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# A Surface Panel Method for the Analysis of Hydrofoils with Emphasis on Local Flows around the Leading and Trailing Edges

by

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# 앞날 및 뒷날 유동 특성을 고려한 표면양력판 이론에 의한 2차원수중익 단면해석

이 진 태*

## Abstract

A basic formulation of the panel method, which is based on the potential field formulation, is reviewed for the case of two-dimensional hydrofoil problems. Numerical procedures to improve the computational efficiency of the panel method are suggested.

By investigating local behavior of the flow around the trailing edge, a *wedge type* Kutta condition is formulated. By subdividing the trailing edge panels, where dipole strengths of the subdivided panels follow the local behavior of the potential values of the flow outside a wedge, the circulation around a hydrofoil is calculated accurately with a relatively small number of panels. The subdividing technique to improve the accuracy of the numerical Kutta condition is proved to be efficient.

A local behavior of the flow around the leading edge is also investigated. By matching the flow around the leading edge with that around a parabola, a very accurate velocity distribution is obtained with relatively small number of panels. An accurate prediction of the stagnation point and the pressure distribution near the leading edge may contribute to improve the accuracy of cavity predictions and boundary layer calculations around hydrofoils.

## 요 약

2차원 날개 단면 주위 유동문제를 포텐셜장에서의 표면양력판이론에 의하여 해석하였고 수치해석 효율을 증대시키기 위한 방법을 제시하였다.

날개 뒷날에서의 유동이 쐐기 주위의 유동과 유사하다는 특성을 이용하여 계산효율을 증대시키기 위한 쐐기형 쿠타 조건(wedge type Kutta condition)을 제시하였다. 또한 쐐기형 쿠타조건의 계산 효율을 증대시키기 위하여 세부 분할방법을 적용하였다. 즉 날개 뒷날 부근의 4개 양력판을 세분하고 세분된 양력판에서의 다이폴세기는 쐐기 주위 유동특성을 따르게 하였다. 세부분할 방법에 의한

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쐐기형 쿠타조건을 2차원 날개단면 문제에 적용하였을 경우 수치계산 효율이 증가됨을 보였다.

날개 앞날에서의 유동은 앞날 반경(leading edge radius)을 갖는 원에 접하는 포물선(osculating parabola) 주위의 유동과 유사하다는 특성을 이용하여 비교적 적은 양력판 갯수에 의한 계산결과로부터 날개 앞날 주위의 유동을 정확히 계산하였다. 날개 앞날 주위의 급격한 유동변화를 정확히 계산함으로써 캐비테이션 발생 문제 및 날개 주위 경계층 문제를 계산하기 위한 정도 높은 입력자료를 제공할 수 있게 되었다.

## 1. Introduction

Panel methods have been in use for certain aerodynamic/hydrodynamic applications since Hess[2] proposed the surface source method in 1964. Since then, various other formulations have appeared which effer advantages in terms of accuracy, computational efficiency or versatility. [6], [5], [3] Although all the properly formulated panel methods are exact in the sense that the numerical solutions converge to the common solution as the number of panels is increased, this does not imply that all the panel methods are equally successful. Indeed, vast differences exist with respect to the prediction accuracy versus computational effort, reliability and simplicity. Recently a comprehensive study of the field of panel methods has been conducted by Lee[4] in order to determine the most suitable formulation for the application to marine propellers.

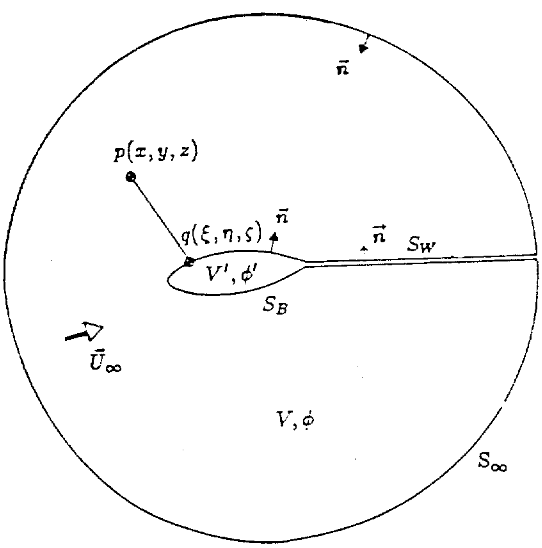


**Fig. 1** Notation for a general body for the application of Green's theorem.

In this paper basic formulations and numerical procedures of the panel method, which is based on the potential field formulation, are reviewed for the case of two-dimensional hydrofoil problems. A special attention is given to the local behavior of the flow near the trailing edge. Magnified flow near the trailing edge resembles the flow outside a wedge. Using the analytic potential expression for the wedge flow, a new numerical implementation of the Kutta condition, which include a correction term to the usual numerical Kutta condition first used by Morino[6], is suggested. (will be described in the subsequent section)

To improve the numerical efficiency of the panel method, a subdividing techniques is applied for the panels near the trailing edge. The four panels near the trailing edge are subdivided into subpanels whose potential values are matched to the local behavior of the wedge flow. As a result the global circulation around a hydrofoil can be calculated accurately with a relatively small number of panels.

The local behavior of surface velocity around the leading edge of a hydrofoil is also investigated, which is similar to that of surface velocity around the parabola osculating a circle with the same leading edge radius. By matching the flow around a leading edge with that around a parabola, pressure distribution around the leading edge can be calculated with great resolution even with small number of panels. The local stagnation point and the pressure peak value nearthe leading edge are calculated very accurately.

## 2. Basic theory

Consider a two dimensional fluid domain $V$ enclosed with boundary $S$, the unit normal vector $\vec{n}$ to $S$ being oriented into $V$, as shown in Fig. 1. The boundary $S$ is composed of the body surface $S_B$, the wake surface $S_W$, and the outer surface $S_\infty$ surrounding the body and wake surface. The body is subject to the inflow velocity $\vec{U}_\infty$. With the assumptions of the incompressible, inviscid fluid, and the irrotational flow in $V$, there exists a perturbation velocity potential $\phi$ which satisfies the Laplace equation,

$$\nabla^2\phi=0. \tag{1}$$

A boundary value problem can be constructed by specifying boundary conditions on the boundary $S$ as follows:

The kinematic boundary condition should be satisfied on the solid body surface $S_B$,

$$\frac{\partial\phi}{\partial n}=-\vec{U}_\infty\cdot\vec{n}. \tag{2}$$

The wake surface $S_W$ is assumed to have zero thickness and has a normal vector defined on the upper surface. The normal velocity jump $\Delta\frac{\partial\phi}{\partial n}$ and the pressure jump $\Delta p$ across $S_W$ is zero, while a jump in the potential, denoted as $\Delta\phi$, is allowed.

$$(\Delta p)_{\text{ on } S_W}\equiv p^+-p^-=0, \tag{3}$$

$$\left(\Delta\frac{\partial\phi}{\partial n}\right)_{\text{on } S_W}\equiv\left(\frac{\partial\phi}{\partial n}\right)^+-\left(\frac{\partial\phi}{\partial n}\right)^-=0, \tag{4}$$

Where the superscripts + and − denote the upper and lower wake surfaces, respectively.

A Kutta condition is required at the trailing edge to uniquely specify the circulation. In its most general form, it states that the flow velocity at the trailing edge remains bounded: i.e.,

$$|\nabla\phi|_{T.E.}<\infty \tag{5}$$

The Kutta condition can be implemented by introducing a cut from the trailing edge to the outer control surface $S_\infty$. By introducing the cut on the wake surface, where the actual cut geometry is not important for a steady case, the potential values around the body becomes unique. The potential jump $\Delta\phi$ across the wake surface is the same as the circulation, $\Gamma$, around the body, and is a constant on $S_W$.

$$(\Delta\phi)_{\text{on } S_W}=\phi^+-\phi^-=\Gamma \tag{6}$$

On the outer control surface $S_\infty$, the perturbation velocity due to the body should vanish in the limit where this surface is an infinite distance from the body.

$$\Delta\phi\rightarrow0,\ \ as\ S_\infty\rightarrow\infty \tag{7}$$

Green's theorem for the velocity potential on the body surface can be stated as (see Lee[4])

$$\frac{1}{2}\phi(p)=-\frac{1}{2\pi}\int_{S_B}\left[\phi(q)\frac{\partial}{\partial n_q}\log R(p,q)-\frac{\partial\phi}{\partial n_q}\log R(p,q)\right]ds-\frac{1}{2\pi}\int_{S_W}\Delta\phi(q)\frac{\partial}{\partial n_q}\log R(p,q)ds, \tag{8}$$

Where $\frac{1}{2\pi}\log R(p,q)$ is the induced velocity potential at field point $p$ due to a source of unit strength located at $q$, and $\frac{1}{2\pi}\frac{\partial}{\partial n_q}\log R(p,q)$ is that due to a doublet of unit strength.

The body geometry is replaced by an $N$-faced inscribed polygon, where $N$ is the number of panels, as shown in Fig. 2. A logical choice for the panel arrangement is a ***cosine*** spacing, where mean line ordinate and thickness are first evaluated at the following points along the nose-tail line,

$$x_j=\frac{c}{2}\left(1+\cos\left(\frac{2\pi(j-1)}{N}\right)\right), j=1,2,\cdots,N/2+1. \tag{9}$$

The panel boundaries are then obtained by adding and subtracting the half thickness of the section at right angles to the mean line. The nodes are numbered in clockwise order, starting at the lower trailing edge, and $j$-th panel, denoted as $C_j$, is one between nodes $j$ and $j+1$.

Singularity strength distribution is assumed to be piecewise constant over the panels. The collocation point, where the discretized integral equation is

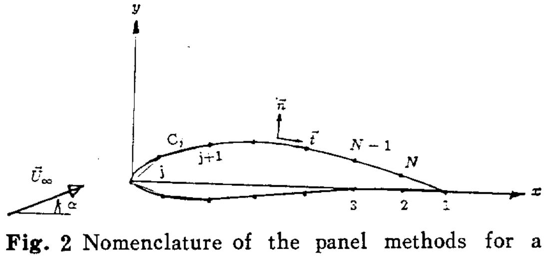


**Fig. 2** Nomenclature of the panel methods for a two-dimensional foil

satisfied, is selected as the midpoint of each panel. Then the discretized form of Equation 8 is

$$\sum_{j=1}^{N} D_{ij}\phi_j + W_i(\Delta\phi)_{wake} = \sum_{j=1}^{N} S_{ij}\left(\frac{\partial\phi}{\partial n}\right)_j \; i=1,2,\cdots,N \tag{10}$$

where

$$D_{ij} = \frac{1}{2\pi}\int_{C_j}\frac{\partial}{\partial n_q}\log R(p_i,q)ds, \text{ if } i\neq j$$

$$= \frac{1}{2}, \text{ if } i=j,$$

$$S_{ij} = \frac{1}{2\pi}\int_{C_j}\log R(p_i,q)ds,$$

$$W_i = \frac{1}{2\pi}\int_{S_W}\frac{\partial}{\partial n_q}\log R(p_i,q)ds.$$

At this point a numerical Kutta condition should be stated to uniquely specify the circulation. According to Morino[6], a Kutta condition may be written as the potential jump in the wake be equal to the difference of potential values of the upper and lower panels at the trailing edge,

$$(\Delta\phi)_{wake} = \phi_N - \phi_1. \tag{11}$$

This numerical implementation of the Kutta condition will be denoted as Morino's Kutta condition afterwards. In Section 3, a different numerical treatment of Kutta condition will be shown to be more efficient.

This results in a system of linear equations for the unknown potential values.

$$\sum_{j=1}^{N} A_{ij}\phi_j = b_i, \; i=1,2,\cdots,N \tag{12}$$

where $A_{ij} = D_{ij}$ , if $j\neq 1$ or $N$,
$= D_{ij} - W_i$ , if $j=1$,
$= D_{ij} + W_i$ , if $j=N$,

and $b_i \equiv \Sigma_{j=1}^{N} S_{ij}\left(\frac{\partial\phi}{\partial n}\right)_j$.

Solution of Equation 12 yields the values of potential on the panels, under the assumption that the potential is constant on each panel. Surface velocity is obtained by numerical differentiation of the potential. A quadratic polynomial to the values of potential at three panel midpoints is assumed, and the velocity at the panel midpoint is obtained by differentiating it with respect to the coordinate that is tangent to the panel. The arclength between two control points is approximated as the sum of half lengths of the adjacent panels.

Once the velocity is known, the pressure is calculated from Bernolli's equation, where the pressure coefficient is defined as

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2_\infty}.$$

Forces and moments are then obtained by summing the force and moment on each panel. An alternative lift coefficient, which is based on Kutta-Joukowsky's law, is calculated by

$$C_L = \frac{L}{\frac{1}{2}\rho U^2_\infty c} = 2\frac{(\Delta\phi)_{wake}}{U_\infty c},$$

and the drage coefficient should be zero.

## 3. Local flow near the trailing edge

To obtain accurate results by panel methods, one should increase the number of panels or use higher-order panel methods. higher-order panel methods use curved panels with singularity of higher-order variations along those. As a consequence, higher-order panel methods need more computing efforts to calculate the influencec oefficients, while the same accuracy can be obtained simply by increasing the number of panels for low order methods.

As the number of panels is increased to obtain an accurate result, it is observed that the global lift force can be predicted more accurately by using finer panel arrangement near the trailing edge for the same number of panels, which suggests modification of Morino's Kutta condition.

If one restricts one's attention to a hydrofoil with finite trailing edge angle, then the local flow at the

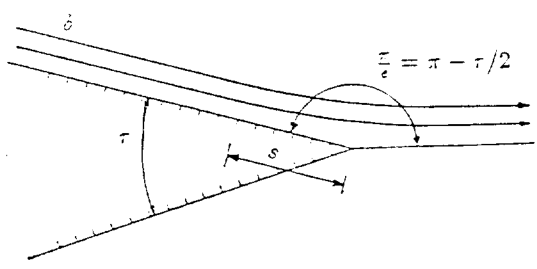

**Fig. 3** Analogy between the flow near trailing edge and the corner flow

trailing edge must resemble the flow about a wedge with an inside angle $\tau$ (see Fig. 3). Since the dividing streamline leaves the foil as a bisector for a steady two-dimensional hydrofoil, the flow outside behaves like a corner flow with an included angle, $\pi-\frac{\tau}{2}$.

Define the distance on the wall from the corner as $s$, then the total potential distribution along the wall of the corner flow can be expressed as,

$$\Phi(s)=bs^e=\phi(s)+\vec{U}_\infty\cdot\vec{s}-C \quad (13)$$

where $b$ is a surface velocity away from the corner, $1\leq e=\frac{\pi}{\pi-\tau/2}\leq 2$ and $C$ is an arbitrary constant.

By an analogy between the flow near the trailing edge and the corner flow, the local behaviors of the perturbation potential values at the upper and lower surfaces near the trailing edge can be expressed as

$$\phi^u(s)=\phi_u{}^w-a^u s+b^u s^e \quad (14)$$

$$\phi^l(s)=\phi_l{}^w-a^l s+b^l s^e, \quad (15)$$

where $a^u=\vec{U}_\infty\cdot\frac{\vec{s}^u}{|\vec{s}^u|}$, $a^l=\vec{U}_\infty\cdot\frac{\vec{s}^l}{|\vec{s}^l|}$ and $\phi_u{}^w, \phi_l{}^w$ are potential values at the upper and lower corner of the trailing edge, and correspond to the value of $C$ in Equation 13.

The (perturbation) potential values at the control points of the last four panels near trailing edge can be expressed as, (see Figure 4)

$$\phi_1=\phi_l{}^w-a^l s_1+b^l s_1{}^e,\quad \phi_2=\phi_l{}^w-a^l s_2+b^l s_2{}^e,$$

$$\phi_N=\phi_u{}^w-a^u s_N+b^u s_N{}^e,$$

$$\phi_{N-1}=\phi_u{}^w-a^u s_{N-1}+b^u s_N{}^e{}_{-1}. \quad (16)$$

Since four more unknowns ($\phi_l{}^w, b^l, \phi_u{}^w$, and $b^u$) have appeared in this formulation, the above four equations are sufficient to complete the matrix equation. This numerical implementation of Kutta condition will be denoted as *wedge type* Kutta condition. Morino's Kutta condition simply equate the potential values at the trailing edge panel to that on the wake surface, i.e.,

$$\phi_1=\phi_l{}^w,\quad \phi_N=\phi_u{}^w. \quad (17)$$

Fig. 5 and 6 show the perturbation potential and pressure distributions around a circular section at 90 degree angle of attack with different numerical implemenations of Kutta condition imposed at the trailing edge. Abscissa of Fig. 5 is an arclength around a foil surface, non-dimensionalized by the total arclength. ($s=0$ denotes the lower trailing edge) With Morino's Kutta condition, the circulation converges to about half the correct value regardless of the panel density. With the *wedge type* Kutta condition, the potential and pressure distributions recover the analytic results. However it should be noted that the difference between the results by these two Kutta conditions becomes very small for thin two-dimensional sections.

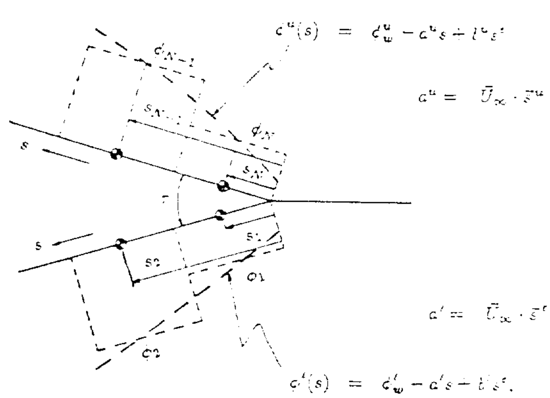

**Fig.** 4 Magnified view of the flow nea trailing edge

The previous example shows that a small discretization error near the trailing edge may influence the Kutta condition, and as a result, may result in a global error in predicting the circulation. In order to get a maximum accuracy for a given number of panels, minimization of the discretization errors near the trailing edge is essential. Since the geometry can be approximated very closely by straight panels near the trailing edge, most of the discretization errors come from the stepwise approximation of the unknown dipole strength.

To minimize the discretization errors due to the stepwise approximation of the dipole strength near the trailing edge, each of the last four panels near the trailing edge is subdivided into an odd number of panels and the dipole strength (hence the potential values) on each subdivided panel follows the behavior of a corner flow.

Fig. 7 shows the case where the last four panels are subdivided into twelve subpanels. Equation 10

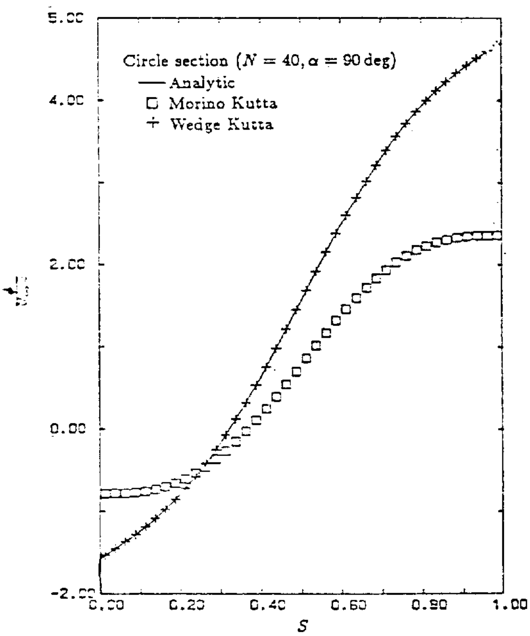


**Fig. 5** Perturbation potential distribution for a circular cylinder at 90 deg angle of attack

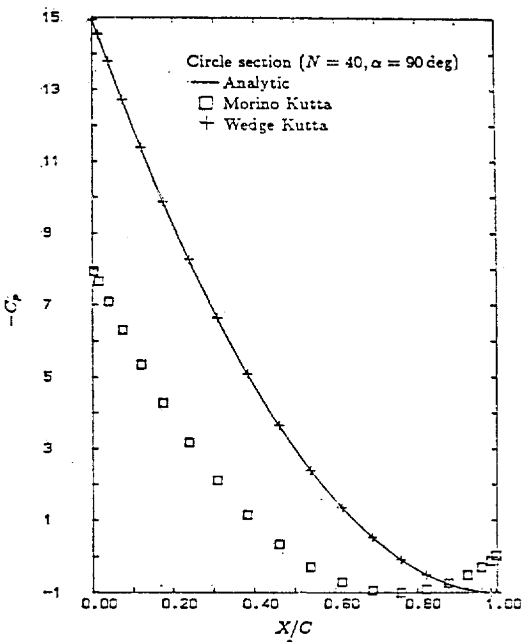


**Fig. 6** Pressure distribution for a circular cylinder at 90 deg angle of attack

for $i=1$ (i.e. for control point no. 1) becomes

$$D_{11}\phi_1+D_{12}\phi_2+\cdots+D_{1N-1}\phi_{N-1}+D_{1N}\phi_N + W_1(\phi_u{}^w-\phi_l{}^w)=b_1. \qquad (18)$$

Since the dipole influence coefficient due to the upper tailing edge panel becomes biggest for the control point in the lower trailing edge panel, $D_{1N-1}\phi_{N-1}$ and $D_{1N}\phi_N$ are replaced by

$$D_{1N}\phi_N=D_{1N}^1\ \phi_N^1+D_{1N}^2\ \phi_N^2+D_{1N}^3\ \phi_N^3$$

$$D_{1N-1}\phi_{N-1}=D_{1N-1}^1\ \phi_{N-1}^1+D_{1N-1}^2\ \phi_{N-1}^2 +D_{1N-1}^3\ \phi_{N-1}^3, \qquad (19)$$

where $\phi_N=\phi_N^2$ and $\phi_{N-1}=\phi_{N-1}^2$ and the superscripts denote the subpanel numbers. (see Fig. 7) Similar equations can be written for three other control points near the trailing edge. Since the dipole strengths of the subpanels are related to those of the main panels near the trailing edge through Equations 14 and 15, number of unknowns does not increase.

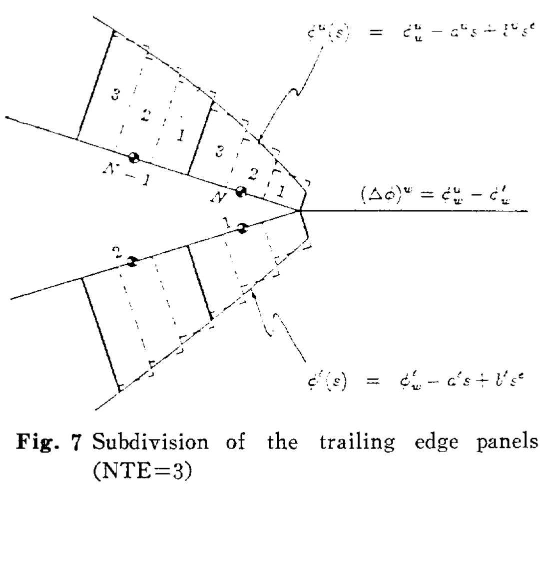


**Fig. 7** Subdivision of the trailing edge panels (NTE=3)

The calculated lift coefficients for a symmetrical Karman-Trefftz section, which has a thickness-chord ratio of 0.17 and a trailing edge angle of 27 degrees, at 5 degree angle of attack, are compared with the analytic lift coefficient in Table 1. The case of NTE=1 means the *wedge type* Kutta condition. As expected, fine discretization near the trailing edge dramatically reduces the error of calculated lift coefficient.

**Table 1** Effect of number of subdividing panels on the lift coefficient for a symmetric Karman-Trefftz section ($t/c=0.17$, $\tau=27$ deg, $\alpha=5$deg, $(C_L)_{anal}=0.3104$)

| NT | NTE | $C_L$ | $\frac{C_L-(C_L)_{anal}}{(C_L)_{anal}}$ |
|---|---|---|---|
| 20 | Morino | 0.287 | −7.4% |
| 20 | 1 | 0.295 | −4.9% |
| 20 | 3 | 0.303 | −2.3% |
| 20 | 9 | 0.305 | −1.8% |
| 40 | Morino | 0.298 | −4.0% |
| 40 | 1 | 0.302 | −2.8% |
| 40 | 3 | 0.306 | −1.5% |
| 40 | 9 | 0.306 | −1.4% |
| 80 | Morino | 0.303 | −2.4% |
| 80 | 1 | 0.306 | −1.4% |
| 80 | 5 | 0.308 | −1.3% |

## 4. Local flow around the leading edge

Detailed knowledge of the flow around the leading edge is very important since the location of stagnation point and pressure distribution near the leading edge are essential inputs for the calculation of boundary layers on hydrofoils. Moreover the pressure peak value is a guidance to decide whether cavitation occurs. One of the important merits of panel methods over vortex lattice methods is the ability to calculate the flow quantities around the leading edge.

Even though global circulation around a hydrofoil can be calculated with a relatively small number of panels within an engineering accuracy, a great number of panels are needed to obtain sufficient resolution to pickup the pressure peak value at the leading edge. A more efficient scheme to calculate the detailed flow quantities around the leading edge is to be devised. This can be achieved by investigating the local behaviour of the flow near the leading edge.

Magnifying the flow near the leading edge, it resembles a flow around a parabola which osculates a circle with leading edge radius, $\rho$(See Fig. 8).

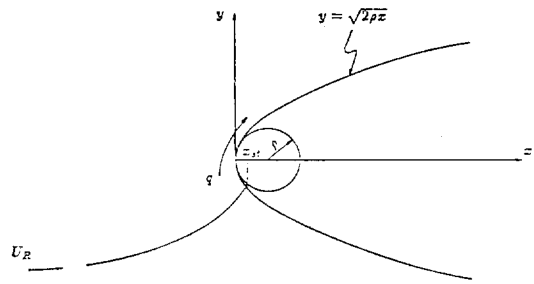


**Fig. 8** Magnified view of the flow around the leading edge

Equation of the parabola is

$$y=\sqrt{2\rho x}. \tag{20}$$

If a parameter $t$ is taken as

$$\frac{x}{\rho}=t^2,$$
$$y=\sqrt{2}\rho t, \tag{21}$$

then the incremental arclength of the foil surface can be expressed as

$$ds=\sqrt{dx^2+dy^2}=2\rho\sqrt{t^2+\frac{1}{2}}dt. \tag{22}$$

The flow outside the parabola has a dividing streamline and has a stagnation point at $x_{st}$ on the foil lower surface. Surface velocity, $q$, around the osculating parabola with stagnation point at $x_{st}$ can be written as [1]

$$\frac{q}{U_R}=\left(1\pm\sqrt{\frac{x_{st}}{x}}\right)\sqrt{\frac{x}{x+\frac{\rho}{2}}}=\sqrt{\frac{t+t_{st}}{t^2+\frac{1}{2}}}. \tag{23}$$

Total potential distribution on the surface of the parabola is expressed as

$$\Phi=\int\frac{\partial\Phi}{\partial s}\frac{ds}{dt}dt=U_R\int\sqrt{\frac{t+t_{st}}{t^2+\frac{1}{2}}}\cdot 2\rho\sqrt{t^2+\frac{1}{2}}dt$$
$$=2\rho U_R\left(\frac{1}{2}t^2+t_{st}t+C\right), \tag{24}$$

where $U_R$ is a reference inflow velocity in the local magnified flow near the leading edge, and $C$ is an integral constant.

Since the potential values are obtained after solving the simultaneous equation (Equation 12), three unknown flow parameters in Equation 24(i.e. $t_{st}$, $U_R$, $C$) can be derived from the obtained potential values at control points near the leading edge. The scheme adopted in this paer to obtain the three

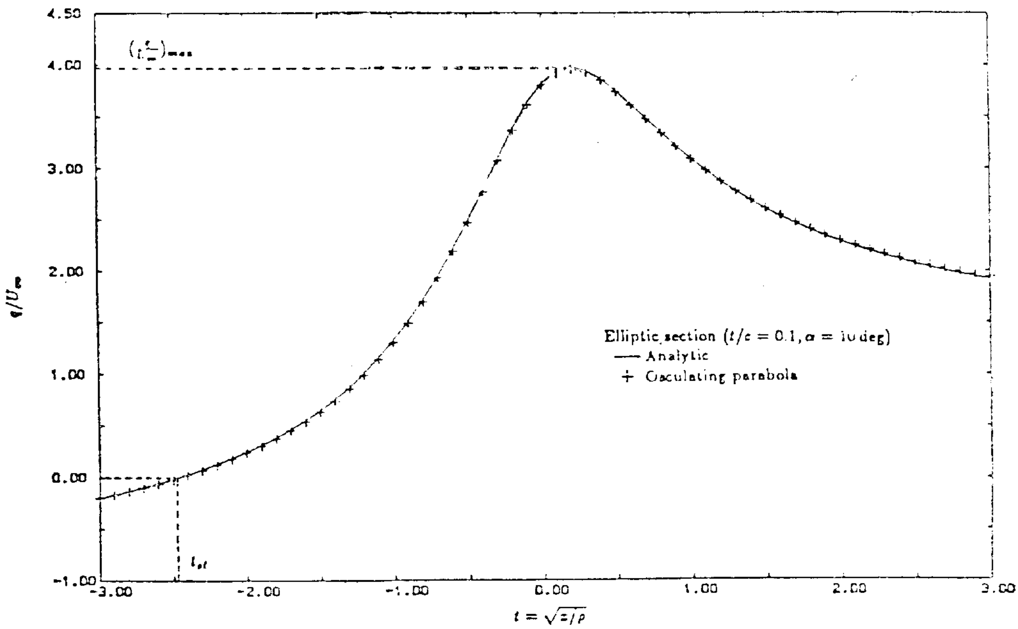


**Fig. 9** Comparition of surface velocities around a leading edge of an elliptic section($t/c$=0.1, $\alpha$=10deg)

unknown parameters is a least square method which uses the potential values at the control points of the four panels adjacent to the leading edge.

The effectiveness of Equation 24 is shown in Fig. 9, where surface velocity distributions near the leading edge of an elliptic section at 10 degree incidence, which has a thickness-chord ratio of 10 percent, are shown. The solid line is an analytic expression of the surface velocity and symbol+is the surface velocity from Equation 23. The three flow parameters are obtained by a least square fitting through the potential values at four control points near the leading edge, which are calculated with 40 panels. Note that the stagnation point and the velocity peak value are predicted very accurately.

Fig. 10 shows an example for a relatively thin cambered section which has a thickness-chord ratio of 4 percent and a camber-chord ratio of 2 percent. This NACA 66 (mod.) section is a typical hydrofoil section used in marine propellers at 0.7 radius. Since no analytic solution for this kind of section is available, a convergence test is carried out by increasing the number of panels. The result by 80 panels is indistinguishable from that by 160 panls. Even the result by 40 panels gives less than 7 percent error in the predicted velocity peak values.

## 5. Conclusions

A basic formulation of the potential based panel method is reviewed for the case of two-dimensional hydrofoil problems. Numerical procedures to improve the compuutational efficiency of the panel method are suggested.

By investigating local behavior of the flow around the trailing edge, a ***wedge type*** Kutta condition is formulated. The usual Morino's Kutta condition is found to have a fundamental error when the trailing edge angle is large, which is fixed by the ***wedge type*** Kutta condition. Numerical results for a circle section demonstrate the effectiveness of the ***wedge type*** Kutta condition for sections with a large trailing edge angle.

An efficient numerical procedure to improve the numerical accuracy is suggested. By subdividing the panels near the trailing edge, where dipole strengths of the subdivided panels follow the behavior of the potential values of the flow outside a

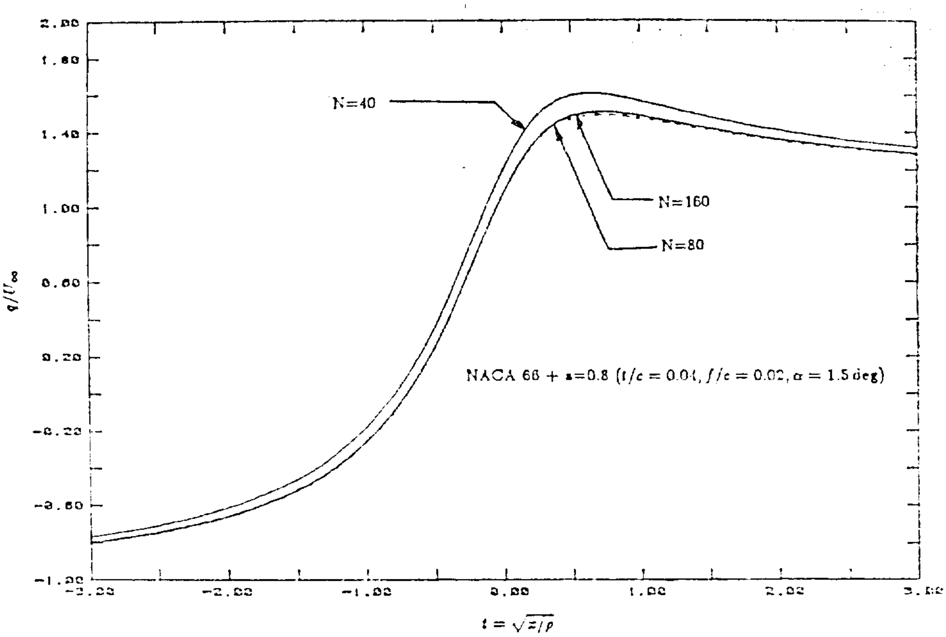


**Fig. 10** Comparison of surface velocities around a leading edge of a typical hydrofoil section (NACA 66 Mod.+a=0.8, $t/c=0.04$, $f/c=0.02$, $\alpha=1.5$deg)

wedge, the circulation around a hydrofoil is calculated accurately with relatively small number of panels. Numerical results for a Karman-Trefftz foil section are given to prove the effectiveness of the subdividing technique coupled with the *wedge type* Kutta condition.

A local behavior of the flow around the leading edge is also investigated. By matching the flow around the leading edge with that around a parabola, a very accurate velocity distribution is obtained with relatively small number of panels. The analytic and calculated velocity distributions around the leading edge is compared to show the effectiveness of the method for an elliptic section with 10 percent thickness-chord ratio. A convergence test is performed for a typical foil section with 4 percent thickness-chord ratio and 2 percent camber-chord ratio. Accurate prediction of the stagnation point and the pressure distribution near the leading edge may contribute to improve the accuracy of the boundary layer calculations around hydrofoils.

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