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A Corrective Maintenance Policy Which Determines Replacement or Repair for the Maintenance of System Failures+

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Abstract

This paper presents a corrective maintenance model to determine either type of maintenance actions upon failure of the system. Types of maintenance actions considered are minimal repair and replacement. Minimal repair cost is assumed to be random, whereas replacement cost is fixed.

A policy, B(t), which determines the type of maintenance action based on the estimated minimal repair cost when the system fails at time t is adopted. To obtain an optimal policy, an expected maintenance cost per unit time is derived and is minimized with respect to B(t).

1. Introduction

Determination of an optimal maintenance policy for systems that are subject to failure has been the subject of considerable investigation.

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The earlist work on optimal preventive replacement policy is done by Barlow and Hunter[1]. In [1], a system is replaced periodically at scheduled times, kT(k=1, 2, ...) and undergoes minimal repair at failures betweeen the periodic replacements. The system failure rate remains undisturbed by any repair of failures. Their models are generalized by Beichelt and Fischer[3] for the case of two types of failures. Morimula[6] and Park[9] propose policies based on the number of minimal repairs performed. Nakagawa and Kowada[8] and Muth[7] discuss replacement policies based both on the cumulative operating time and the number of minimal repairs performed. Menipaz[5] introduces a model which is applicable when maintenance costs change over time.

In this paper, a corrective maintenance model is presented to determine either type of maintenance actions upon failure of the system. Types of maintenance actions considered are minimal repair and replacement. Minimal repair cost is assumed to be random, whereas replacement cost is fixed. A policy, B(t), which determines the type of maintenance actions based on the estimated minimal repair cost when the sytem fails at time t is adopted. Under this policy, an expected maintenance cost per unit time is derived and is minimized with respect to B(t).

2. A Corrective Maintenance Model

2.1 Definition

Minimal repair	:	A maintenance action which restores the system to the failure rate it has when it failed; there is no change in system time. This is often called "bad as old".
Replacement	:	A maintenance action which restores the system time to zero; the failure rate curve is that of a new system. This is often called "good as new".
Minor failure	:	A failure whose maintenance requires minimal repair.
Catastrophic failure	:	A failure whose maintenance requires replacement.

2.2 Notation

t : time $t \geqq 0$

$B(t)$: a function of time which classifies maintenance actions into two groups. If minimal repair cost for a failure occurred at time t exceeds $B(t)$, replacement is undertaken. Otherwise minimal repair is undertaken.

$h(t)$: failure rate time t

$h_m(t)$: failure rate of minor failure at time t

$h_c(t)$: failure rate of catastrophic failure at time t

y : random variable for minimal repair cost

$f(y, t)$: pdf of minimal repair cost at time t. See fig 1

a : replacement cost

$P_c(b, t)$: probability that a failure occurred at time t is catastrophic under policy $B(t)$. See fig 1.

$C_{mc}(B, t)$: expected minimal repair cost when the failure occurred at time t is minor

$P_o(t)$: probability of no catastrophic failure during time interval $[o, t]$

$f_o(t)$: pdf of time to the first catastrophic failure

$E(c, B)$: expected maintenance cost per unit time under policy $B(t)$

2.3 Assumptions

1) The planning time horizon is infinite.
2) The number of failure at $[o, t]$ follows Nonhomogenous Poisson Process (NHPP) with parameter $\int_o^t h(x)\,dx$.

   This implies that failure rate at time t is $h(t)$ and successive failures are mutually stochastically independent.
3) Only replacements and minimal repairs are performed for failures.
4) Failure rate function, $h(t)$, is increasing.

2.4 Expected Cost Per Unit Time

To determine either type of maintenance actions upon a failure of a system at time t, a poiicy, B(t), is introduced. Here, a policy is a function of time which determines the type of maintenance actions based on the estimated minimal repair cost when the system fails at time t. If we adopt this policy, the following maintenance actions can be followed:

If minimal repair cost for a failure occurred at time t exceeds B(t), replacement is undertaken. Otherwise minimal repair is undertaken.

The policy is illustrated in fig 1. Under this policy, the expected maintenance cost per unit time can be derived.

Now, the probability that a failure occurred at time t will be catastrophic is

$$P_c(B,t) = 1 - \int_0^{B(t)} f(y,t)\,dy \tag{1}$$

So, the failure rates of minor failure and catastrophic failure are, respectively

$$h_m(t) = [1 - P_c(B,t)]\, h(t) \tag{2}$$

$$h_c(t) = P_c(B,t)\, h(t) \tag{3}$$

and the expected minimal repair cost when the failure occurred at time t is minor is

$$C_{mc}(B,t) = \frac{\int_0^{B(t)} yf(y,t)\,dy}{1 - P_c(B,t)} \tag{4}$$

Now, we formulate an expected maintenance cost per unit time when the first catastrophic failure occurs at time t. If the first catastrophic failure occurs at time t, the expected maintenance cost during the time interval [o, t] is

$$\int_0^t h_m(x)\, C_{mc}(B,x)\, dx + a \tag{5}$$

The first term of (5) represents minimal repair cost during the time interval [o, t] and the second term represents replacement cost for the catastrophic failure at time t. So the expected maintenance cost per unit time is

$$\frac{\int_0^t h_m(x)\ C_{mc}(x)\ dx + a}{t} \tag{6}$$

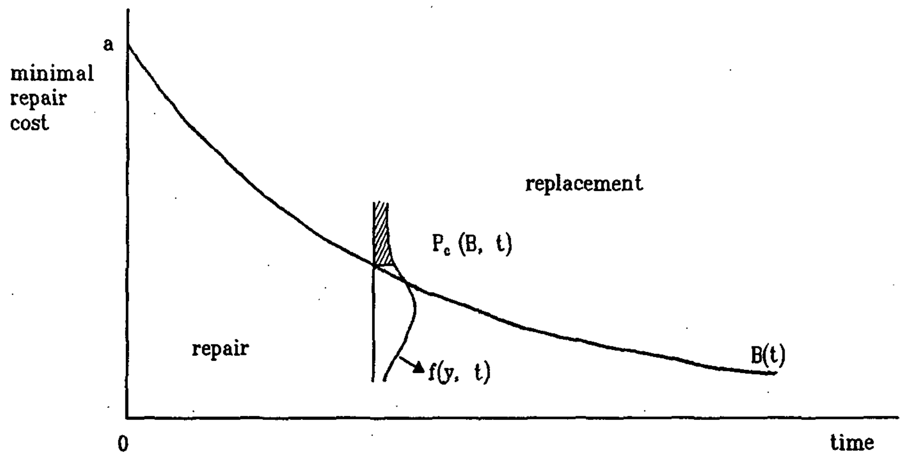

Fig 1. Maintenance Policy, B(t)

Now, we derive pdf of time to the first catastrophic failure. The probability of no catastrophic failure during time interval [o, t] is [2]

$$P_o(t) = \exp\left[-\int_0^t h_c(x)\ dx\right] \tag{7}$$

Then, the pdf of time to the first catastrophic failure is

$$\begin{aligned} f_o(t) &= -\frac{dP_o(t)}{dt} \\ &= h_c(t) \exp\left[-\int_0^t h_c(x)\ dx\right] \end{aligned} \tag{8}$$

Now, the expected maintenance cost per unit time under policy B(t) is

$$E(c,B) = \int_0^\infty f_o(t) \frac{\int_0^t h_m(x)\ C_{mc}(B,x)\ dx + a}{t}\ dt \tag{9}$$

If we substitute (2), (3), (4) and (8) into (9), we have

$$E(c,B) = \int_0^\infty \frac{1}{t} h(t)\ P_c(B,t)\ \exp\left[-\int_0^t h(x)\ P_c(B,x)\ dx\right] \cdot \left[\int_0^t h(x)\ C_e(B,x)\ dx + a\right] dt \tag{10}$$

where

$$\begin{aligned} C_e(B,x) &= \{1-P_c(B,x)\}\ C_{mc}(B,x) \\ &= \int_0^{B(x)} yf(y,t)\ dy \end{aligned} \tag{11}$$

Now, we wish to obtain the policy B(t) which minimizes the objective function given by (10).

This problem is turned out to be a function optimization problem. So, the necessary conditions of the optimum B(t) can be derived from the Pontryagin's maximum principle [4]. The conditions, however, consist of the differential equations, initial conditions, and terminal conditions. It is usually difficult to solve these differential equations and the boundary conditions. Therefore, an approximate solution procedure is considered, which transforms this function optimization problem into a point optimization problem.

3. Approximate Solution Procedure

If minimal repair cost for a failure occurred at time t exceeds B(t), replacement is undertaken, where replacement cost is a. Therefore, $0 \leqq B(t) \leqq a$ and $B(0) = a$. Since the failure rate function, h(t), is increasing, B(t) is decreasing in t. Such characteristics of B(t) can be summarized as follows:

$$B(0) = a \tag{12}$$

$$0 \leqq B(t) \leqq a \ \text{ for all } t \geqq 0 \tag{13}$$

$$\frac{dB(t)}{dt} < 0 \qquad \text{for all } t > 0 \tag{14}$$

and

$$\lim_{t \to \infty} B(t) = 0 \tag{15}$$

The shape of B(t) is illustrated in fig 1. Two such simple functions of B(t) which satisfy (12) − (15) are as follows :

$$B(t) = a \frac{s}{t+s} \tag{16}$$

$$B(t) = a \exp(-st), \quad s > 0 \tag{17}$$

If a specific function of B(t) such as (16) or (17) is substituted into (1) − (10), the objective function becomes a function of s. Then, the optimal value of s, s^*, can be found by using either an analytic or numerical method.

These solution procedures can be summarized as follows :

Step 1. Guess the shape of B(t) and determine a specific function of B(t) which satisfies (12) − (15).

Step 2. Calculate (1) − (8) by substituting B(t) obtained in step 1 into (1) − (8).

Step 3. Using step 2, calculate the objective function, E(c; B).
Then E(c, B) becomes a function of s.

Step 4. obtain the optimal value of s, s^*, by using either differentiation or numerical method.

4. Numerical Example

To illustrate an approximate solution procedure, an example is presented and solved by using steps 1−4.

Let

$$h(t) = t \tag{18}$$

$$a = 20000 \tag{19}$$

$$f(y,t) = \begin{cases} \frac{1}{20000}, & 0 < y < 20000 \\ 0 & \text{elsewhere} \end{cases} \tag{20}$$

From step 1,

$$B(t) = 20000 \frac{s}{t+s} \quad , s > 0 \tag{21}$$

From step 2 and section 2.4, the following terms are obtained:

$$P_c (B,t) = \frac{s}{t+s} \tag{22}$$

$$C_e (B,t) = 10000 \left(\frac{s}{t+s} \right)^2 \tag{23}$$

Also,

$$\int_0^t h(x) P_c (B,x) \, dx = \frac{1}{2}t^2 - st + s^2 \log \left(\frac{s+t}{s} \right) \tag{24}$$

and

$$\int_0^t h(x) \, C_e(B,x) \, dx = 10000 \, s^2 \left[\log \left(\frac{s+t}{s} \right) - \frac{-t}{s+t} \right] \tag{25}$$

From step 3, we have the expected maintenance cost per unit time as follows:

$$E(c,B) = \int_0^\infty 10000 \left(\frac{t}{s+t}\right) \exp \left[-\frac{1}{2}t^2 + st - s^2 \log \left(\frac{s+t}{s} \right) \right] \cdot \left[s^2 \left\{ \log \left(\frac{s+t}{s}\right) - \left(\frac{t}{s+t}\right) \right\} + 2 \right] dt \tag{26}$$

(22) is a funtion of s. From step 4, by numerical method we know that the value of s which minimizes (22) is 2.5 approximately. So from (21), trial policy B(t) is

$$B(t) = 20000 \frac{2.5}{t+2.5} \tag{27}$$

With s=2.5 the value of (22) can be shown to be 16483 approximately by numercial integration.

5. Concluding Remarks

In this paper the general problem of machine maintenance and replacemant has been examined by taking into account the inherent failures and repairs.

The objective function of corrective maintenance policy is presented, which can be modified under various assumptions. In the problem considered in this paper there are still some questions to be solved. For example exact solution procedures for the optimal solution should be studied further.

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