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How to Analyze an Unreliable Two-Machine, Two-Stage Transfer Line with Interstage Buffer+

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Abstract

This paper discusses an analytical method to derive the efficiency and the average storage level of an unreliable two-machine, two-stage automatic transfer line with interstage buffer storage. Extensive numerical experiments and simulations are also conducted to see the behavior of the system under the wide range of parameter values as well as various distributions.

1. Introduction

In recent years, as computer technology has been developing very rapidly, the automation of maufacturing systems is one of the most active research topics in many industries. Many benefits can be derived from the automation of manufacturing systems. However, the automated systems have disadvantageous aspects, too. The most important one is to require large investments for the installation of new systems or for upgrading the existing manual or semi-automated systems. Another disadvantage is that there is a strong interdependency between the stages of a system. Since the failure of one stage causes the failure of whole system, the system efficiency can be low in spite of large investments. Therefore, economic analysis should be carried out for automation before implementation.

At the same time, we have to explore the possible way to improve the efficiency of automated manufacturing system. One way to compensate for the failure of one stage is to provide an alternate machine which will be brought into use when the main machine fail especially, at bottleneck stage. That is, control is switched over to the backup or idle machine as soon as the other machine, which is running, fails. Another possible way to imporve the

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efficiency is to reduce the interdependency between the stages by providing buffer storages between the stages.

Many researches have been conducted for the analysis of manufacturing systems with inter-stage buffer. Buzacott[1], Okamura and Yamashina[9], Ignall and Silver[6], Gershwin and Schick[4] derived analytical solutions by modelling the system as a discrete time Markov process. But discrete time approach has a restriction that it cannot be applied to the analysis of the systems with different production rates, which is common in real world. Considering above restriction, we take a continuous time Markov process for the analysis of the system. Malathronas et. al.[8] and Wijngaard[10] derived an analytical solution with this continuous time approach.

Meantime, few researches has been done on how the alternate machine can improve the efficiency of the system. Fox and Zerbe[2] discussed the case where a stage consists of two identical machines, alternatingly acting as spares for each other.

In this paper, the analytical method is developed to derive the efficiency and the average storage level of an unreliable two-stage automatic transfer line, which consists of main and alternate machines in each stage and the interstage buffer storage.

2. Description of the Problem

Consider two-stage automatic transfer line shown in Fig.1. This system has two stages in series separated by inter-stage buffer storage with finite capacity.

Each stage consists of two machines(main and alternate machines). Each machine has own unique production rate and is subject to failure. It is assumed that uptime and downtime of each machine are exponentially distributed with known parameters. Also, main and alternate machines in each stage are assumed to have equal production rates.

Raw units come from outside the system and are processed at the first stage, then move to buffer, and pass the second stage following the completion of process. There is an infinite supply of raw units as well as an infinite demand for completed units.

In each stage, the main machine operates unitl it fails. If it fails, the alternate machine is brought to operate if it is up. If the alternate fails again, the main machine takes over if it is repaired already. This alternating process is repeated continuously. It is also assumed that there are enough repair facilities. Fox and Zerbe[2] assumed that only one repair facility is available. In this system, if the second stages goes down while the first stage is processing units, the storage level increases. Eventually, buffer becomes full, and the first stage cannot process the units any further because no space is available for processed units. We call this "blocking". Similarly, the second stage will be starved when no units are available in buffer due to failure of the first stage. We call this "starvation". We assume that the idle(blocked or starved) machine never fails. Also, when buffer is empty, production at the second stage can be partially restricted by the first stage in case production rate of the second stage is larger than that of the first stage. Similarly, production at the first stage may be partially restricted when the first stage can run faster than the second stage. Obviously, increasing buffer capacity results better efficiency at the cost of capital investment for buffer. Therefore, we should always take into account trade-off between the system efficiency and buffer capacity.

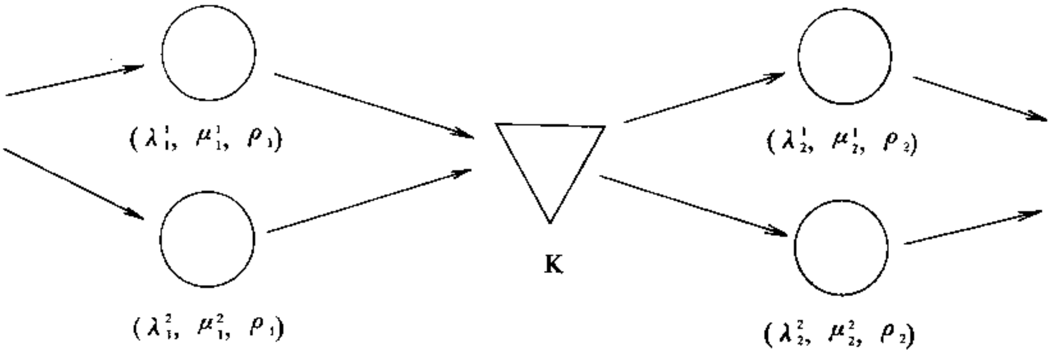

Fig. 1. Two-Stage Automatic Transfer Line

In Fig.1, define system parameters as follow:

λ_i^1:Failure rate of main machine in stage i, i=1,2
λ_i^2:Failure rate of alternate machine in stage i, i=1,2
μ_i^1:Repair rate of main machine in stage i, i=1,2
μ_i^2:Repair rate of alternate machine in stage i, i=1,2
ρ_i:Production rate of stage i, i=1,2
K:Buffer capacity

In section 3, it is shown that how a stage with main and alternate machines can be approximated by a stage with one machine. Section 4 discusses the analytical method to study the behavior of an unreliable two-stage system. Section 5 shows the results of numerical analysis and simulation results is presented in section 6.

3. Representation of Two-Machine-Stage as One-Machine-Stage

In this section, we discuss how to represent a stage of main and alternate machines as a stage of one machine. The behavior of the stage of two machines is modelled as an alternating renewal process. The instant when both machines fail is considered as a starting point of an off-period, and when one machine is repaired, an on-period begins and remains until both machines fail again. Each time the process goes off, everything starts over again. That is the process starts over again after a complete cycle consisting of an on and an off interval. In other words, a renewal occurs whenever a cycle is completed. This alternating renewal process is depicted in Fig.2

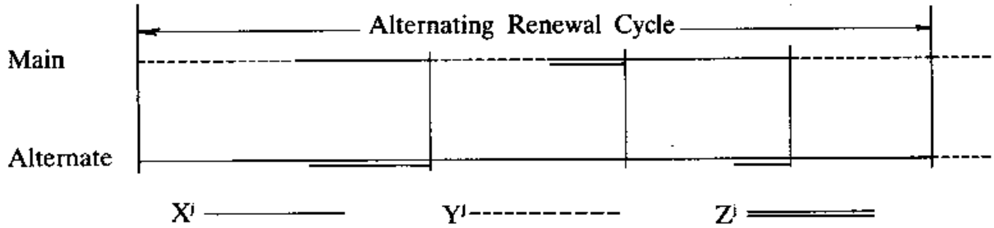

Fig. 2. Alternating Renewal Process

Define

X_i^j:Running time of machine j of stage i, $\quad j=1,2$

Y_i^j:Repair time of machine j of stage i, $\quad j=1,2$

Z_i^j:Idle time of machine j of stage i, $\quad j=1,2$

where

$$j=\begin{cases}1 & \text{Main machine}\\ 2 & \text{Alternate Machine}\end{cases}$$

In Fig.2, the stage i starts a new running period when either main or alternate machine gets repaired. Therefore, the repair time of the stage i is a minimum of the repair times of main machine and alternate machine. Since the repair time of both machines are exponentially distributed with parameter $(\mu_i^1+\mu_i^2)$ as Eq.(1).

(1) $\quad Y_i=\text{Min}(Y^1,\ Y^2)\sim \text{Exp}(\mu_i^1+\mu_i^2)$

If the failed machine is repaired before the running machine fails, the stage i can continue to run. When the main machine gets repaired first, running time of stage i, T_1, is expressed as:

$$T_1=X_1^1+X_2^2+X_3^1+X_4^2+X_5^1+X_6^2+\cdots\cdots+X_N^j$$

In equation above,$(X_1^1,\ X_3^1,\ X_5^1\cdots\cdots)$are independent random variables with commom exponential distribution with parmeter λ_i^1, and $(X_2^2,\ X_4^2,\ X_6^2,\cdots\cdots)$are also independent random variables with common exponential distribution with parameter λ_i^2. The running time, T_1 will be increased by X^2 with probability P_{12} when the main machine fails. Likewise, T_1 will also be increased by X^1 with probability P_{21} when the alternate one fails. This alternating process continues until the current working machine fails before the standby one gets repaired. P_{12} and P_{21} are defined as follow:

$$P_{11}=P(X^1>Y^2)=\frac{\mu_i^2}{\lambda_i^1+\mu_i^2},\ P_{21}=P(X^2>Y^1)=\frac{\mu_i^1}{\lambda_i^2+\mu_i^1}$$

The expected running time of stage i, $E(T_1)$, is:

$$E(T_1)=\frac{(\lambda_i^2+\mu_i^1)\{(\lambda_i^1+\mu_i^2)E(X^1)+\mu_i^2E(X^2)\}}{\lambda_i^1\lambda_i^2+\lambda_i^1\mu_i^1+\lambda_i^2\mu_i^2}$$

If the alternate machine gets repaired first, the expected running time of stage i can be expressed similarly:

$$E(T_2)=\frac{(\lambda_i^1+\mu_i^2)\{\mu_i^1E(X^1)+(\lambda_i^1+\mu_i^2)E(X^2)\}}{\lambda_i^1\lambda_i^2+\lambda_i^1\mu_i^1+\lambda_i^2\mu_i^2}$$

Consequently, the expected running time of stage i, $E(X_i)$, is:

$$E(X_i)=E(T_1)P(Y^1<Y^2)+E(T_2)P(Y^1>Y^2)$$

where

$$P(Y^1 < Y^2) = \frac{\mu_i^1}{\mu_i^1 + \mu_i^2}, \quad P(Y^1 > Y^2) = \frac{\mu_i^2}{\mu_i^1 + \mu_i^2}$$

Therefore,

$$\text{(2)} \qquad E(X_i) = \frac{(\mu_i^1/\lambda_i^1)(\lambda_i^1 + \mu_i^2)(\lambda_i^2 + \mu_i^1 + \mu_i^2) + (\mu_i^2/\lambda_i^2)(\lambda_i^2 + \mu_i^1)(\lambda_i^1 + \mu_i^1 + \mu_i^2)}{(\mu_i^1 + \mu_i^2)(\lambda_i^1\lambda_i^2 + \lambda_i^1\mu_i^1 + \lambda_i^2\mu_i^2)}$$

As a special case, when both machines have the same parameters, the running time of stage i can be written in terms of main machine as follow:

$$X_i = X_1^1 + X_2^1 + X_3^1 + X_4^1 + \cdots\cdots\cdots\cdots X_N^1$$

We see that X_1^1, X_2^1, $\cdots\cdots$, X_N^1 are independent random variables with common exponential distribution, and N is geometrically distributed with parameter P_{12}, which is equal to P_{21}. Therefore, we will call that X_i has the compound geometric distribution with the mean, $E(X_i)$ as given in Eq.(3).

$$\text{(3)} \qquad E(X_i) = \frac{\lambda_i^1 + \mu_i^1}{(\lambda_i^1)^2}$$

As in Eq. (1), the repair time of stage i has an exponential distribution with parameter, $(\mu_i^1 + \mu_i^2)$, while the distribution of the running time of stage i has a mean, $E(X_i)$, as given in Eq. (2). However, simulation shows that, as long as the mean of the running time remains the same, it's distribution is not significant to the efficieney of the stage. Therefore, the running time of stage i can be approximated to have an exponential distribution with mean, $E(X_i)$.

4. Two-Stage Automatic Transfer System

In section 3, it is shown how to represent a stage with two machines can be approximated as a stage with one machine. This section discusses an analytical method to evaluate the efficiency and the average storage level of a two-stage system, which is shown in Fig. 3, where each stage is consisting of a machine having exponentially distributed uptimes and downtimes.

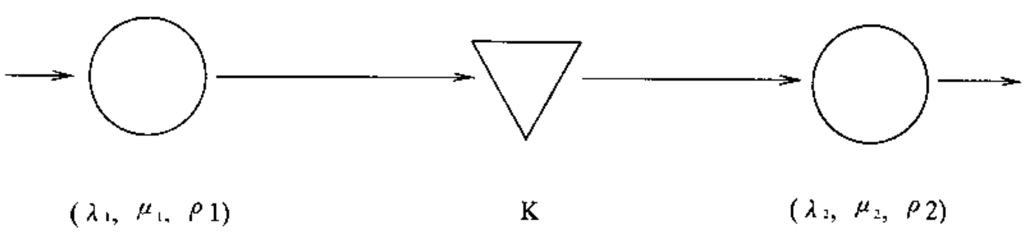

Fig. 3. Two-Stage Automatic Transfer Line with One Machine in Each Stage

Define the failure rates and repair rates of machines:

λ_i:Failure rate of a machine in stage i, i=1,2
μ_i:Repair rate of a machine in stage i, i=1,2

The variables denoting the storage level and the state of the system are defined as follow:

x_i:Storage level of buffer
m_i:State of stage i

$$= \begin{cases} U & \text{if stage i is up} \\ D & \text{if stage i is down} \end{cases}$$

With the definitions as above, the state of the system can be expressed by three component vector, such as (U,U,x), which means that the first stage and the second stage are processing units while storage level is equal to x. Also, the blocking state of the first stage is expressed as (U,D,K). Similarly, the starvation state of the second stage can be expressed as (D,U,0).

This two-stage system produces its output at the rate of ρ_2 as long as the second stage is working and the buffer is not empty. But if the second stage is faster than the first stage in processing units, when buffer is empty, the production at the second stage is restricted by the first stage.

For our computational convenience, the time scale is changed through the introduction of the relative production rate, ρ, and the relative storage capacity, K as shown below.

$$\rho = \frac{\rho_2}{\rho_1}, \ \overline{K} = \frac{K}{\rho_1}$$

Suppose that the system is in state (D,U,0) at time 0, then there exist time points when the system returns to state (D,U,0). at which the process probabilistically restarts itself. That is, with probability 1, there exists a time such that the continuation of the process beyond that time is a probabilistic replica of the whole process starting at time 0. Therefore, we can model this two-stage system as a regenerative process in which the regeneration point is defined as the beginning of the state of the system (D,U,0), and the time between the two regeneration points is called a regeneration cycle.

The quantities of interest including the expected cycle length, the expected system efficiency per cycle, the average storage level per cycle and the expected time of various states of the system, can be determined by defining appropriate cost functions, and then calculating the expected cost per cycle. In general, the cost functions depend on the state of the system, which are defined by

$c_1(x)$: the cost/unit time in state (D,D,x), $0 \leq x \leq \overline{K}$
$c_2(x)$: the cost/unit time in state (D,U,x), $0 \leq x \leq \overline{K}$
$c_3(x)$: the cost/unit time in state (U,D,x), $0 \leq x \leq \overline{K}$
$c_4(x)$: the cost/unit time in state (U,U,x), $0 \leq x \leq \overline{K}$

In order to calculate the expected cost per cycle, the following four functions are introduced to represent the expected cost until the end of the cycle depending upon the present state of the system.

$f_1(x)$: the expected cost until the end of the cycle if the system is now in state (D,D,x), $0 \leq x \leq \overline{K}$
$f_2(x)$: the expected cost until the end of the cycle if the system is now in state (D,U,x), $0 \leq x \leq \overline{K}$
$f_3(x)$: the expected cost until the end of the cycle if the system is now in state (U,D,x), $0 \leq x \leq \overline{K}$
$f_4(x)$: the expected cost until the end of the cycle if the system is now in state (U,U,x), $0 \leq x \leq \overline{K}$

Then, the expected cost per cycle(EC) is follow:

$$(4) \qquad EC = \frac{c_2(0)}{\mu_1} + f_4(0)$$

In Eq.(4), first term is the expected cost until the state (D,U,0) is changed to the state (U,U,0), and second term represents the expected cost from the beginning of the state (U,U,0) until the end of the cycle.

In order to find $f_4(0)$ in Eq. (4), we set up differential equations of stationary probabilities which shows how the function f_1, f_2, f_3, f_4 can be determined. The cost $f_1(x)$ can be divided into the costs during the first small time interval δ and the rest of the costs until the end of the cycle.

$$f_1(x) = c_1(x)\delta + (1-\mu_1\delta)(1-\mu_2\delta)f_1(x) + \mu_1\delta(1-\mu_2\delta)f_3(x) + (1-\mu_1\delta)\mu_2\delta f_2(x)$$

By taking the limit as δ goes to zero,

$$0 = c_1(x) - (\mu_1 + \mu_2)f_1(x) + \mu_1 f_3(x) + \mu_2 f_2(x), \quad 0 \leq x \leq \overline{K}$$

Similarly, we can derive

$$\rho f_2'(x) = c_2(x) - (\lambda_2 + \mu_1)f_2(x) + \lambda_2 f_1(x) + \mu_1 f_4(x), \quad 0 \leq x \leq \overline{K}$$

$$-f_3'(x) = c_3(x) - (\lambda_1 + \mu_2)f_3(x) + \lambda_1 f_1(x) + \mu_2 f_4(x), \quad 0 \leq x \leq \overline{K}$$

$$(\rho - 1)f_4'(x) = c_4(x) - (\lambda_1 + \lambda_2)f_4(x) + \lambda_1 f_2(x) + \lambda_2 f_3(x), \quad 0 \leq x \leq \overline{K}$$

From above four equations, a system of three differential equations can be derived as shown in Eq.(5).

$$(5) \qquad \begin{pmatrix} f_2'(x) \\ f_3'(x) \\ f_4'(x) \end{pmatrix} = A \begin{pmatrix} f_2(x) \\ f_3(x) \\ f_4(x) \end{pmatrix} + B(x)$$

where

$$A=\begin{pmatrix} \frac{-1}{\rho}\left(\mu_1+\frac{\lambda_2\mu_1}{\mu_1+\mu_2}\right) & \frac{\lambda_2\mu_1}{\rho(\mu_1+\mu_2)} & \frac{\mu_1}{\rho} \\ \frac{-\lambda_1\mu_2}{\mu_1+\mu_2} & \mu_2+\frac{\lambda_2\mu_2}{\mu_1+\mu_2} & -\mu_2 \\ \frac{\lambda_1}{\rho-1} & \frac{\lambda_2}{\rho-1} & -\frac{\lambda_1+\lambda_2}{\rho-1} \end{pmatrix} \qquad B(x)=\begin{pmatrix} \frac{1}{\rho}\left\{\frac{\lambda_2 c_1(x)}{\mu_1+\mu_2}+c_2(x)\right\} \\ -\left\{\frac{\lambda_1 c_1(x)}{\mu_1+\mu_2}+c_3(x)\right\} \\ \frac{c_4(x)}{\rho-1} \end{pmatrix}$$

The boundary conditions are derived as follows:

For $x=\overline{K}$, $0=c_3(\overline{K})-\mu_2\{f_3(\overline{K})-f_4(\overline{K})\}$

If $\rho\leq 1$ and when $x=\overline{K}$

$$0=c_4(\overline{K})-(\lambda_1+\lambda_2)f_4(\overline{K})+\lambda_1 f_2(\overline{K})+\lambda_2 f_3(\overline{K})$$

If $\rho\geq 1$ and when $x=0$

$$0=c_4(0)-(\lambda_1+\lambda_2)f_4(0)+\lambda_1 f_2(0)+\lambda_2 f_3(0)$$

For $x=0$, $\quad 0=f_2(0)$

In Eq.(5), If $\frac{\mu_1}{\lambda_1+\mu_1}\neq\frac{\rho\mu_2}{\lambda_2+\mu_2}$, matrix A has three distinct eigenvalues, and one of these is always 0. If $\frac{\mu_1}{\lambda_1+\mu_1}=\frac{\rho\mu_2}{\lambda_2+\mu_2}$, A has two different eigenvalues, and 0 is always the eigenvalue of algebraic multiplicity of two. Wijngaard [10] stated that the nonzero eigenvalue has index two, which is not true.

In order to find the expected cost per cycle, solution procedure can be summarized as follows:

Step 1:Specify the value of Ci(x) as given below depending on the quantity of interest

Expected Cycle Length,	$c_1(x)=c_2(x)=c_3(x)=c_4(x)=0\ \ 0\leq x\leq\overline{K}$
System Efficiency,	$c_1(x)=c_3(x)=0,\ c_2(x)=c_4(x)=\rho\ \ 0<x\leq\overline{K}$
	$c_1(0)=c_2(0)=c_3(0)=0,\ c_4(0)=\min(1,\rho)$
Average Storage Level,	$c_1(x)=c_2(x)=c_3(x)=c_4(x)=x,\ 0\leq x\leq\overline{K}$

Step 2:Solve the system of linear differential equations, Eq.(5) to find $f_4(0)$

Step 3:Calculate the expected cost per cycle.

Through the procedure explained above, we can find the necessary values for the analysis of two-stage transfer system. Note that only numerical solutions are available. However, in case of $\rho=1$ ($\rho_1=\rho_2$), closed forms of solutions can be derived.

4. Numerical Solutions

In designing a manufacturing system, the choice of the machines at each stage is likely to be determined by the technology of the process and the product to be manufactured. However, the amount of buffer capacity to be allocated is influenced by the trade-off between the cost of providing the buffer space(and perhaps by the cost of holding storage) and the gain of the system efficiency. We are primarily interested in how the system efficiency and the average storage level depend on the buffer capacity.

Before we discuss the numerical experiment results, the properties of symmetry and reversibility is discussed. It is simple to prove these properties for our model by analysis of sample paths. The reversibility principle is: if the direction of flow in a manufacturing line is reversed, the system efficiency remains same, but the average storage levels are complementary, i.e. in each buffer, the new average storage level is the buffer capacity less the old average level. As a consequence of this fact, in a symmetric N-stage system, we know that the average storage levels of ith and(N-i)th buffer are complementary to each other. In a system composed of two identical stages, the average storage level is half the buffer capacity. In an unbalanced line, if the second stage is the bottleneck, storage tends to pile up in front of it. Hence we see that the average storage level is a convex function of buffer capacity. In the reversed system, with the first stage bottleneck, we of course observe that the average storage level is concave in buffer capacity.

In experiments 1 and 2, we explore how the parameters of the stage influence the system efficiency. In both experiments, we assume that the first stage is the bottleneck and has fixed parameters, Clearly, any change in the parameters of the second stage that improve it's effective production rate, $r_i = \frac{\rho_i \mu_i}{\lambda_i + \mu_i}$, (incerase the production rate or repair rate or reduce failure rate) will increase the system efficiency. In these experiments, we look at various parameter sets for the second stage that keep its effective production rate fixed, i.e.we keep r_2 constant.

In experiment 1, we vary the failure and repair rates while keeping reliability$\left(\frac{\mu_i}{\lambda_i + \mu_i}\right)$ of the second stage constant. We see that increasing the repair rate (which reduces mean downtime) will increase the system efficiency at any positive buffer capacity, even though the failure rate is increased proportionally. This is because with the reduced mean downtime, there is less chance to fill the buffer, thus blocking the bottleneck stage. Therefore, we see that the efficiency is more sensitive to the repair rate than the failure rate. However, the average storage level is essentially independent of the repair rate under these experimental conditions.

In experiment 2, we examine three values of the production rate(ρ_2). For each of the production rates, we consider two ways of adjusting the reliability of the second stage to keep the effective production rate constant, i.e. change the failure rate(cases 2, and 3) and the repair rate(cases 4, and 5). We note that, with the same effective production rates and failure or repair rates, a more reliable but slower machine gives more system efficiency. When $\rho_1 > \rho_2$, as in cases 3 and 5, the production rate of the second stage is never restricted even though the buffer is empty, as long as the first stage works. Furthermore, since the reliability of the second stage is comparatively high, the blocking chance of the first stage(bottleneck stage) is pretty small, and the system efficiency reaches to the possible maximum value(effective production rate of the bottleneck stage)at relatively small buffer capacity. Given machines of the same reliability, as in experiment 1, the machine with the larger repair rate is better(compare case 2 with 4,3 with 5).

Experiment 3 is a sensitivity analysis, in which failure rates and production rates of both stages are improved by 10 percent respectively, based on the parameters in case 1. Note that the system efficiency is more sensitive to the effective production rate of the bottleneck stage, which is obvious in the limiting case of infinite buffer capacity. At smaller buffer capacity, increasing the production rate of the bottleneck stage is not as productive as increasing its reliability. This is because the small buffer is often full, limiting the production rate of the first stage, which is the bottleneck stage, to the instantaneous production rate of the second stage. But at larger capacity, blocking of the first stage is rare and the system efficiency approaches to the effective production rate of the first stage. Since this is directly proportional to the production rate of the first stage, case 3 gives the largest system efficiency. The system efficiency curves of cases 1,4, and 5 have the same asymptotes. That means improving the effective production rate of the second(non-bottleneck) stage improves the system efficiency a little at large buffer capacity. At smaller buffer capacity, the system efficiency is more sensitive to the production rate of the second stage than to its reliability because the effective production rate of the second stage is directly proportional to it's production rate.

6. Simulation

In this section, simulation experiments are carried out to see how the system behaves when the rapair times are not exponentially distributed. Throughtout the simulation experiments, the Antithetic Variates technique and Common Random Numbers[7] are used to reduce the variances of the simulation results.

Five types of repair distributions are tested to evaluate the effect of repair time distribution on the behavior of the system; Exponential(case 1), 2–Erlang(case 2), Uniform(case 3), 4–Erlang(case 4), and Deterministic(case 5), while case 0 shows the analytical solution from section 4. Sequences of Exponential failure times are exactly identical in all five cases. As for the repair times, Exponential and Uniform repair times are generated from the same streams of Uniform (0,1) random numbers, while 2–Erlang and 4–Erlang repair times are generated from different streams of Uniform(0,1) random numbers. Even though only partial synchronization is maintained, Common Random Numbers gives us a considerable gain in precision. Mean repair times of all five distributions are same, while variances are different; variances of Deterministic repair times are zero, variances of Uniform and 4–Erlang repair times are one fourth, and variances of 2–Erlang repair times are one half of variances of Exponential repair times respectively.

For each set of parameters, five pairs of simulation runs, in which each run is performed for 30,000 time units, are carried out to have enough observations for statistical analysis of simulation results.

Throughout the experiments, parameters of the first stage are never changed and the first stage is always remained as the bottleneck stage. Based on the parameters in Simulation 1, the repair rate of the second stage is varied in Simulation 2, while five percent faster production rate is chosen in Simulation 3. In Simulation 4 through 6, same parameters are chosen as used in on Simulation 1 through 3 respectively, except that the buffer capacity is increased from 5 to 50, to investigate the effect of the buffer capacity to the behavior of the system under the different repair time distributions.

When buffer capacity is small (K=5) compared with the expected production loss during the repair time, the differences between the system efficiencies of five repair time distributions are statistically insignificant(refer Table 4 through Table 6). This is because the storage level hits boundary and the non-failed stage turns into the idle state most of the time when one stage fails. Therefore, we conclude that the behavior of the system is affected very little by the variances or the distributions of the repair times, when buffer capacity is comparatively small.

On the contrary, when buffer capacity is large (K=50) campared with the expected production loss during the repair times, we see the significant statistical differences in the system efficiencies of cases 1,2,3, and 5, while the efficiencies of case 3 and 4 are statistically insignificant(refer Table 7 through Table 9). From above, we say that the system efficiency is significantly affected by the variances of the repair times rather than the distributions. The reason is; when the variances of the repair times are small, there are very little chances that the non-failed stage goes into the idle stage. But as the variances get larger, not only the probability that the non-failed stage turns into the idle state gets larger, but the idle time gets longer. Therefore, it is our conclusion that the system efficiency deteriorates as the variances of the repair times get larger. However, there is no indication of any statistical differences in the average storage levels. This indicates that the average storage level is not so sensitive to the variances of the repair times as the system efficiency is, as long as buffer capacity is large enough.

Finally, when buffer capacity is extremely larger compared with the expected production loss during the repair times, the probability that the non-failed stage turns into the idle state when one stage fails are very small. Accordingly, we know that the behavior of the system is less dependent on the variances of the repair times, because both stage are more decoupled each other.

Table 1. Parameters for Experiment 1

Case	λ_1	μ_1	ρ_1	r_1	λ_2	μ_2	ρ_2	r_2	r_2/r_1
1	0.01	0.09	1.0	0.9	0.012	0.12	1.0	0.9091	1.01
2	0.1	0.09	1.0	0.9	0.008	0.08	1.0	0.9091	1.01

Table 2. Parameters for Experiment 2

Case	λ_1	μ_1	ρ_1	r_1	λ_2	μ_2	ρ_2	r_2	r_2/r_1
1	0.01	0.09	1.0	0.9	0.004	0.096	1.0	0.96	1.0667
2	0.01	0.09	1.0	0.9	0.012	0.096	1.08	0.96	1.0667
3	0.01	0.09	1.0	0.9	0.002	0.096	0.98	0.96	1.0667
4	0.01	0.09	1.0	0.9	0.004	0.032	1.08	0.96	1.0667
5	0.01	0.09	1.0	0.9	0.004	0.192	0.98	0.96	1.0667

Table 3. Parameters for Experiment 3

Case	λ_1	μ_1	ρ_1	r_1	λ_2	μ_2	ρ_2	r_2	r_2/r_1
1	0.02	0.08	1.0	0.8	0.01	0.09	1.0	0.9	1.125
2	0.018	0.08	1.0	0.8163	0.01	0.09	1.0	0.9	1.1025
3	0.02	0.08	1.1	0.88	0.01	0.09	1.0	0.9	1.1045
4	0.02	0.08	1.0	0.8	0.009	0.09	1.0	0.9091	1.1364
5	0.02	0.08	1.0	0.8	0.01	0.09	1.1	0.99	1.2375

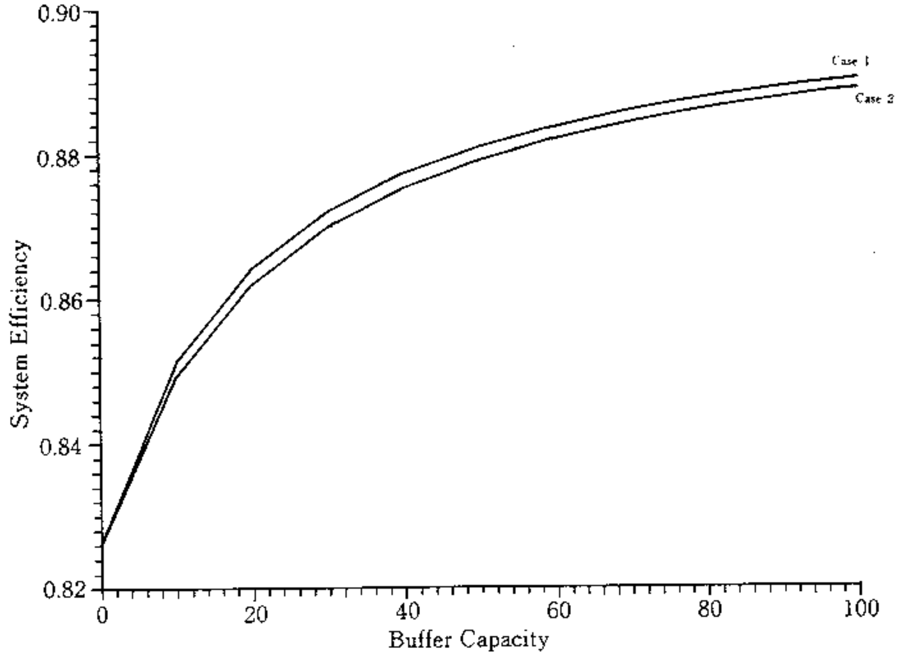

Fig.4. Experiment 1

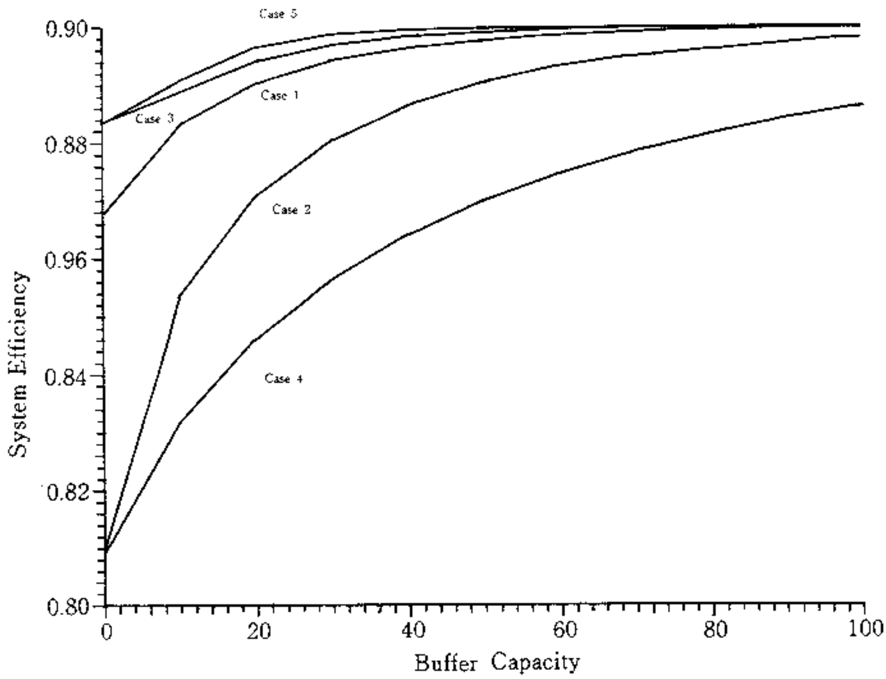

Fig.5. Experiment 2

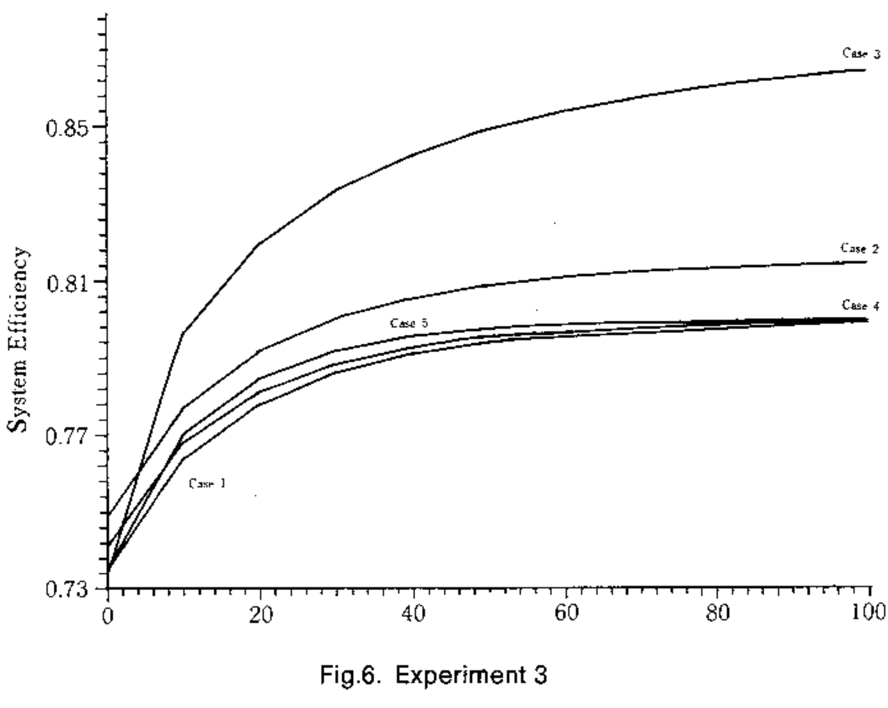

Fig.6. Experiment 3

Table 4. Result for Simulation 1

Parameters:(0.01,0.09,1),(0.01,0.09,1),K=5								
	System Efficiency				Average Storage Level			
Case	Mean	σ	M−2σ	M+2σ	Mean	σ	M−2σ	M+2σ
0	0.8383	–	–	–	2.5000	–	–	–
1	0.8337	0.0027	0.8284	0.8391	2.5297	0.1128	2.3041	2.7552
2	0.8358	0.0029	0.8299	0.8416	2.5092	0.0931	2.3229	2.6955
3	0.8348	0.0017	8.8314	0.8382	2.4999	0.0915	2.3169	2.6829
4	0.8368	0.0028	0.8313	0.8424	2.5144	0.1086	2.2973	2.7315
5	0.8365	0.0019	0.8327	0.8403	2.5089	0.1180	2.2730	2.7448

Table 5. Result for Simulation 2

Parameters:(0.01,0.09,1),(0.01,0.10,1),K=5								
	System Efficiency				Average Storage Level			
Case	Mean	σ	M−2σ	M+2σ	Mean	σ	M−2σ	M+2σ
0	0.8409	–	–	–	2.4514	–	–	–
1	0.8412	0.0027	0.8359	0.8465	2.4794	0.1098	2.2598	2.6991
2	0.8433	0.0031	0.8372	0.8495	2.4630	0.0936	2.2759	2.6501
3	0.8425	0.0017	0.8391	0.8459	2.4547	0.0929	2.2689	2.6404
4	0.8448	0.0028	0.8389	0.8502	2.4740	0.1107	2.2526	2.6955
5	0.8442	0.0020	0.8403	0.8481	2.4745	0.1169	2.2407	2.7083

Table 6. Result for Simulation 3

Parameters:(0.01,0.09,1),(0.01,0.09,1.05),K=5								
	System Efficiency				Average Storage Level			
Case	Mean	σ	M−2σ	M+2σ	Mean	σ	M−2σ	M+2σ
0	0.8420	–	–	–	1.3850	–	–	–
1	0.8414	0.0038	0.8339	0.8490	1.4103	0.0380	1.3342	1.4863
2	0.8440	0.0024	0.8392	0.8488	1.5085	0.0576	1.3933	1.6237
3	0.8435	0.0022	0.8391	0.8478	1.5280	0.0551	1.4177	1.6382
4	0.8448	0.0023	0.8402	0.8495	1.5591	0.0589	1.4413	1.6768
5	0.8451	0.0023	0.8405	0.8498	1.5651	0.0513	1.4626	1.6677

Table 7. Result for Simulation 4

Parameters:(0.01,0.09,1),(0.01,0.09,1),K=50								
	System Efficiency				Average Storage Level			
Case	Mean	σ	M−2σ	M+2σ	Mean	σ	M−2σ	M+2σ
0	0.8750	–	–	–	25.000	–	–	–
1	0.8751	0.0022	0.8707	0.8796	24.912	0.9801	22.952	26.872
2	0.8805	0.0021	0.8762	0.8848	25.001	1.5489	21.904	28.099
3	0.8820	0.0015	0.8790	0.8849	25.523	1.1516	23.219	27.826
4	0.8834	0.0023	0.8789	0.8880	25.711	1.0693	23.572	27.849
5	0.8851	0.0008	0.8834	0.8868	25.952	1.1741	23.605	28.300

Table 8. Result for Simulation 5

Parameters:(0.01,0.09,1),(0.01,0.10,1),K=50								
	System Efficiency				Average Storage Level			
Case	Mean	σ	$M-2\sigma$	$M+2\sigma$	Mean	σ	$M-2\sigma$	$M+2\sigma$
0	0.8810	–	–	–	23.108	–	–	–
1	0.8815	0.0020	0.8775	0.8855	22.964	1.0031	20.958	24.971
2	0.8861	0.0027	0.8807	0.8914	22.504	1.8860	18.733	26.276
3	0.8876	0.0013	0.8850	0.8903	22.782	1.1996	20.383	25.181
4	0.8890	0.0024	0.8842	0.8938	22.913	1.1285	20.662	25.177
5	0.8904	0.0011	0.8883	0.8925	22.435	1.3246	19.786	25.085

Table 9. Result for Simulation 6

Parameters:(0.01,0.09,1),(0.01,0.09,1.05),K=50								
	System Efficiency				Average Storage Level			
Case	Mean	σ	$M-2\sigma$	$M+2\sigma$	Mean	σ	$M-2\sigma$	$M+2\sigma$
0	0.8886	–	–	–	15.764	–	–	–
1	0.8889	0.0045	0.8800	0.8978	15.964	0.7240	14.516	17.412
2	0.8929	0.0045	0.8900	0.8959	15.086	1.2587	12.569	17.604
3	0.8951	0.0013	0.8925	0.8977	14.352	0.8209	12.711	15.994
4	0.8947	0.0017	0.8913	0.8981	14.145	0.9571	12.231	16.059
5	0.8968	0.0007	0.8955	0.8982	13.152	0.9188	11.315	14.990

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