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—Research Paper—

Cooperative Determination of Economic Order Quantity to Reduce a Supplier's Freight Cost
(공급자의 운송비용을 절감하기 위한 경제적 발주량의 상호협동적 결정)

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ABSTRACT

A joint economic ordering problem between supplier and customer has been formulated where a supplier has to pay the freight cost which follows the principle of economy of scale. An algorithm is given to determine order size and price simultaneously which give gains to both parties. A scenario is presented within which both parties come to a mutual agreement on the revised order size and price by utilizing quantity discount schedule.

1. Introduction

In most of researches on inventory problem, price and order size are assumed to be determined by supplier and buyer in isolation, respectively. However, in case that the order size influences the supplier's cost significantly, it may be wise for the supplier to induce the buyer to boost his order size by suggesting the price discount. A typical case is where the supplier has to pay a freight cost which follows the principle of economy of scale and so wants buyer's order size to qualify freight rate breaks.

In this paper, a model is analyzed to determine price and order size which give the minimum joint cost of supplier and buyer. And it is discussed how to share the cost reduction amount between supplier and buyer. We assumed that the supplier is in charge of a freight cost.

Recently, several studies [1, 2, 3, 4, 5, 6, 8, 9, 10] dealt with price scheduling problem for quantity discount to increase supplier's profit. And C.Y. Lee [7] analyzed how a buyer deter-

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mines economic order quantity considering freight discount costs.

This paper is distinctive from the past studies in three respects. First, the sum of gains of both supplier and buyer is maximized and is allowed to be splitted in a predetermined ratio between them. Second, the supplier's freight cost is considered which follows the principle of economy of scale. Third, it is presented how the supplier may induce the buyer to place orders of the size which qualifies the freight rate breaks.

NOTATIONS

The following notations will be used :

For buyer,

D = annual quantity of demand

Q = order size

C_1 = ordering cost (\$ /order)

C_2 = yearly inventory carrying cost, expressed as a percentage of thė value of the item(% / year)

For supplier,

S(Q) = set-up cost per order including freight cost

C = variable unit cost

P = original base price

A = price discount coefficient ; discounted price per unit becomes PA

2. Development of the model

Let supplier's set-up cost including freight cost be expressed as follows ;

$$S(Q)=K_j+K, \text{ if } N_{j-1}<Q\leq N_j \quad (1)$$

j=1, 2, ···, J where

$$K_j<K_{j+1} \text{ and } K_{j+1}/N_{j+1}\leq K_j/N_j \quad (2)$$

K is the fixed cost and K_j is the freight cost for Q if $N_{j-1}<Q\leq N_j$. The condition (2) implies that the freight cost follows the principle of economy of scale and has N_1, ···, N_{J-1} as rate breaks.

The supplier's annual cost incurred by the order of the customer may be expressed as S(Q) (D/Q) + CD. Thus, the supplier's annual net profit is given by

$$PAD-S(Q)(D/Q)-CD \quad (3)$$

The last term in the profit function is independent of price and order size, and so we will simplify the profit function as follows ;

$$F(A, Q)=PAD-S(Q)(D/Q) \quad (4)$$

And the total annual inventory related cost of the customer is expressed as follows ;

$$E(A, Q)=C_1 D/Q+C_2 PAQ/2+PAD \quad (5)$$

With no discount available, the current order size becomes Q_a from the EOQ formula and

$$Q_a = \sqrt{2C_1 D/(C_2 P)} \tag{6}$$

The current order size is assumed to be determined optimally by the customer under the given price level, P.

If the order size affects the supplier's set-up costs significantly, the supplier will want to increase the order size to qualify for freight rate breaks even if he admits the price discount. Since the current order size and price are determined by buyer and supplier in isolation, respectively, simultaneous determination of price and order size from the viewpoint of joint cost function will result in extra benefits to both sides.

The next problem is how to share the gain resultant from the revision of price and order size. When one between two agents engrosses the gain, the other may not be willing to take the risk of changing the trade terms. Therefore, the one who initiates the bargaining for revising (A, Q) needs to provide economic incentives for the other to accept his suggestion.

Suppose that the supplier makes up his mind to take only a predetermined portion, r $(0 \leq r \leq 1)$, of the gain and is willing to concede the remaining portion to the buyer. Then, it is reasonable to assume that the supplier will try to maximize the total amount of the gain so that his own share may also be maximized. Thus, the problem that the supplier is confronted with is the maximization of the total gain, not only the profit of his own, which may be expressed as the following ;

$$\min_{A,Q} \; H(A, Q) = E(A, Q) - F(A, Q)$$
$$= C_1 D/Q + S(Q)D/Q + C_2 PAQ/2 \tag{7}$$

subject to

$$(1-r)\{F(A, Q) - F(1, Q_a)\} = r\{E(1, Q_a) - E(A, Q)\} \tag{8}$$

Solving the equation (8), we get

$$A = 2\{rQE(1, Q_a) + (1-r)QF(1, Q_a) + (1-r)S(Q)D - rC_1 D\}/\{PQ(rC_2 Q + 2D)\} \tag{9}$$

Replacing A in (7) with (9), H(A, Q) becomes a function of only Q(Denote it as H(Q)). And let $H_j(Q)$ be the expression of H(Q) for $N_{j-1} < Q \leq N_j$. Then, $H_j(Q)$ can be shown to be strongly unimodal with respect to Q and is minimized on

$$Q_j = (-Y + \sqrt{Y^2 - 4XZ})/(2X) \tag{10}$$

where $X = 2C_2\{rE(1, Q_a) + (1-r)F(1, Q_a)\} - rC_2^2(K+K_j)$

$Y = -4rC_2(C_1 + K + K_j)D$

$Z = -4(C_1 + K + K_j)D^2$

Note that Q_j increases monotonically with respect to $(K+K_j)$ and Q_j can be reduced to Q_a by setting $(K+K_j)=0$.

The curve of joint cost function, H(Q), is illustrated in Figure 1.

3. Algorithm to determine the optimal order size and price

To obtain the optimal (A, Q), an algorithm is provided.

Step 1. Compute Q with $K_j = 0$ in equation (10) and denote it as Q_0. Let i be the largest index such that $Q_0 > N_i$. If $i = J$, then $Q^* = N_J$ and stop. Otherwise, go to step 2.

Step 2. Compute Q_j and compare Q_j with N_j for $j = i+1, i+2, \cdots$ until the first index $k \leqq J$ is found such that $Q_k \leqq N_k$, then go to step 3. If $Q_j > N_j$ for all $i+1 \leqq j \leqq J$, then set $k = J+1$ and go to step 4.

Step 3. Compute the cost H(A, Q) by (7), (9), (10) for $Q = N_i, N_{i+1}, \cdots, N_{k-1}$ and Q_k. Select the one that yields the minimum cost as (A^*, Q^*) and stop.

Step 4. Compute the cost H(A, Q) for $Q = N_i, N_{i+1}, \cdots, N_{k-1}$.

Select the one that yields the minimum cost as (A^*, Q^*) and stop.

Examining figure 1 to illustrate the indices mentioned above, we see that $i = 2$, $k = 4$.

PROPERTY 1 : The values of (A^*, Q^*) found in the above algorithm is optimal of (7) and (8) when the supplier's set up cost is expressed as equation (1).

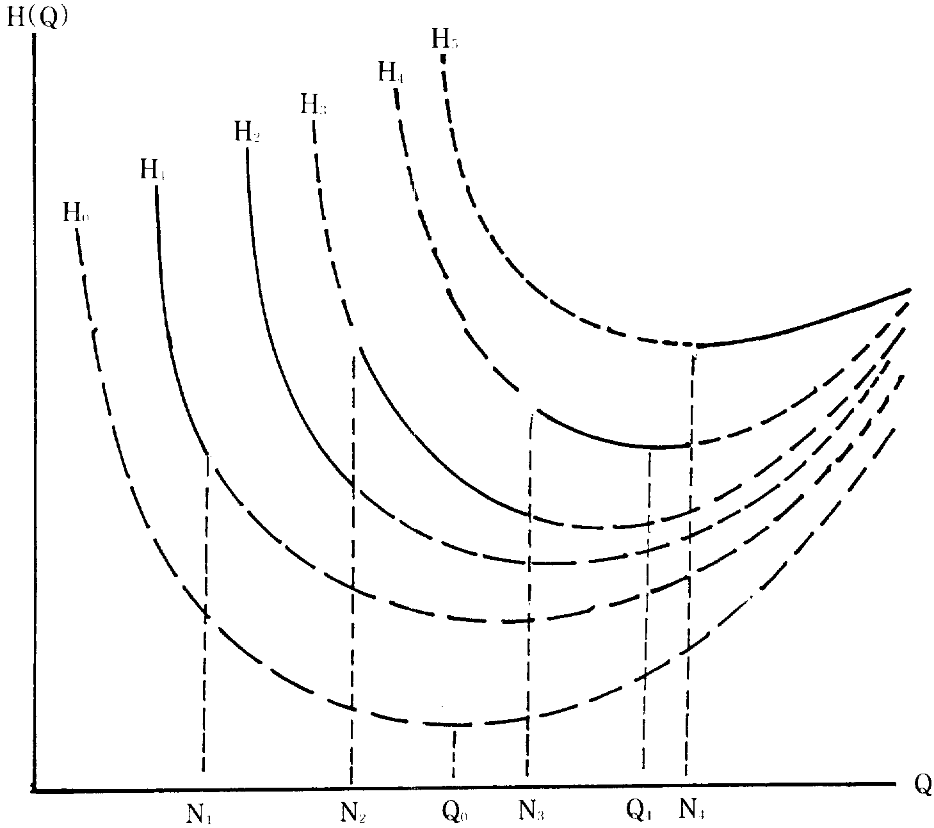

Figure 1. Illustration of a total cost curve

Proof of this property is in Appendix

4. Inducing the buyer by utilizing quantity discount

There may be many methods for the supplier to induce the buyer to increase order sizes. One of the most effective and common methods is quantity discount.

Two questions should be answered in introducing quantity discount. The first one is whether a given quantity discount system is capable of inducing the buyer to an order-size and price predetermined in the previous section. The second one is how the supplier can determine parameters of a quantity discount system.

Consider all-unit quantity discount with a single price break. Then, discount coefficient will have different value according to order-size, Q, as follows :

$$A = \begin{cases} 1 & \text{for } Q<B \\ R & \text{for } Q \geqq B \end{cases}$$

where $0<R<1$ and B is the price break point.

Curves of $E(1, Q)$ and $E(R, Q)$ are illustrated as in Fig. 2 and the optimal ordering quantity Q^* of the buyer will be one of Q_a, B and Q_b.

It can be shown $F(R, Q_b) < F(1, Q_a)$. Thus, the following property holds.

Property 2 : A supplier can entice a customer to increase the order size to Q by offering the all-unit quantity discount pricing schedule ($R=A$, $B=Q$) as long as both $F(A, Q) \geqq F(1, Q_a)$ and $E(A, Q) \leqq E(1, Q_a)$ hold.

5. A numerical example

Considering the problem with the following parameters :

$P = \$5$

$D = 2000$ units/year

$K = \$10$

$K_j = 8j(1-0.02(j-1))$

$N_0 = 0$

$N_j = 30j$ for $j = 1, 2, \cdots, 25$

$C_1 = 30$

$C_2 = 0.3$

Note that N_j and K_j satisfy (1). Applying the algorithm, we have $Q_a = 283$, $E(1, Q_a) = 10424$, $F(1, Q_a) = 9465$ and $H(1, Q_a) = 959$. Table 1 shows the optimal solutions for the cases $r=0$, 0.5, and 1. Notice that the joint cost, $H(A, Q)$, is improved significantly while maintaining a sharing ratio of the gain as prescribed. When the supplier entice the buyer by means of all-unit quantity discount, the terms of quantity discount becomes $R = A^*$ and $B = Q^*$.

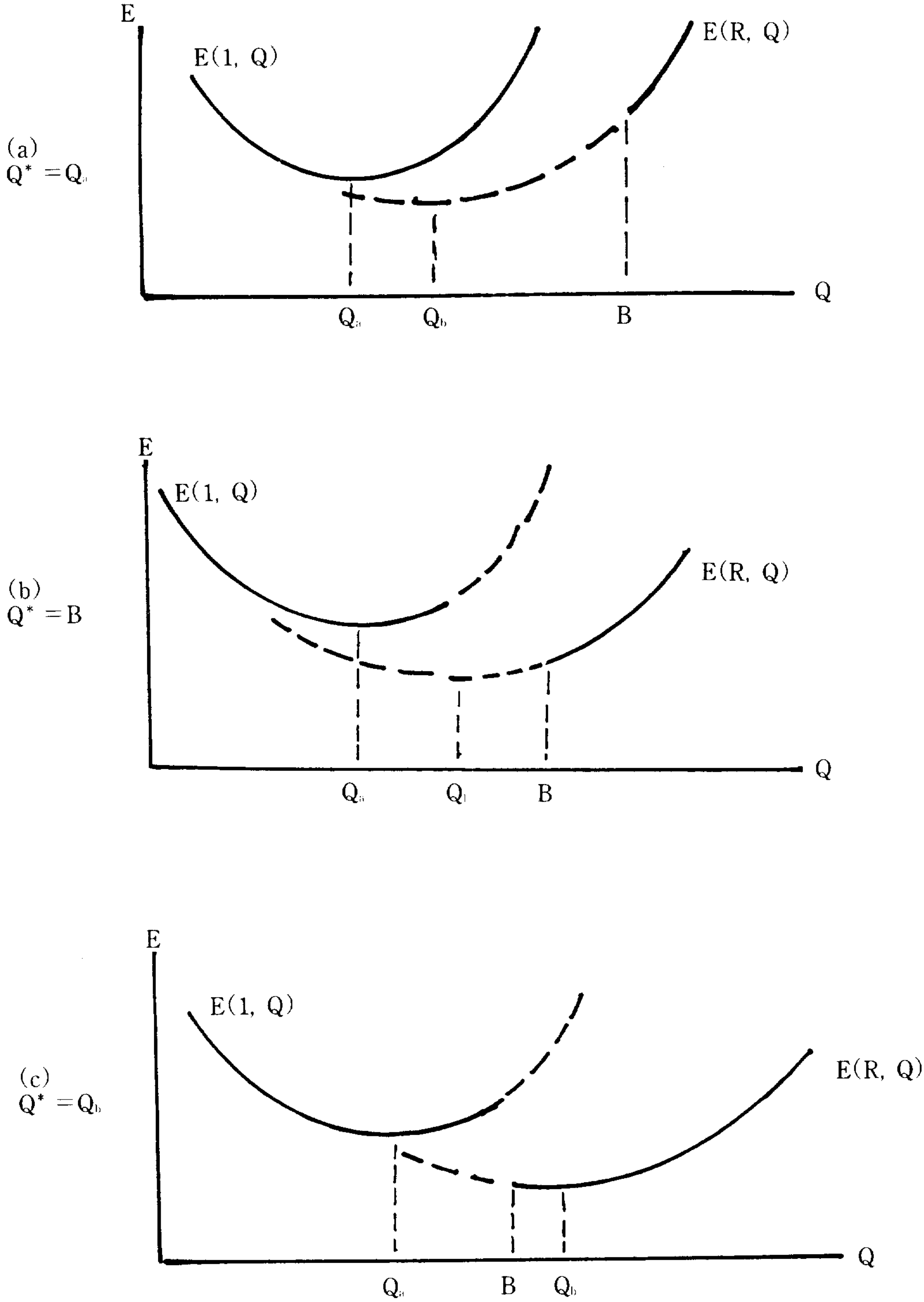

Figure 2. Costomer's cost curves and the optimal order quantity

Table 1 The optimal solutions for a numerical example

r	0	0.5	1
Q^*	480	450	450
A^*	0.988	0.992	0.995
$E(A^*,Q^*)$	10361	10393	10424
$F(A^*,Q^*)$	9465	9496	9527
$H(A^*,Q^*)$	896	897	898

APPENDIX

Proof of Property 1

If $Q_0 \leqq N_J$ in step 1, we will consider three possible regions for the order quantity Q : $0<Q\leqq N_i$, $N_i<Q\leqq N_{k-1}$, and $N_{k-1}<Q$, where i and k are found in step 1 and step 2.

The function $H_j(Q)=(C_1+K+K_j)D/Q+C_2PA(Q)Q/2$ is strongly unimodal in Q and has its minimum point at $Q=Q_j$ of (10). Note that $A(Q)$ represents equation (9). Let $H_0(Q)$ and Q_0 be $H_j(Q)$ and Q_j for $S(Q)=K$.

For the first region, $0<Q\leqq N_i$, note that $H_0(Q)$ is decreasing in Q for $0<Q\leqq N_i<Q_0$, where i is found in step 1. Thus for any $Q<N_i$, we have $N_{j-1}<Q\leqq N_j$ for some $1\leqq j\leqq i$ and

$$\begin{aligned} H(Q) &= H_j(Q) \\ &=(C_1+K+K_j)D/Q+C_2PA(Q)Q/2 \\ &=K_jD/Q+H_0(Q) \\ &>K_jD/Q+H_0(N_j) \\ &\geqq K_jD/N_j+H_0(N_j) \\ &\geqq K_iD/N_i+H_0(N_i) \\ &=H_i(N_i) \end{aligned} \quad \text{(A1)}$$

Hence, we see that any $0<Q<N_i$ cannot be an optimal solution. In other words, in the first region, $0<Q\leqq N_i$, only $Q=N_i$ is possible to be an optimal solution.

For the second region, $N_i<Q\leqq N_{k-1}$, note that $Q_j>N_j$ for any $i<j\leqq k-1$ by the definition of k. Thus, $H_j(Q)$ is a decreasing function of Q for $Q\leqq N_j$. And so, for any $N_i<Q\leqq N_{k-1}$, let $N_{j-1}<Q<N_j$, where $i<j\leqq k-1$, we have

$$H(Q)=H_j(Q)\geqq H_j(N_j)=H(N_j)$$

Hence, we see that in the second region, $N_i<Q\leqq N_{k-1}$, we need only consider $Q=N_{i+1}, \cdots, N_{k-1}$ as candidates for the optimal solution.

Now, consider the third region, $Q>N_{k-1}$. First note that $N_{k-1}<Q_k\leqq N_k$, where k is found in step 2. Note also that for any $1\leqq j\leqq J$, $H_j(Q)<H_{j+1}(Q)$ for all Q. Hence, we have

$$H(Q_k) = H_k(Q_k)$$
$$\leq H_k(Q) \text{ for all } Q$$
$$\leq H_j(Q) \text{ for all } j>k$$

Hence, if k>J, then in region 3 only Q_k can possibly be an optimal solution. Thus, the value of Q^* found in step 3 is the optimal quantity.

If k=J+1, then it is obvious that any $Q>N_J$ is not feasible solution. Hence the value of Q^* found in step 4 is the optimal solution.

Finally, if $Q_0>N_J$, then for any $0<Q<N_J$, we can show that $H(Q)>H(N_J)$ by the same argument as that of (A1). Hence the optimal order quantity is N_J. (Q.E.D.)

REFERENCES

1. Banerjee, A.," "A Joint Economic-lot-size Model for Purchaser and Vendor", Decision Science, Vol. 17, pp. 292-311, 1986.
2. Banerjee, A.," On "A Quantity Discount Pricing Model to Increase Vendor Profits", Management Science, Vol. 32, No. 11, pp. 1513-1517, 1986.
3. Crowther, J. F., "Rationale for Quantity Discounts", Harvard Business Review, pp. 121-127, Mar.-Apr., 1964.
4. Goyal, S. K., "Determination of a Supplier's Economic Ordering Policy", Journal of the Operaional Research Society, Vol. 38, No. 9, pp. 853-857, 1987.
5. Hwang, H. and K. H. Kim, "Supplier's Discount Policy with a Single Price-break Point", Engineering Costs and Production Economics, Vol. 10, pp. 279-286, 1986.
6. Kim, K. H. and H. Hwang, "An Incremental Discount Pricing Schedule with Multiple Customers and Single Price Break", European J. of Operational Research Vol. 35, pp. 71-79, 1988.
7. Lee, C. Y., "The Economic Order Quantity for Freight Discount Costs", IIE Transactions, pp. 318-320, September, 1986.
8. Lee, H. L. and M. J. Rosenblatt, "A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits", Management Science, Vol. 32, No. 9, pp. 1177-1185, 1986.
9. Monahan, J. P., "A Quantity Discount Pricing Model to Increase Vendor Profits", Management Science, Vol. 30, No. 6, pp. 720-726, 1984.
10. Rosenblatt, M. J. and H. L. Lee, "Improving Profitability with Quantity Discounts under Fixed Demand", IIE Transactions, Vol. 17, No. 4, pp. 388-395, 1985.