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—Research Paper—

Travel-Time Models for Class-Based AS/RS Systems

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ABSTRACT

This paper presents average travel time models for automated warehousing system where the stacker crane transports only one pallet at a time with the tchebychev travel, I/O point is located at the corner of the racks, and items are stored by the class-based storage assignment rule. In this study, the racks are treated as the continuous rectangle in time and a statistical approach was used to develop the models. In order to test the proposed models, average travel times determined by the models are compared with the true values for various rack shapes.

1. Introduction

Automatic warehousing systems are rapidly replacing conventional warehouses for the storage and movement of high-volume goods. An automatic warehousing system differs from nonautomated systems in that the storage and retrieval of items is performed by automatic stacker cranes rather than by manually-operated devices (e.g., forklift trucks). For each arriving pallet, a minicomputer selects an open location in the storage racks and directs the stacker crane to store the pallet in a specified location. When a request is made for the stored item, the minicomputer, recalling from memory where the item's pallet is stored, directs the crane to the rack location for the retrieval of the specified item. A conveyor system provides the link between the warehouse and the source or destination for the items (e.g., production line, receiving or shipping dock). The exchange of pallets from the stacker crane to the conveyor occurs at the input/output(I/O) point.

In this paper, analytical expressions are derived for travel times for the stacker crane in an automatic warehousing system. A variety storage rack shapes are examined. Both single and dual command systems are included. The principal assumptions for the analysis are : a continuous approximation to the discrete rack face, class-based storage assignment, and Tchebychev travel for the storage and retieval. (Note that this is simultaneous horizontal and vertical travel

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in contrast to rectilinear or sequential travel, as performed by a lift truck.)

Derived travel-time expressions extend the results by Graves, Hausman, and Schwarz (GHS) for class-based storage in their papers [3, 5, 6, 7, 8]. GHS considered a variety of storage methods (random, dedicated, and class-based) and they assumed the rack to be square in time. But empirical experience indicates that the optimum design for automatic warehousing system frequently is not square in time. Furthermore, many systems that have been installed are not square in time. For this reason it is desirable to determine the expected travel time for a rack that is not necessarily square in time.

2. General Model Formulation

Assumptions

The model is developed under the following assumptions :

1) Each pallet holds only one part number or item type.
2) All storage locations are the same size, as are the pallet themselves. Therefore all storage locations are candidates for storing any pallet load.
3) The rack is considered to be a continuous rectangular pick face where the I/O point is located at the lower left-hand corner.
4) The stacker crane operates either on a single or dual command basis, i.e., multiple stops in the aisle are not allowed.
5) The rack length and height, as well as the stacker crane speed in the horizontal and vertical directions, are known.
6) Class-based storage is used.
7) The stacker crane travels simultaneously in the horizontal and vertical directions. In calculating the travel time, constant velocities are used for horizontal and vetical travel.
8) Pick-up and deposit (P/D) times associated with load handling are ignored. The P/D time is generally independent of the rack shape and the travel velocity of the stacker crane. Furthermore, given the load characteristics, the P/D time is deterministic. Hence, it is a straightforward matter to include the P/D time after the average travel time has been computed.
9) Storage and retrieval requests are serviced FCFS, first-come-first-served.
10) The turnover frequency of each item is known. Turnover frequency is the number of times a given item is requested in some time period : e.g., day, month, year, etc. Alternatively, it is the reciprocal of the length of storage time for the item.

Notations

The following notations are used throughout this paper :

S_h : speed of the stacker crane in the horizontal direction
S_v : speed of the stacker crane in the vertical direction
M : length of the rack

H : height of the rack

t_h : horizontal travel time required to go to the farthest column from the I/O station (t_h = M/S_h)

t_v : vertical travel time required to go to the farthest row from the I/O station (t_v = H/S_v)

b : shape factor

$D = \text{Max}\{t_h, t_v\}$ and $b = \text{Min}\{t_h/D, t_v/D\}$

R1 : boundary between Class I and II

R2 : boundary between Class II and III

Tr : expected travel time from I/O point to storage locations under randomized storage assignment

Tc : expected travel time from I/O point to storage locations under class-based storage assignment

Lr : expected interleaving time between the two randomly selected points under randomized storage assignment

Lc : expected interleaving time between the two randomly selected points under class-based storage assignment

Under the above assumptions and notations, the expected single command travel time for class-based storage, E(SC), is given by

$$E(SC) = 2Tc$$

and the expected dual command travel time for class-based storage, E(DC), is given by

$$E(DC) = 2Tc + Lc.$$

For the above equations, we must derive Tc and Lc to calculate the expected travel time. Because Tc and Lc for three-class system are in exactly the same manner as for the corresponding two-class system, we will consider only two-class system here.

Under two-class system, Tc is given by

$$Tc = p(\text{I}) * T(\text{I}) + p(\text{II}) * T(\text{II}) \quad (1)$$

where p(i) is the probability of a storage or retrieval into class i, T(i) is the expected one-way travel time to class i.

Under two-class system, Lc is given by

$$Lc = p(\text{I},\text{I}) * L(\text{I}) + p(\text{II},\text{II}) * L(\text{II}) + 2p(\text{I},\text{II}) * L(\text{I},\text{II}) \quad (2)$$

where p(i,j) is the probability of an interleaving from a location in class i to a location in class j, L(i) is the expected interleaving time within class i, and L(i,j) is the expected interleaving time between locations in class i and j.

Equation (1) and (2) can be calculated for any specified boundary between class I and II. Before proceeding, we need to introduce various class shapes with boundaries. These class-shapes are divided into 3 categories (see Figure 1 and Figure 2) as follow :

1) TYPE I : All class-shapes are rectangle-shape
2) TYPE II : class-shapes are rectangle-shape and elbow-shape
3) TYPE III : All class-shapes are elbow-shape except for class I .

Consequently, class-shapes are composed of rectangle-shapes and elbow-shapes.

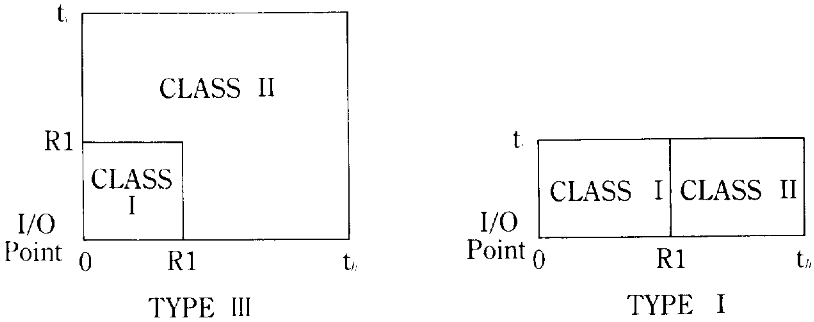

Fig. 1 Class-Shapes for Two-Class System

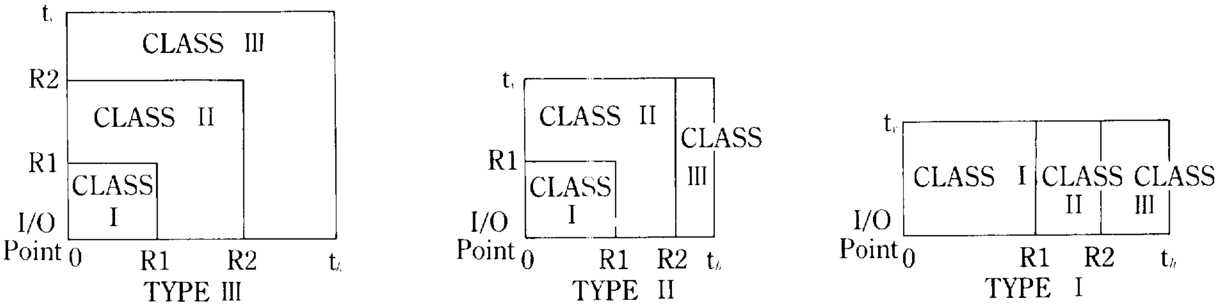

Fig. 2 Class-Shapes for Three-Class System

2.1. Expected Travel Time for Rectangle-Shape

Let the storage (or retrieval) point be represented by (x,y) in time, where $0 \leq x \leq d$ and $0 \leq y \leq f$ (see Fig.3). Travel from (0,0) to (x,y), say t_u, will be $t_u = \mathrm{Max}(x,y)$. Now let G(z) denote the probability that travel time to (x,y) is less than or equal to z. If assuming the x, y corrdinates are independently generated, then G(z) can be represented as follows :

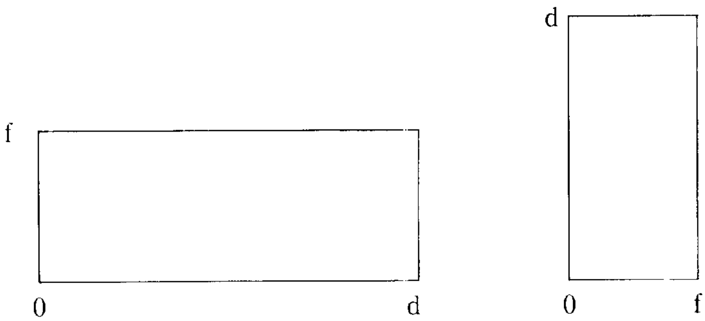

Fig.3 A Rectangle-Shape

$$G(z)=Pr(t_{xy}\leq z)=Pr(x\leq z)Pr(y\leq z).$$

Furthermore, for randomized storage the coordinate locations are assumed to be uniformly distributed. Thus,

$$Pr(x\leq z)=z/d \qquad \text{if } 0\leq z\leq d$$

and

$$Pr(y\leq z)=\begin{cases} z/f & \text{if } 0\leq z\leq f, \\ 1 & \text{if } f<z\leq d. \end{cases}$$

Hence

$$G(z)=\begin{cases} z^2/(df) & \text{for } 0\leq z\leq f \\ z/d & \text{for } f<z\leq d. \end{cases}$$

Therefore the probability density function, $g(z)$, will be

$$g(z)=\begin{cases} 2z/(df) & \text{for } 0\leq z\leq f \\ 1/d & \text{for } f<z\leq d. \end{cases}$$

Consequently, the following result is obtained ;

$$Tr=\int_0^d zg(z)dz=d/2+f^2/(6d) \tag{3}$$

2.2. Expected Interleaving Time for Rectangle-Shape

To analyze the expected interleaving time between the two points, recall that any point is represented as (x,y) in time and $0\leq x\leq d$ and $0\leq y\leq f$. Let t_B be the time required to travel between storage and retrieval of locations. Also, let

$$\begin{aligned} F(z)&=Pr(t_B\leq z) \\ &=Pr(|x_1-x_2|\leq z)Pr(|y_1-y_2|\leq z), \end{aligned}$$

where (x_1,y_1) and (x_2,y_2) are the two random points. The Lr for rectangle-shape is derived in exactly the same manner as for the corresponding Tr for rectangle-shape. Consequently, the following result is obtained.

$$Lr=d/3+f^2/(6d)-f^3/(30d^2) \tag{4}$$

2.3. Expected Travel Time for Elbow-Shape

Let R be the boundary between class I and II and assume $C\geqq B$ (see Fig. 4) . For randomized storage, from equation (1)

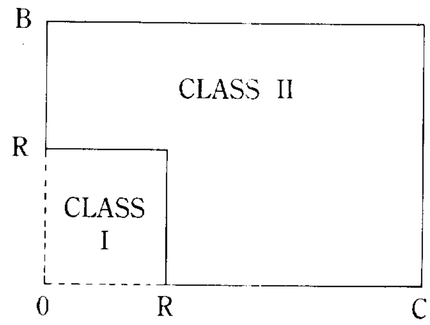

Fig. 4 A Elbow-Shape.

$$Tr = T(\text{I}) * p(\text{I}) + T(\text{II}) * p(\text{II}),$$

and $p(\text{I}) = R^2/(BC)$,

$$p(\text{II}) = (BC - R^2)/(BC),$$

from equation (3)

$$Tr = C/2 + B^2/(6C),$$
$$T(\text{I}) = 2R/3.$$

Consequently, the following result is obtained :

$$T(\text{II}) = \{Tr - p(\text{I}) * T(\text{I})\}/p(\text{II})$$
$$= (3BC^2 + B^3 - 4R^3)/\{6(BC - R^2)\} \tag{5}$$

2.4. Expected Interleaving Time for Elbow-Shape

In order to derive expected interleaving time for elbow-shape, Lr, we consider the elbow-shape with two rectangles as in Fig. 5. From equation (2)

$$Lr = L1 * \left(\frac{R(B-R)}{BC-R^2}\right)^2 + L2 * \left(\frac{B(C-R)}{BC-R^2}\right)^2 + 2 * L12 * \left(\frac{BR(B-R)(C-R)}{(BC-R^2)^2}\right)$$

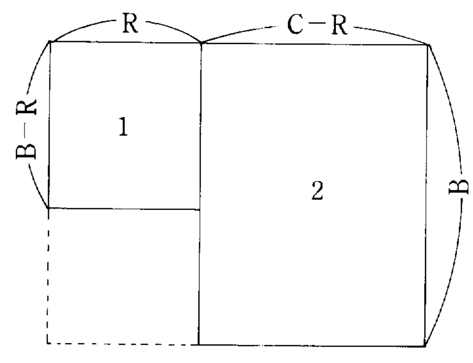

Fig. 5 Two Rectangle-Shape

where L1 is expected interleaving time in zone 1 in Fig. 5, L2 is in zone 2 and L12 is from zone 1 to zone 2.

L12 is derived with exactly the same manner as for the corresponding expected travel time for rectangle-shape. The results are as follow.

1) $C-R \leqq B,\ R<B/2$

$$L12=1/\{120BR(B-R)(C-R)\}*[30R^5+5CR^4-10C^2R^3+10C^3R^2-5C^4R-85BR^4+80BCR^3-90BC^2R^2+20BC^3R+70B^2R^3-50B^3R^2-30B^2CR^2+5B^4R+20B^3CR+30B^2C^2R-B^5+5B^4C-10B^3C^2+10B^2C^3-5BC^4+C^5]$$

2) $C-R \leqq B,\ B/2 \leqq R \leqq C/2$

$$L12=1/\{120BR(B-R)(C-R)\}*[-2R^5+5CR^4-10C^2R^3+10C^3R^2-5C^4R-5BR^4+80BCR^3-90BC^2R^2+20BC^3R-10B^2R^3-30B^2CR^2-5B^4R+20B^3CR+30B^2C^2R+5B^4C-10B^3C^2+10B^2C^3-5BC^4+C^5]$$

3) $C-R \leqq B,\ R>C/2$

$$L12=1/\{120BR(B-R)(C-R)\}*[30R^5-75CR^4+70C^2R^3-30C^3R^2+5C^4R-5BR^4+80BCR^3-90BC^2R^2+20BC^3R-10B^3R^2-10B^2R^3-30B^2CR^2-5B^4R+20B^3CR+30B^2C^2R+5B^4C-10B^3C^2+10B^2C^3-5BC^4]$$

4) $C-R>B,\ R<B/2$

$$L12=1/\{120BR(B-R)(C-R)\}*[31R^5-80BR^4+80B^2R^3-40B^3R^2+10B^4R+60BCR^3-60BC^2R^2-60B^2CR^2+60B^2C^2R]$$

5) $C-R>B,\ B/2 \leqq R \leqq C/2$

$$L12=1/\{120BR(B-R)(C-R)\}*[-R^5+B^5+60BCR^3-60BC^2R^2-60B^2CR^2+60B^2C^2R]$$

Consequently, the following results of expected interleaving times for elbow-shape, Lr are obtained.

1) $C-R \leqq B,\ R<B/2$

$$Lr=1/\{60(BC-R^2)^2\}*[-5CR^4+10C^2R^3-10C^3R^2+5C^4R-5BR^4+40BCR^3-30BC^2R^2-20BC^3R-10B^3R^2+10B^2R^3-30B^2CR^2+5B^4R-20B^3CR+30B^2C^2R-B^5+5B^4C+10B^3C^2+10B^2C^3+5BC^4+C^5] \quad (6)$$

2) $C-R \leqq B,\ B/2 \leqq R \leqq C/2$

$$Lr=1/\{60(BC-R^2)^2\}*[32R^5-5CR^4+10C^2R^3-10C^3R^2+5C^4R-85BR^4+40BCR^3-30BC^2R^2-20BC^3R-50B^3R^2+90B^2R^3-30B^2CR^2+15B^4R-20B^3CR+30B^2C^2R-2B^5+5B^4C+10B^3C^2+10B^2C^3+5BC^4-C^5] \quad (7)$$

3) $C-R \leqq B,\ R>C/2$

$$Lr = 1/\{60(BC-R^2)^2\} * [64R^5 - 85CR^4 + 90C^2R^3 - 50C^3R^2 + 15C^4R - 85BR^4 + 40BCR^3 - 30BC^2R^2 - 20BC^3R + 90B^2R^3 - 50B^3R^2 - 30B^2CR^2 + 15B^4R - 20B^3CR + 30B^2C^2R - 2B^5 + 5B^4C + 10B^3C^2 + 10B^2C^3 + 5BC^4 - 2C^5] \quad (8)$$

4) $C-R>B$, $R<B/2$

$$Lr = 1/\{60(BC-R^2)^2\} * [-R^5 - 10BR^4 + 54B^2R^3 - 20B^3R^2 - 2B^5 + 60BCR^3 - 60BC^2R^2 + 10B^4C + 20B^2C^3] \quad (9)$$

5) $C-R>B$, $B/2 \leqq R \leqq C/2$

$$Lr = 1/\{60(BC-R^2)^2\} * [31R^5 - 90BR^4 + 80B^2R^3 - 60B^3R^2 + 10B^4R - 3B^5 + 60BCR^3 - 60BC^2R^2 + 10B^4C + 20B^2C^3] \quad (10)$$

3. Evaluation of the Models

The developed continuous model is now empirically evaluated for accuracy. More specifically, the results obtained from the model are compared with those obtained from a discrete rack. The time function from the I/O point to location (i,j) for the discrete rack is defined as

$$t_{i,j} = \mathrm{Max}\left\{\frac{2i-1}{2R}, \frac{2j-1}{2C}\right\} \qquad \text{for} \quad \begin{array}{l} i=1, \cdots, R \\ j=1, \cdots, C \end{array}$$

A configuration of AS/RS was developed for the purpose of evaluation. The configuration has a rack with 50 columns and 2 rows, which yields 100 openings. For this configuration, it is assumed that the width and the height of each rack opening is 1 m and the stacker crane travels at 20 m/min and 5 m/min in the horizontal and vertical directions respectively. Subsequently, the number of rows and columns in each rack is varied to obtain different shape factors while keeping the number of openings approximately equal. The results obtained for the above condition are presented in Tables 1 and 2. Based on Tables 1 and 2, the following results are obtained :

1) For Type I, it can be observed that the continuous model displays a satisfactory performance with the largest percentage deviation being 0.4938355 %.
2) For Type II, as the weight of class I increases, the percentage deviation increases. The reason is that the increasing importance given to the first few pallets in the class I.

Table 1. Comparison of Travel-Times (b=0.16)

$t_h=2.5$ $t_v=0.4$ b=0.16 No. of columns=50 No. of rows=2 $s_h=20$ $s_v=5$

R1	R2	P1	P2	P3	DS	DD	CS	CD	% Deviation		type
0.5		0.20	0.80		5.040000	3.366694	5.042667	3.364992	0.0529160	0.0505349	TYPE I
		0.4	0.6		4.08	2.920399	4.085334	2.921622	0.1307239	0.041889	
		0.6	0.4		3.12	2.345606	3.128	2.349355	0.2564146	0.1598466	
		0.8	0.2		2.16	1.642315	2.170667	1.648192	0.4938355	0.357886	
0.3		0.09	0.91		5.180001	3.417564	5.042668	3.364993	2.651205	1.538268	TYPE III
		0.2	0.8		4.666667	3.220382	4.529818	3.157052	2.932477	1.96652	
		0.4	0.6		3.733333	2.763416	3.597368	2.684893	3.642044	2.841542	
		0.8	0.2		1.866666	1.468376	1.732454	1.376366	7.18989	6.266099	
0.5	1.6	0.2	0.44	0.36	5.04	3.364129	5.042667	3.364992	0.0529157	0.0256482	TYPE I
		0.4	0.4	0.2	3.8	2.710419	3.80533	2.716637	0.1403533	0.0964824	
		0.6	0.3	0.1	2.8	2.078802	2.808	2.083184	0.2857191	0.2107779	
		0.7	0.2	0.1	2.5	1.888469	2.509333	1.893707	0.3733349	0.2773895	
0.3	0.9	0.09	0.27	0.64	5.084	3.368391	5.042667	3.364992	0.8130055	0.1008915	TYPE II
		0.2	0.2	0.6	4.746667	3.261099	4.698272	3.254653	1.019561	0.1976453	
		0.5	0.2	0.3	2.986666	2.281363	2.898272	2.233498	2.959635	2.098084	
		0.7	0.2	0.1	1.813333	1.389917	1.698272	1.317326	6.345281	5.222665	
0.2	0.3	0.04	0.05	0.91	5.1735	3.415054	5.042668	3.364993	2.528891	1.465889	TYPE III
		0.2	0.1	0.7	4.135	2.982453	4.031591	2.92468	2.50083	1.937131	
		0.4	0.2	0.4	2.67	2.116005	2.600909	2.06158	2.58769	2.572097	
		0.6	0.2	0.2	1.65	1.34612	1.615121	1.311893	2.113877	2.542704	

P1 : turnover frequency in the Class I
P2 : turnover frequency in the Class II
P3 : turnover frequency in the Class III
DS : expected single command travel time for discrete rack
DD : expected dual command travel time for discrete rack
CS : expected single command travel time for continuous rack
CD : expected dual command travel time for continuous rack
TYPE I : $R1 \geqq t_v$
TYPE II : $R1 < t_v$, $R2 \geqq t_v$
TYPE III : $R2 < t_v$

Table 2. Comparison of Travel-Times (b=0.3636, 0.64, 1)

t_h =1.65 t_v =0.6 b=0.3636 No. of columns=33 No. of rows=3 s_h =20 s_v =5

R1	R2	P1	P2	P3	DS	DD	CS	CD	% Deviation		type
0.9		0.54	0.46		3.458	2.307389	3.462	2.314957	0.1156723	0.3280054	TYPE I
		0.8	0.2		2.667408	1.837554	2.673334	1.844039	0.2221414	0.3529465	
		0.9	0.1		2.36334	1.622145	2.37	1.627432	0.2820771	0.3259444	
0.3		0.09	0.91		3.540494	2.339559	3.4481	2.307589	2.333771	1.366498	TYPE III
		0.2	0.8		3.216551	2.211279	3.128	2.160272	2.752988	2.306665	
		0.4	0.6		2.645747	1.922853	2.546	1.845678	3.770069	4.013557	
		0.6	0.4		2.074942	1.563215	1.964	1.470768	5.346736	5.913883	
0.7	1.1	0.42	0.24	0.34	3.462	2.314639	3.466	2.317872	0.1155593	0.139664	TYPE I
		0.6	0.2	0.2	2.859999	1.979134	2.865715	1.983096	0.1998382	0.2002268	
		0.7	0.2	0.1	2.483333	1.722644	2.49	1.727103	0.2684946	0.258199	
0.4	1.0	0.16	0.44	0.4	3.46	2.313375	3.464	2.316375	0.1156123	0.1297125	TYPE II
		0.3	0.4	0.3	2.970455	2.073028	2.976667	2.077363	0.2091422	0.2090994	
		0.5	0.3	0.2	2.384091	1.734808	2.393334	1.740946	0.3876754	0.3538193	
		0.7	0.2	0.1	1.797727	1.32519	1.81	1.333072	0.6826995	0.5948005	
0.2	0.5	0.04	0.21	0.75	3.565	2.357927	3.45259	2.309532	3.153165	2.127142	TYPE III
		0.2	0.2	0.6	2.968572	2.117975	2.899215	2.058882	2.336374	2.790071	
		0.4	0.2	0.4	2.188572	1.689228	2.17408	1.64319	0.6621579	2.725403	
		0.6	0.2	0.2	1.408571	1.134951	1.448944	1.129928	2.866244	0.4425744	

t_h =1.25 t_v =0.8 b=0.64 No. of columns=25 No. of rows=4 s_h =20 s_v =5

R1	R2	P1	P2	P3	DS	DD	CS	CD	% Deviation		type
0.9		0.72	0.28		2.836001	1.907997	2.841334	1.911744	0.1880278	0.1963832	TYPE I
		0.9	0.1		2.470001	1.676159	2.476667	1.680266	0.2698569	0.2451955	
0.6		0.36	0.64		2.836	1.908101	2.841334	1.911744	0.1880699	0.1909184	TYPE III
		0.5	0.5		2.563194	1.759621	2.569792	1.764238	0.2574037	0.2623915	
		0.7	0.3		2.173472	1.514424	2.181875	1.520061	0.3866523	0.3722714	
		0.9	0.1		1.783749	1.230305	1.793958	1.236527	0.5723385	0.5057001	
0.8	1.1	0.64	0.24	0.12	2.836	1.907738	2.841333	1.911744	0.1880615	0.2099696	TYPE I
		0.8	0.1	0.1	2.55	1.743191	2.556667	1.747516	0.2614377	0.2481512	
0.5	1.0	0.25	0.55	0.2	2.912267	1.939113	2.841334	1.911748	2.435691	1.411186	TYPE II
		0.4	0.4	0.2	2.683467	1.839085	2.602788	1.800804	3.006528	2.081535	
		0.6	0.3	0.1	2.227601	1.560524	2.127091	1.503245	4.512013	3.670504	
0.2	0.5	0.04	0.21	0.75	2.907142	1.947759	2.841334	1.911744	2.263677	1.849068	TYPE III
		0.2	0.2	0.6	2.442285	1.747051	2.41021	1.700523	1.313336	2.66321	
		0.4	0.2	0.4	1.837714	1.405275	1.848076	1.369098	0.5638803	2.574366	
		0.6	0.2	0.2	1.233142	0.974639	1.285943	0.975289	4.281778	0.066678	

$t_h = 1$ $t_v = 1$ b=1 No. of columns=20 No. of rows=5 $s_h = 20$ $s_v = 5$

R1	R2	P1	P2	P3	DS	DD	CS	CD	% Deviation		type
0.5		0.25	0.75		2.524621	1.734312	2.666668	1.8	5.626447	3.787583	TYPE III
		0.4	0.6		2.31303	1.616664	2.400001	1.657334	3.760039	2.515647	
		0.6	0.4		2.030309	1.428112	2.044446	1.438075	0.6665337	0.6969142	
		0.8	0.2		1.748788	1.203375	1.68889	1.18563	3.425103	1.474594	
0.4	0.7	0.16	0.33	0.51	2.647904	1.829711	2.666667	1.8	0.7086098	1.623796	TYPE III
		0.3	0.3	0.4	2.346731	1.675537	2.370482	1.645878	1.01209	1.770135	
		0.5	0.3	0.2	1.856731	1.353667	1.896756	1.345841	2.155703	0.5780872	
		0.7	0.2	0.1	1.506154	1.08294	1.541106	1.097652	2.320609	1.358533	

3) For Type III, in the two-class system, as the weight of class I increases, the precentage deviation increases. However, in the three-class system, as the weight of class I increases, the percentage deviation does not increase because of the offset between classes.
4) As the shape factor of rack decreases, the travel-time increases.

4. Concluding Remarks

We have derived analytical expressions for travel time models in class-bassed AS/RS systems. Various storage rack shapes were considered. Both single and dual command systems are also included.

The travel-time expressions presented could be used to establish throughput standards on existing systems. The results for evaluating the models are also quite useful in estimating throughput performance for first-cut evaluations of automatic warehousing system design configurations.

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