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The Solution of Vehicle Scheduling Problems with Multiple Objectives in a Probabilistic Environment

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Abstract

Vehicle Scheduling Problem (VSP) is a generic name given to a whole class of problems involving the visiting of "stations" by "vehicles," where a time is associated with each activity. The studies performed to date have the common feature of a single objective while satisfying a set of restrictions and known customer supplies or demands. However, VSPs may involve relevant multiple objectives and probabilistic supplies or demands at stations, creating multicriteria stochastic VSPs.

This paper proposes a heuristic algorithm based on goal programming approach to schedule the most satisfactory vehicle routes of a bicriteria VSP with probabilistic supplies at stations. The two relevant objectives are the minimization of the expected travel distance of vehicles and the minimization of the due time violation for collection service at stations by vehicles.

The algorithm developed consists of three major stages. In the first stage, an artificial capacity of vehicle is determined, on the basis of decision maker's subjective estimates. The second one clusters a set of stations into subsets by applying an efficient cluster method developed. In the third one, the stations in each subset are scheduled by applying an iterative goal programming heuristic procedure to each cluster.

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1. Introduction

Vehicle Scheduling Problem (VSP) is a generic name given to a whole class of problems involving the visiting of "stations" by "vehicles," where a time is associated with each activity. The objective is to build up a schedule of routes minimizing an objective such as the travel distance (cost or time) of vehicles, while meeting the restrictions given, such as vehicle capacity and / or travel distance. The operation in all VSPs may be one of collection, delivery, both collection and delivery, or one involving neither. The VSP is also referred to as time-constrained Vehicle Routing Problem (VRP).

Manifestations of this problem appear in many diverse sectors of the economy, in both the public and private sectors. In recent years, many researchers have been concerned not only with obtaining an optimal solution but also with developing practical and economical heuristic methods for VSP which is NP-complete.

The detailed survey about VSP compiled by Bodin et al. [1] contains over 200 references. The optimal seeking algorithms have been developed on the basis of the branch-and-bound procedure, dynamic programming, and integer programming. The algorithms have been proposed by Christofides, Mingozzi and Toth [2,3], Soumis [4], Kolen, Kan and Trienekens [5] and many other researchers. In recent years, many good heuristics have been developed following the works of Clarke and Wright [6] and Gillet and Miller [7]. However, no algorithm always showed a dominant performance in VSPs.

The studies performed to date have the common features of a single objective while satisfying a set of restrictions and known customer supplies or demands. However, the collection or delivery problems inherent in VSPs may not lend themselves to a model construction concerning only one objective and deterministic supplies or demands. They may involve relevant multiple objectives and probabilistic supplies or demands at stations, creating multicriteria stochastic VSP.

It is an important objective to minimize total travel distance (time) of vehicles in VSP. This objective can be understood as the minimization of total cost for vehicle operation since minimizing the travel distance of vehicles results in the minimization of time for operation and the minimization of the number of vehicles needed. The most researchers have adapted the distance objective in VSPs.

Stations to be serviced may have a preference on the service time by vehicles, duing to their production schedule or warehouse operation. Then each station can set a due time (or a specific time interval) by which collection or delivery service at stations must be completed. For example, a particular collection may be constrained to complete till 11 : 00 AM.

These timing constraints may arise in industrial refuse collection, scheduled mail pick-up and delivery, and many collection or delivery problems in which the facility is accessible only by a specified time. Minimizing the violation of due times for collection service at stations will improve the serviceability for customers.

It may not be practically easy to know an exact amount of supplies or demands at stations in advance of the actual vehicle operation, duing to the station's dynamic situation. Instead, the quantities at stations may be expressed by an appropriate probability distribution with a known mean and variance. For example, in VSP for the collection of raw materials, the station's supply capability may be changed dynamically depending upon its operation progress or inventory policy, and described by a specific beta distribution. The stochastic property of supplies or demands will bring about a serious uncertainty on the service of customers by vehicles.

This paper proposes a heuristic algorithm based on goal programming approach to schedule the most satisfactory vehicle routes of the bicriteria VSP with probabilistic supplies at stations.

2. Statement of Bicriteria Stochastic VSP Model

The specific bicriteria stochastic VSP model to be studied in this paper can be stated as follows:

The number of stations with probabilistic supplies at known locations are to be serviced exactly once by a set of vehicles with both capacity and distance restrictions, starting from a central depot and eventually returning to the depot through stations. The basic problem is to schedule a set of vehicle routes considering two relevant objectives which are the minimization of the expected total travel distance of vehicles and the minimization of total due time violation for collection service at stations by vehicles, while keeping the restrictions. The objectives are, more often than not, conflicting. Routes must be designed before the actual supplies at stations become known. An additional trip must be made if some stations cannot be serviced on a particular trip.

Conflicts arise because improvement in one objective can only be made to the detriment of the other objective.

The presences of nonlinearity in the objective function for the minimization of due time violation and of the stochastic property in the capacity constraint, in fact, make the problem considerably difficult to solve. Transformation of a stochastic VSP to a deterministic VSP will make it possible to solve the problem much easier. Because the complexity inherent in the problem is also dependent on the number of stations in the prospect, a set of stations needs to be partitioned into smaller subsets, that enables to apply a multi-objective decision making technique to each subset. The heuristic algorithm developed for the bicriteria stochastic VSPs consists of three major stages.

In the first stage, an artificial capacity of vehicle is determined on the basis of decision maker (DM)'s subjective allowances for two events that total vehicle load on a route is less than a specified quantity and a route cannot be serviced completely at a particular trip. Then the bicriteria stochastic VSP is transformed to a deterministic bicriteria VSP by viewing mean supplies at stations as true supplies and the artificial vehicle capacity asa true capacity.

The second stage of the algorithm clusters a set of stations into subsets by applying an efficient clustering procedure. Each subset ultimately comprises the stations for a single route. In the clustering procedure, a function is employed to choose stations to be clustered into subsets that uses information about the distances and the polar coordinate angles.

The third stage schedules the stations in each subset by applying a goal programming procedure iteratively to each cluster. This scheduling procedure has an important capability of taking into account the DM's preference regarding the goal priority structure and the target values of the goal constraints.

3. Transforming the Problem into a Deterministic Bicriteria VSP

If the supplies at stations q_i, are independent and identically distributed random variables with mean μ_i and standard deviation s_i, then by applying the central limit theorem, total route supply is given by

$$x = q_1 + q_2 + \cdots\cdots + q_k$$

and is approximately normally distributed with mean and standard deviation

$$\mu = \mu_1 + \mu_2 + \cdots\cdots + \mu_k$$

$$s = \sqrt{s_1^2 + s_2^2 + \cdots\cdots s_k^2}$$

where k represents a total number of stations on the route. It is pointed out that using a probability distribution for the supplies at stations should preclude the possibility of generating a negative supply for a station.

Consider two constraints of the form

$$\text{Prob}\{x \leq Qr \mid \mu\} \leq \alpha$$

and

$$\text{Prob}\{x \geq Q \mid \mu\} \leq \beta$$

where Q is a vehicle capacity, r is a ratio to be assigned by a DM, and α and β are subjective estimates of vehicle operation in terms of probability.

The first form expresses the personal allowance for the event that total vehicle load on a route is less than a specified quantity Qr. The parameter α can be considered as a producer's risk. The second one represents the personal allowance for the event that a route cannot be serviced completely at a particular trip. The parameter β can be considered as a consumer's risk. It is the probability that an additional trip is made to complete a service for a route.

A closed-form expression for the artificial capacity $\bar{a}$ can be obtained by treating the above two chance constraints as an equality each, solving the two equations simultaneously, and replacing μ by $\bar{a}$.

$$\bar{a} = \frac{Q\,(Z_{1-\beta}\, r - Z_\alpha)}{Z_{1-\beta} - Z_\alpha}$$

where Z is a unit normal variate.

The artificial capacity refers to a capacity to be used in a stochastic VSP in place of the true capacity. This artificial capacity will be less than the true capacity. Then routing is performed by viewing mean supplies at stations as true supplies and filling vehicles up to their artificial capacities.

The equation for $\bar{a}$ is applicable to all probability distributions, excepting the ones that generate negative supplies at stations-like the normal distribution. Golden and Yee [8] developed some expressions for $\bar{a}$, solely based on a subjective estimate of customer service in terms of the probability of route failure. However, the expressions do not exist for many probability distributions. This is because of the requirement that the variance of a probability distribution for the supplies should be expressed as a function of its mean.

4. Clustering Procedure

A cluster method was developed for grouping a set of stations. The technique is based on the heuristics of Gillet and Miller [], Clarke and Wright [6], and Williams [9].

The clustering procedure starts with an unassigned station at an extreme point in the area in order to form the beginning of a feasible link. A feasible link is a route of one or more stations which does not violate the restrictions of the vehicle travel distance and capacity. The link has two ends to which stations can be assigned. The ends represent two stations newly assigned to this link and temporarily connected to the depot. At the beginning of the feasible link, only the end that is the farthest station from the depot exists.

In the procedure, each end of the link pseudo-assigns (temporarily assigns) the closest two feasible stations. A station under competition from two different ends is pseudo-assigned to the closer end. The losing end pseudo-assigns the next closest feasible station. The total number of pseudo-assigned stations may be less than four, depending upon the number of feasible stations.

A function CR(i) is then computed for the pseudo-assigned stations and the station with the highest value of CR(i) is added to the link. This adds a station that is far from the depot in terms of distance, and also close to the end of the link in terms of the polar coordinate angle. After the addition, the remaining pseudo-assigned stations are released from their respective ends.

The function CR(i) developed by Park and Jun [10] is as follow:

$$CR(i)=w\frac{d_{0i}}{d_{0m}}+(1-w)\ (1-\frac{|\theta_{ij}|}{\pi})$$

where

d_{ij}=distance between stations i and j,

0=central depot,

m=farthest station from the depot,

w=shape parameter,

θ_{ij}=angular difference between the end j and its pseudo-assigned station i.

In the function, the shape parameter w represents a weight of distance relative to angular difference.When w is close to zero, emphasis is placed on the polar coordinate angle of the station. This is the basic concept of Gillet and Miller's SWEEP algorithm. On the other hand, a large w emphasizes the distance from a depot to a station. This is the concept of Clarke and Wright's savings method. A simple simulation may be used to find the best w for a minimum number of routes (required vehicles) simply by altering w.

The travel distance of the link, for the purpose of the feasibility test, is determined by computing the distance increase when a station is assigned to the link. If this tentative travel distance of the link is D_t, then, new D_t=old $D_t+(d_{ji}+d_{0i}-d_{0j})$.

The total load of the link is updated simply by adding the service quantity of the newly assigned station.

This process is continued until no feasible station can be assigned to the link. A cluster is then formed. The whole process is repeated until all stations have been assigned to the links. The solution is the set of created subsets. This cluster method does not consider the service timing preferences of stations in its procedure.

The procedural steps can be summarized as follows.

Step 1 : Initialization.

(1) Evaluate the polar coordinates for stations with the depot.

(2) Construct a symmetrical distance matrix which gives the distance of stations from one another.

Step 2 : Search for the farthest unassigned station from the depot to form a new feasible link

Step 3 : Pseudo-assign the closest two feasible stations to the farthest station.

(1) If no feasible station exists, go to Step 5.

(2) Otherwise, compute CR(i) for the stations and assign the station with the highest value of CR(i) to the link. The link now has two ends.

Step 4 : Pseudo-assign the closest two feasible stations to each end of the link.

(1) If no feasible station exists, go to Step 5.

(2) Otherwise, compute CR(i) for the station(s) and assign the station with the highest value of CR(i) to the link. Repeat Step 4.

Step 5 : Form a cluster. The completed subset is part of the final solution in the clustering stage and need not be considered during further clustering procedure.

Step 6 : Go to Step 2 if there are unassigned station(s). Otherwise, stop.

5. Scheduling Procedure

The goal programming (GP) approach is utilized to determine a vehicle schedule for each subset clustered. The GP approach allows consideration of the desired goals while permitting multiple conflicting objectives. It is also useful in evaluating different strategies under various assumed goal levels and / or goal priority structures. The notation used to describe the scheduling procedure of the algorithm is listed as follows:

S=set of stations in a route, including a central depot.

N=number of stations in a route, excluding a central depot.

T_d=target value of the travel distance on a route.

T_u=target value of the due time violation for service on a route.

U_i=due time for service at station i.

5.1 Initial Development of an Exact GP Model

Model objective and their priorities

Two objectives and their priorities are shown in Table 1. In the table, W_1 and W_2

represent the preemptive priorities or weights of the two objectives.

Table 1. Priority Structures of Two Models

Objectives	Model I	Model II
Minimization of total travel distance	W_1	W_2
Minimization of total due time violation	W_2	W_1

Decision variables

The decision variables $x_{ij}=1$ if the vehicle visits station j immediately after visiting station i; otherwise $x_{ij}=0$.

Model constraints

The GP model usually has two types of constraints, i.e. system and goal constraints.

(1) Only one station must immediately follow station i in a given route. The system constraints are:

$$\sum_{\substack{j \in S \\ j \neq i}} x_{ij} + n_{(1)} - p_{(1)} = 1, \text{ for } i \in S.$$

These constraints can be achieved by minimizing both negative ($n_{(1)}$) and positive ($p_{(1)}$) deviations for each station i.

(2) Only one station must immediately precede station j in a given route. The system constraints are:

$$\sum_{\substack{i \in S \\ i \neq j}} x_{ij} + n_{(2)} - p_{(2)} = 1, \text{ for } j \in S.$$

These constraints can be achieved by minimizing both $n_{(2)}$ and $p_{(2)}$ for each station j.

(3) A constraint must be imposed to ensure that a selection of x_{ij} actually represents a feasible, complete route without subtours. To accomplish this task, N additional variables, y_i, are defined. The system constraints are:

$$y_i - y_i + (N+1)\ x_{ij} + n_{(3)} - p_{(3)} = N,$$

for i, $j \in S$, $i \neq j$, and i, $j \neq 0$

where y_i, $i=1,2,\ldots,N$, are arbitrary real numbers.

The desired results can be achieved by minimizing $p_{(3)}$ from the constraints.

(4) One objective of the VSP is the minimization of the total distance traveled by vehicles. Its target value is set with the consideration of the legal or contractual condition, goods deterioration, maintenance, and / or production schedule. This goal constraint can be expressed by

$$\sum_{i \in S} \sum_{\substack{j \in S \\ j \neq i}} d_{ij} x_{ij} + n_{(4)} - p_{(4)} = T_d.$$

The minimiation of total travel distance can be achieved by assuming the bound as zero and minimizing $p_{(4)}$.

(5) Another objective of the VSP is the minimization of total due time violation for collection service at stations by vehicles. Its target value is set with the consideration of customer service. The due time for service at station is expressed by distance unit. The goal constraint can be expressed by

$$\max\{\sum_{\substack{j\in S\\ j\neq 0}} (d_{0j}-u_j)\, x_{0j},\ 0\} + \max\{\sum_{\substack{j\in S\\ j\neq 0}} \sum_{\substack{l\in S\\ l\neq j\\ l\neq 0}} (d_{0j}+d_{jl}-u_l)\, x_{0j}\, x_{jl},\ 0\} + \cdots \max\{\sum_{\substack{j\in S\\ j\neq 0}} \sum_{\substack{l\in S\\ l\neq j\\ l\neq 0}} \cdots \sum_{q} \sum_{\substack{r\in S\\ r\neq q\neq\cdots\neq l\neq j\\ r\neq 0}} (d_{0j}+d_{jl}+\cdots\cdots+d_{qr}-u_r)\, x_{0j}\, x_{jl}\cdots\cdots x_{qr},\ 0\} + n_{(5)} - p_{(5)} = T_u.$$

The minimization of total due time violation for service at stations can be achieved by assuming the bound as zero and minimizing $p_{(5)}$.

(6) Since the decision variables require 0 or 1 integar values, the system constraints for integrality have to be provided. This is accomplished by minimizing $p_{(6)}$ from the system constraints

$$x_{ij} + n_{(6)} - p_{(6)} = 1, \text{ for } i,\ j \in S \text{ and } i \neq j.$$

However, these constraints may not be expressed explicitly in the GP model when a computer code for integer programming is utilized.

Achievement function

The achievement function of the GP model includes minimizing deviation(s), either negative or positive, or both, from a set of goals, with certain preemptive priority weights assigned by the DM. However, a primal priority should be given to the first three system constraints, because those are the basic constraints for defining the VSP. The remaining three goal constraints may be assigned certain preemptive priorities by the DM. The achievement functions for the two models are formulated as follows:
For Model I,

$$\min.\ W_1[n_{(1)}+p_{(1)}+n_{(2)}+p_{(2)}+p_{(3)}] + W_2[p_{(4)}] + W_3[p_{(5)}].$$

For Model II,

$$\min.\ W_1[n_{(1)}+p_{(1)}+n_{(2)}+p_{(2)}+p_{(3)}] + W_2[p_{(5)}] + W_3[p_{(4)}].$$

5.2 Heuristic Procedure

The GP formulation for an exact solution has a serious computational difficulty in its application due to constraint (5). That is, the GP model is a nonlinear mixed-integer GP for which no efficient and practical solution procedure has been developed. Though a nonlinear mixed-integer GP may be at least theoretically solved by transforming it into a linear mixed integer GP, its size increases rather dramatically and quickly gets out of hand [11]. Furthermore, for constraint (5), the number of possible partial routes to be enumerated is greatly increased as the number of stations is increased, which requires tremendous effort in formulating the constraints.

To overcome such problems, an iterative procedure with GP applications was developed. This heuristic procedure is based on the theory that, in a single machine scheduling problem, the minimization of total due time violation of jobs may be achieved by the earliest due time rule [12]. In the VSP the service by a vehicle can be considered as the process by a machine, and the sum of the travel time of a vehicle from the immediately previous station visited to the next station to visit and the actual service time at the next station can be considered as the job processing time.

With each iteration in the algorithm, the next station to service is determined by solving a linear mixed-integer GP model constructed on the basis of the known schedule of the stations determined at previous iterations, instead of generating a complete route schedule at one time as in the exact GP method. The constraint (5) in the exact GP model is simplified to a linear 0–1 integer GP constraint for determining a station with the earliest due time among the stations unscheduled. Consequently, the GP model can be solved in practice without the tremendous effort otherwise required.

The procedure is repeated until a complete route schedule is obtained. The schedule obtained at the last iteration is considered the most satisfactory solution. The steps of the procedure can be stated as follows:

Let [k] be the kth station to be visited on a route and [0] be the central depot.

Step 1. Let k=0.

Setp 2. Solve the following GP model with the achievement function based on the DM's preference on the goal priority structure.

For Model I:

$$\min.\ W_1[n_{(1)}+p_{(1)}+n_{(2)}+p_{(2)}+p_{(3)}]+W_2[p_{(4)}]+W_3[p_{(5)}].$$

For Model II:

$$\min.\ W_1[n_{(1)}+p_{(1)}+n_{(2)}+p_{(2)}+p_{(3)}]+W_2[p_{(5)}]+W_3[p_{(4)}].$$

subject to

$$\sum_{\substack{j\in S\\ j\neq i}} x_{ij}+n_{(1)}-p_{(1)}=1,\ \text{for } i\in S;$$

$$\sum_{\substack{i\in S\\ i\neq j}} x_{ij}+n_{(2)}-p_{(2)}=1,\ \text{for } j\in S;$$

$$y_i-y_j+(N+1)\ x_{ij}+n_{(3)}-p_{(3)}=N,\ \text{for } i,j\in S,\ i\neq j,\ \text{and } i,j\neq 0;$$

$$\sum_{i\in S}\sum_{\substack{j\in S\\ j\neq i}} d_{ij}\ x_{ij}+n_{(4)}-p_{(4)}=T_d;$$

$\sum_{\substack{j \in s \\ j \neq 0 \\ j \neq k}} u_j \; x_{[k]j} + n_{(5)} - p_{(5)} = 0;$

$x_{ij} = 0$ or 1, for $i,j \in S$ and $i \neq j$;

$n_{(i)}, p_{(i)} \geq 0$, for $i=1,2,3,4,5$.

Setp 3. Let $k=k+1$.

Step 4. If $k=N$, accept the current route sequence as the most satisfactory solution and stop. Otherwise, (i) [k] is determined and (ii) let $x_{[k-1][k]}=1$.

Step 5. Change one of the system constraints according to the following principle: $x_{[k-1][k]} =1$, thus the achievement function should minimize both n and p from the corresponding constraint. Solve the newly defined GP model and go to Step 3.

In applying the heuristic procedure to each subset, a total of N^2+N+4 model constraints, with a total of N^2+2N decision variables, should be formulated at each iteration. However, the effort of constraint formulation is actually limited to the first iteration only, because for the remaining iterations until termination only very slight changes are required in the GP model. Once the GP model is formulated at each iteration, it can be solved using mixed-integer GP [13].

6. EXAMPLE PROBLEM

The algorithm proposed for the bicriteria stochastic VSPs is illustrated by a simple example problem, The problem involves a single depot and six stations to serve by vehicles, In Figure 1, the rectangular coordinates of the stations and depot are expressed on the right side of the corresponding node denoted by the number inside each circle. It is assumed that the supplies at stations follow a beta distribution with a known mean each. The distances between stations, and the mean supplies and the due times for service at stations are given in Table 2. In the table the due times are expressed by the travel distance of vehicle. The following conditions are given :

(i) The maximum allowable vehicle travel distance is limited to 190 units.

(ii) There are 200-unit capacity vehicles available.

(iii) α and β are 10% and 5% each.

The proposed algorithm is now applied to determine the most satisfactory vehicle schedules of the example problem. With $\alpha=0.1$, $\beta=0.05$, $Z_\alpha \doteqdot 1.28$, $Z_{1-\beta} \doteqdot 1.64$, and $r=0.5$, the artificial capacity of vehicle $\bar{a}$ is computed as 144 units. Assuming $w=0.2$, two subsets {3,2,4,1} and {6,5} are obtained in the clustering stage. In the scheduling stage, the target value of vehicle travel distance is reasonably determined by adding 20 units to the minimal travel distance of each route.

Table 3 shows the results of the example problem for the two models with different goal priority structures. As one would expect, the outcomes for the models differ. It shows that the proposed algorithm successfully performs the trade-off between the achievement levels of the two objectives, based on the DM's goal priority sturcture.

7. CONCLUSION

A heuristic algorithm based on goal programming approach was proposed to determine the most satisfactory vehicle schedules for bicriteria VSPs with probabilistic supplies at stations. The two relevant objectives are the minimization of the expected total travel distance of vehicles and the minimization of total due time violation for collection service at stations by vehicles. The algorithm consists of three major stages ; transforming procedure into the deterministic bicriteria VSP, clustering procedure for stations, and scheduling procedure the subsets of stations.

The algorithm has three important features ; First, the transforming technique is valid to all probability distributions only with the information of mean supplies at stations, excepting the ones that generate negative supplies at stations. Second, the clustering method can manipulate simply weights on angle and distance of stations. Third, the scheduling procedure has the capability of taking into account the DM's preferences with respect to the goal priority structure and the tatget values of the goal constraints.

It is noted that the approach applied in the heuristic algorithm could be extended to include any number of possible objectives that would make the model more realistic and adaptable. Finally, the development of an interactive procedure is suggested, which implements the proposed algorithm in order to reach the most favorable vehicle schedules from the DM's viewpoint.

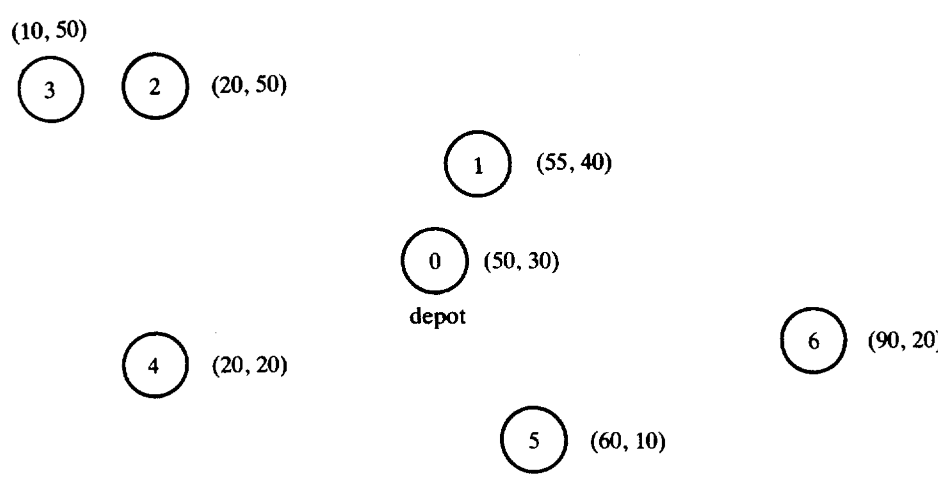

Figure 1. Graphical Configuration of a Depot and Stations in Example Problem

Table 2. Data for Example Problem

distance matrix

	0	1	2	3	4	5	6
0	–	11	36	44	31	22	41
1	11	–	36	46	40	30	40
2	36	36	–	10	30	56	76
3	44	46	10	–	31	64	85
4	31	40	30	31	–	41	70
5	22	30	56	64	41	–	31
6	41	40	76	85	70	31	–

	mean supplys	due times for service
0	—	—
1	15	100
2	80	80
3	30	140
4	70	30
5	85	20
6	25	50

Table 3. Computational Results of Example Problem

Model No.	I	II
Vehicle Schedule	0–4–2–3–1–0 0–5–6–0	0–4–2–1–3–0 0–5–6–0
Total Travel Distance of Vehicles	222	281
Total Due Time Violation	23	9

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