minimization of environmental hazard becomes the premier factor. Moreover, nuclear power piping systems are very complicated system which has various types of connections and supports. Especially the stress concentration near a pipe joint results from the vibration which power plant operation, abrupt failure and earthquake induce[2]. Thus, a piping system demands the high level of structural safety during a limited lifetime. And accurate behavior of nuclear power piping systems must be estimated in both the static and dynamic analysis against an accident.

In the present analytic method of piping systems, there are one method which uses shell elements and the other method which uses beam elements. In the former case, even if the behavior of piping systems can be accurately estimated, it is inappropriate and uneconomical to analyse entire piping system because of difficult modelling and long computation time. In the latter case, saving in computation time is significant and analysis model is simple. But it neglects the flexibility effect such as ovalization of a pipe section which results from a discordance between the main pipe center line and the pipe joint. Thus, we can obtain merely approximate analysis result by this method.

In a pipe joint, the local deformation effect depends on the ratios of diameter and thickness of interconnected pipes. Especially the flexibility effect of a pipe joint becomes the main factor in the static and dynamic analysis when the ratios of diameter and thickness are large.

Thus, a method using beam elements in consideration of the flexibility effect of a pipe joint is more efficient than a finite element method using shell elements and more accurate than a simplified method using beam elements only. In the present study, an accurate and efficient analysis method is developed considering the local deformation effect of a pipe joint by introducing pipe joint elements.

2. DEVELOPMENT OF PIPE JOINT ELEMENT

A pipe joint shown in Fig.1 is an interconnection of a main pipe and a slender pipe. The slender pipe is connected at the perimeter of the main pipe. In general, a main pipe has relatively large diameter compared to thickness and the local deformation of pipe joint as shown in Fig.2. When the conventional beam model is used for analysis of a piping system, it is assumed pipes are interconnected at the center line of a main pipe and ignore the effect of local deformation of a pipe joint. In this study, pipe joint elements are proposed to take the pipe joint flexibility into consideration.

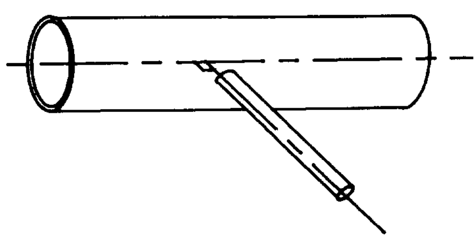


Figure 1. General shape of pipe joint

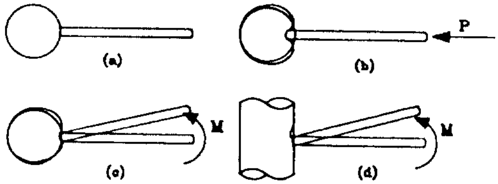


Figure 2. Local deformations of pipe joint

Pipe joint elements are composed of two sub-elements as shown in Fig.3. Element II is used to represent pipe joint deformations shown in Fig.2(b) and element I is employed to account for those shown in Figs.2(c) and 2(d). The length of element II is same as the radius of the main pipe and the length of element I is zero. The pipe joint shown in Fig.1 can be modelled as shown in Fig.4 using the proposed pipe joint elements.

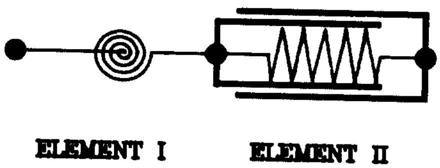


Figure 3. Proposed pipe joint elements

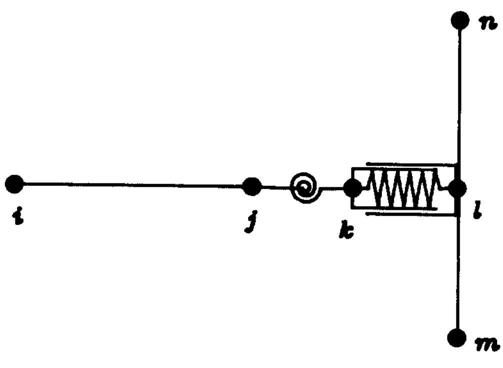


Figure 4. Pipe joint model using the proposed pipe joint elements

2.1 Formulation of Stiffness Matrix

The element $i\ell$ shown in Fig.4 consists of beam element ij, rotational deformation element jk and axial deformation element $k\ell$. In the first stage of stiffness matrix formulation, element ij and jk are combined to form element ik as shown in Fig.5. The rotational springs in two directions that have stiffnesses, K_{sy} and K_{sz}, respectively are installed at node j of the beam element. Thus, the member stiffness matrix of the element ik is expressed as follows :

$$[K_M]=\begin{pmatrix}[K_{ii}] & [K_{ik}] \\ [K_{ki}] & [K_{kk}]\end{pmatrix} \tag{1}$$

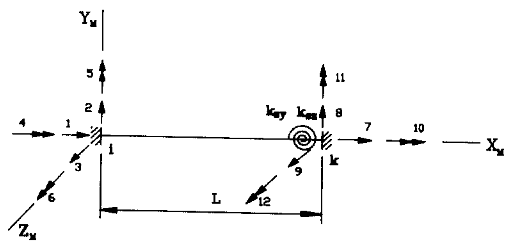


Figure 5. Member with elastic springs at one end

The submatrix $[K_{kk}]$ can be obtained by finding the inverse matrix of the flexibility matrix $[F_{kk}]$.

$$[K_{kk}]=[F_{kk}]^{-1} \tag{2}$$

Terms in the submatrix $[K_{kk}]$ are defined as the reactions at node k of the member due to unit displacements at the same node. Statically equivalent actions at node i may be computed using the transformation matrix[7].

$$[T_{ik}]=\begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -L & 0 & 1 & 0 \\ 0 & L & 0 & 0 & 0 & 1\end{pmatrix} \tag{3}$$

Then, terms in the submatrix $[K_{ik}]$ can be computed as the static equilibrants of those in $[K_{kk}]$, as follows :

$$[K_{ik}]=-[T_{ik}]\ [K_{kk}] \tag{4}$$

Because the member stiffness matrix $[K_M]$ is symmetric, the submatrix $[K_{ki}]$ can be obtained by transposing $[K_{ik}]$.

$$[K_{ki}]=[K_{ik}]^T=-[K_{kk}]\ [T_{ik}]^T \quad (5)$$

And the submatrix $[K_{ii}]$, similar to Eq.(4), can be found from $[K_{ki}]$. That is,

$$[K_{ii}]=-[T_{ik}]\ [K_{ki}]=[T_{ik}][K_{kk}]\ [T_{ik}]^T \quad (6)$$

As stated above, the member stiffness matrix which is considered to include rotational springs at one end of the beam element is entirely derived from the flexibility matrices.

$$[K_M]=\begin{pmatrix} [T_{ik}][K_{kk}][T_{ik}]^T & -[T_{ik}][K_{kk}] \\ -[K_{kk}][T_{ik}]^T & [K_{kk}] \end{pmatrix} \quad (7)$$

Considering the elastic spring of the element II in Fig.3, several terms related to axial deformation of Eq.(7) in the right side must be modified. That is easily derived from the motion of series of springs.

To be compatible with the stiffness matrix at center line of the main pipe, the equilibrium with respect to force at nodes i and k, is presented as

$$\{P_i\}=[K_{ii}]\ \{U_i\}+[K_{ik}]\ \{U_k\} \quad (8)$$

$$\{P_k\}=[K_{ki}]\ \{U_i\}+[K_{kk}]\ \{U_k\} \quad (9)$$

And, using the transformation matrix $[T_{k\ell}]$ with respect to nodes k and ℓ, the load-displacement relations between these nodes are given by

$$\{P_k\}=[T_{k\ell}]\ \{P_\ell\} \quad (10)$$

$$\{U_\ell\}=[T_{k\ell}]^T\ \{U_k\} \quad (11)$$

The transformation matrix $[T_{k\ell}]$ is formed as follows[7]:

$$[T_{k\ell}]=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -z_{k\ell} & y_{k\ell} & 1 & 0 & 0 \\ z_{k\ell} & 0 & -x_{k\ell} & 0 & 1 & 0 \\ -y_{k\ell} & x_{k\ell} & 0 & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

where $x_{k\ell}$ is a distance between two nodes k and ℓ with respect to x-axis, and $y_{k\ell}$, $z_{k\ell}$ are one with respect to y-axis and z-axis respectively. From Eq.(11), it can be rewritten by $\{U_k\}$ as shown in Eq(13).

$$\{U_k\}=[T_{k\ell}]^{-T}\ \{U_\ell\} \quad (13)$$

Then, substituting Eq.(13) into Eqs.(8) & (9), the following equations are derived.

$$\{P_i\}=[K_{ii}]\ \{U_i\}+[K_{ik}]\ [T_{k\ell}]^{-T}\ \{U_\ell\} \quad (14)$$

$$\{P_k\}=[K_{ki}]\ \{U_i\}+[K_{kk}]\ [T_{k\ell}]^{-T}\ \{U_\ell\} \quad (15)$$

And if Eq.(10) is substituted into Eq.(15), we can obtain Eq.(16) with respect to $\{P_\ell\}$.

$$\{P_\ell\}=[T_{k\ell}]^{-1}[K_{ki}]\{U_i\}+[T_{k\ell}]^{-1}[K_{kk}][T_{k\ell}]^{-T}\{U_\ell\} \quad (16)$$

In matrix forms, with respect to nodes i and ℓ, it can be written as follows :

$$\begin{Bmatrix} P_i \\ P_\ell \end{Bmatrix}=\begin{bmatrix} [K_{ii}] & [K_{ik}]\ [T_{k\ell}]^{-T} \\ [T_{k\ell}]^{-1}[K_{ki}] & [T_{k\ell}]^{-1}[K_{kk}][T_{k\ell}] \end{bmatrix}\begin{Bmatrix} \{U_i\} \\ \{U_\ell\} \end{Bmatrix} \quad (17)$$

Ultimately, the member stiffness matrix which includes the characteristics of the proposed pipe joint elements is shown as

$$[K_M^*]=\begin{pmatrix} [T_{ik}]\ [K_{kk}]\ [T_{ik}]^T & -[T_{ik}]\ [K_{kk}]\ [T_{k\ell}]^{-T} \\ -[T_{k\ell}]^{-1}\ [K_{kk}]\ [T_{ik}]^T & [T_{k\ell}]^{-1}\ [K_{kk}]\ [T_{k\ell}]^{-T} \end{pmatrix} \quad (18)$$

2.2 Determination of Stiffness Values

The proposed pipe joint elements using three springs play an important role in an efficient analysis of a piping system because the flexibility effect comes from them on a pipe joint. Their stiffness values depend on parameters such as length, thickness and diameter etc. of the main pipe with a pipe joint, and how they change according to the parameters shall be helpful to a simplified analysis of piping systems. But, if a pipe joint is not typical in shape, it is very difficult to find them through a theoretical study. So, we have got approximate values through a parameter study. This parameter study is in process. In the present study, the stiffness values of the pipe joint model as shown in Fig. 6 have been obtained by the load-displacement relationships when the ratio of length to diameter is egual to 4.

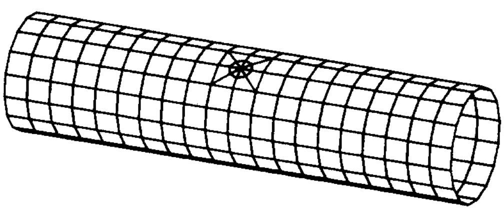


Figure 6. Main pipe with a pipe joint to determine stiffness values

2.3 Formulation of Mass Matrix

In the formulation of the mass matrix with a pipe joint, using the concept of a rigid body motion as stated above, mass matrices corresponding to the pipe joint elements can be acted on the center line of the main pipe. The lumped mass matrix $[M]$ of the element ik, shown in Fig. 5, can be obtained as follows :

$$[M]=\begin{bmatrix}[M_i] & O \\ O & [M_k]\end{bmatrix} \tag{19}$$

The inertia force with respect to node k is

$$\{P_k\}=[M_k]\ \{\ddot{U}_k\} \tag{20}$$

Differentiating the displacement vector $\{U_\ell\}$ of Eq.(11) twice with respect to time t, the acceleration vector can be obtained as follows :

$$\{\ddot{U}_\ell\}=[T_{k\ell}]^T\ \{\ddot{U}_k\} \tag{21}$$

Substituting Eqs.(10) & (21) into Eq.(20), the following relationship is obtained.

$$\{P_\ell\}=[T_{k\ell}]^{-1}\ [M_k]\ [T_{k\ell}]^{-T}\ \{\ddot{U}_\ell\} \tag{22}$$

Thus, the mass matrix with respect to node ℓ is

$$[M_\ell]=[T_{k\ell}]^{-1}\ [M_k]\ [T_{k\ell}]^{-T} \tag{23}$$

Consequently, the mass matrix for the element $i\ell$ can be expressed as

$$[M]=\begin{pmatrix}[M_i] & O \\ O & [M_\ell]\end{pmatrix} \tag{24}$$

3. NUMERICAL EXAMPLE

3.1 Example Structures

One out of several example structures used to verify the performance of the proposed pipe joint elements and analysis model is shown Fig. 7. Length, diameter and thickness of the main pipe are 550cm, 50cm and 0.5cm respectively and two slender pipes are joined to the main pipe at the pipe joint 150cm and 350cm apart from the left end of the main pipe respectively. Both ends of the main pipe are fixed to rigid base. Diameter and thickness of slender pipes connected to the main pipe are 10cm and 0.5cm respectively and lengths are shown in Fig.7. The material for all of the pipes is assumed to be structural steel.

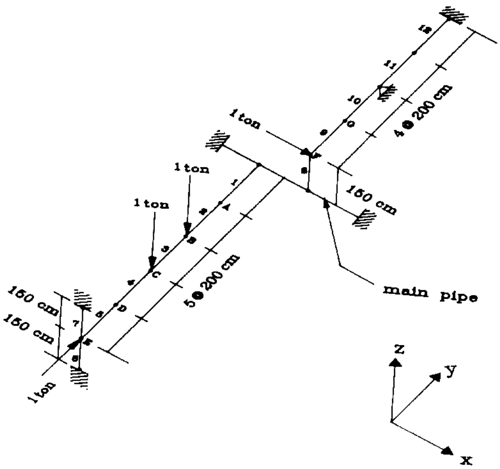


Figure 7. Example structure

Three types of analysis model are used in this study, Model C is the conventional beam model that cannot account for the effect of local deformation of pipe joint, while Model P is the proposed model which has pipe joint elements in addition to beam elements. A three dimentional finite element model, named Model F, consists of plate /shell elements representing the main pipe and beam elements employed to model slender pipes. The numbers of nodes and elements used for these models are listed in Table 1. For a complicated piping system, the numbers of nodes and elements to be used for the Model F will be increased significantly while those for the joint models used for the Model P will be limited by the number of pipe joint types.

Table 1. Nodes and elements of models

	Number of Nodes	Number of Beam Elements	Number of Shell Elements
Model C	26	25	0
Model F	912	12	904
Model P	26	25	0
Pipe Joint Model used for Model P	328	0	332

3.2 Static Analysis Results

Loads applied to the piping system are shown in Fig. 7. Analyses for Model C and P were performed using computer program developed in this

Table 2. Translation and rotation obtained by static analysis [cm, rad]

Node No	Translation in x−dir.			Translation in y−dir.			Translation in z−dir.		
	Model P	Model F	Model C	Model P	Model F	Model C	Model P	Model F	Model C
A	.003	.043	.006	.045	.084	.016	−20.892	−21.274	−10.823
B	.004	.050	.006	.050	.088	.021	−35.514	−36.046	−23.953
C	.003	.036	.004	.055	.092	.026	−31.630	−32.017	−23.688
D	.001	.015	.002	.060	.096	.032	−13.123	−13.267	−10.253
E	.000	.000	.000	.065	.101	.037	−.002	−.002	−.002
F	6.001	6.159	3.546	.000	.000	−.000	−.001	.024	−.002
G	2.803	2.858	1.631	.000	.000	−.000	−.000	.013	−.006
Node No	Rotation in x−dir.			Rotation in y−dir.			Rotation in z−dir.		
	Model P	Model F	Model C	Model P	Model F	Model C	Model P	Model F	Model C
A	.1016	.1035	.0753	.0000	−.0004	.0000	.0000	.0001	.0000
B	.0317	.0315	.0400	.0000	−.0003	.0000	.0000	.0000	.0000
C	−.0673	−.0684	−.0425	.0000	−.0002	.0000	.0000	−.0001	.0000
D	−.0984	−.0996	−.0755	.0000	−.0001	.0000	.0000	−.0001	.0000
E	−.0134	−.0136	−.0107	.0000	.0000	.0000	.0000	.0000	.0000
F	.0000	.0000	−.0001	.0445	.0453	.0291	.0122	.0128	.0076
G	.0000	−.0001	.0000	.0334	.0340	.0218	.0174	.0178	.0102

Table 3. Member forces obtained by static analysis [ton, ton · cm]

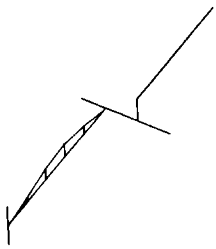
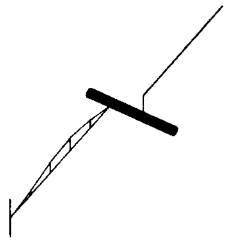
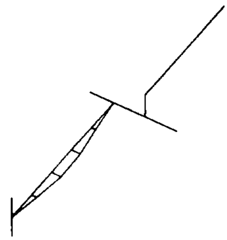
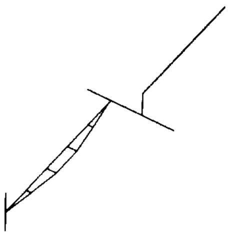
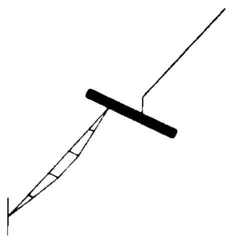
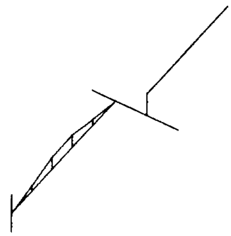
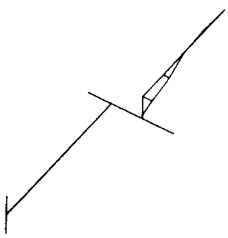
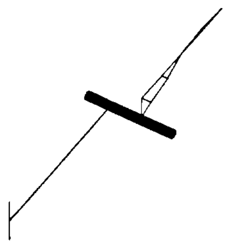
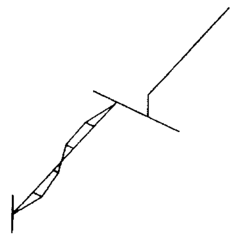
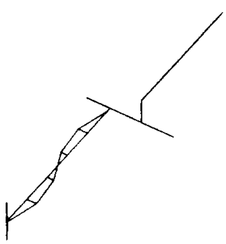
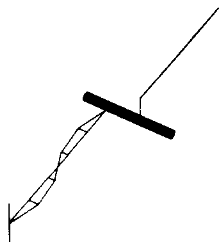
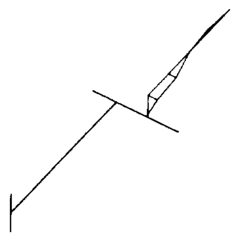
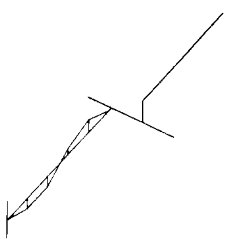
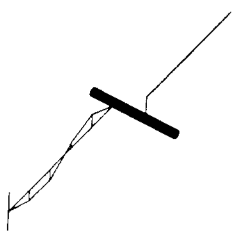
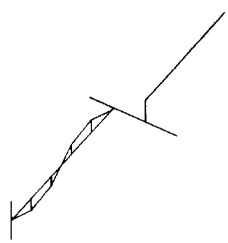
Beam No	Axial Force			Shear Force in y-dir			Shear Forcen in z-dir		
	Model P	Model F	Model C	Model P	Model F	Model C	Model P	Model F	Model C
1	.808	.702	.889	.80	.79	.99	.000	−.001	.000
	−.808	−.702	−.889	−.80	−.79	.99	.000	.001	.000
5	.808	.702	.889	−1.20	−1.21	−1.01	.000	−.001	.000
	−.808	−.702	−.889	1.20	1.21	1.01	.000	.001	.000
6	.600	.605	.506	1.38	1.35	1.12	.000	−.001	.000
	−.600	−.605	−.506	−1.38	−1.35	−1.12	.000	.001	.000
7	−.600	−.605	−.506	−1.58	−1.64	−1.23	.000	.000	.000
	.600	.605	.506	1.58	1.64	1.23	.000	.000	.000
8	.000	.001	−.001	.00	−.03	.01	−.851	−.850	−.916
	.000	−.001	.001	.00	.03	−.01	.851	.850	.916
9	−.001	.026	−.014	.00	.00	.00	.149	.150	.084
	.001	−.026	.014	.00	.00	.00	−.149	−.150	−.084
Beam No	**Torsion**			**Bending Moment in y-dir**			**Bending Moment in z-dir**		
	Model P	Model F	Model C	Model P	Model F	Model C	Model P	Model F	Model C
1	.00	.14	.00	.0	.5	.1	94.4	88.1	222.6
	.00	−.14	.00	.0	−.3	.0	64.7	69.8	−25.9
5	.00	−.14	.00	.0	−.1	.0	55.7	56.7	32.9
	.00	−.14	.00	.0	.2	.0	−295.8	−298.8	−235.2
6	.01	.09	.01	.0	.1	.0	140.7	138.3	113.5
	−.01	−.09	−.01	.0	.1	.0	66.7	63.5	54.6
7	−.01	−.09	−.01	.0	.0	.0	−155.1	−160.6	−121.8
	.01	.09	.01	.0	.0	.0	−81.2	−85.9	−63.0
8	−25.81	−25.38	−13.82	109.1	109.5	125.0	.1	−3.6	1.8
	25.81	25.38	13.82	17.7	18.0	11.6	.0	−.2	.3
9	17.69	18.02	11.58	−25.8	−25.4	−13.8	.0	.2	−.3
	−17.69	−18.02	−11.58	−4.1	−4.6	−2.9	.0	.0	.1

study in 16−bit IBM PC, and for Model F, SAP IV was used in a super computer(CRAY 2S). Nodal displacements of slender pipes are compared in Table 2. Displacements obtained using the Model P are very close to those of the Model F while the Model C underestimated significantly because the flexibility of pipe joints is ignored. Member forces of slender pipes are listed in Table 3. The effect of flexible pipe joints tends to reduce axial forces and bending moments in a slender pipe near the pipe joint and increase bending moments and shear forces at the other end of a slender pipe. Consequently, a piping system designed based on the analysis results which are obtained by using a conventional beam model may result in over−design near the pipe joint and under−design around the other end of a slender pipe.

3.3 Dynamic Analysis Results

The main purpose of dynamic analysis of example structures is to verify the equivalence of structural properties of two beam models and the finite element model because dynamic response of a structure is mainly dependent on eigenvalues and eigenvectors of these systems. The first five

Table 5. Mode shapes of models

Mode	Model P	Model F	Model C
1			
2			
3			
4			
5			

frequencies for models C, F and P are listed in Table 4 and corresponding mode shapes are shown in Fig.8. Frequencies of model F and P are very close to each other and much lower than those of model C which can not account for the flexibility of pipe joints. Mode shapes of the example structure demonstrate that dynamic behavior of two slender pipes are almost independent and the main pipe can be considered as a flexible support to a slender pipe. Discrepancy in frequencies will result in significant differences in dynamic response prediction.

Table 4. Frequencies(cycle / sec)

Mode	Model P	Model F	Model C
1	4.823	4.788	5.378
2	4.947	4.933	5.919
3	11.916	11.810	14.956
4	14.118	14.080	14.987
5	14.411	14.400	16.180

Therefore, addition of the proposed pipe joint elements to a beam model is expected to improve the accuracy of dynamic response prediction for a piping system. Consequently, it turns out to be essential to account for the flexibility of pipe joints in the response prediction of piping systems.

4. DISCUSSION & CONCLUSIONS

The proposed pipe joint elements are introduced in the present study to account for the local deformation of a pipe joint where a slender pipe is connected to another pipe with relatively larger diameter. The static and dynamic analyses of several example piping systems have been performed and the following conclusions are drawn from comparison of analysis results obtained using three types of analysis model.

1. Beam models with the pipe joint elements result in a very accurate response prediction of a piping system those while those without the pipe joint elements lead to underestimation of displacements and overestimation of bending moments in the slender pipe and stresses in the main pipe around the pipe joint for the static loads.
2. Beam models with the proposed pipe joint elements lead to an accurate prediction of frequencies and mode shapes which is essential for accurate estimation of dynamic response of a piping system subjected to a severe earthquake.
3. Time history of stresses near a pipe joint can be obtained efficiently using the proposed model and fatigue analysis of the pipe joint of a complicated piping system can be performed on a personal computer.
4. The proposed analysis models can provide the static or dynamic response prediction of a piping system with an accuracy comparable to the one obtained using a complicated three dimensional finite element model while the simplicity of the proposed model and requrirements for the memory size and computational time are similar to those of the conventional beam model.
5. The procedure used to estimate the equivalent stiffness of fictitious springs used in the proposed model need to be improved and simplified using the theory of thin shell.

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