〈硏究論文〉

Journal of the Korean Institute of Industrial Engineers
Vol. 14, No. 2, Dec. 1988

M/G/1 Queue With Two Vacation Missions

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Abstract

We consider a vacation system in which the server takes two different types of vacations alternately. We obtain the server idle probability and derive the system size distribution and the waiting time distribution by defining supplementary variables. We show that the decomposition property works for these mixed-vacation queues. We also propose a method directly to obtain the waiting time distribution without resorting to the system equations. The T-policy is revisited and is shown that the cost is minimized when the length of vacations are the same.

1. Introduction

In this paper we consider a queueing system in which the server leaves for a vacation as soon as the system becomes empty. When he returns from the vacation to find no customers, he immediately leaves for another vacation, but this time differently distributed from the first one. We call the first vacation the 'first type' and the second vacation the 'second type'. These first and second types of vacations alternate until the server finally finds at least one customer. We can view this queueing system as the one in which the server has two different missions to fulfill during his vacations. If the first and second types of vacations are distributed the same, this vacation model corresponds to the second model of Levy and Yechiali [4]. If both types of vacations are constants of fixed length T, this is exactly the system with T-policy introduced by Heyman[3].

For the M/G/1 queue with server vacation, Fuhrmann and Cooper[3] proved that the waiting time can be decomposed into the waiting time in the ordinary M/G/1 queue and the residual vacation time. We show that this decomposition property is still valid for two types of vacations and even for multi-types of vacations by deriving the system size and waiting time distributions. In section 2, we derive the server idle probability. To state the

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result, the server idle probabbility(i.e., the probaility that the server is on vacation) is not affected by the vacation policy. In section 3, we define the supplementary variables to set up the system equations and derive the system size distribution. In section 4, we obtain the waiting time distribution and propose a method that enables one to directly derive the waiting time distribution of any combinations of vacations without resorting to the complicated system equations. In the last section, we revisit the T-policy and show that the system is optimized when the two vacations are of the same length under the cost structure of Heyman[3].

Server Idle Probability

Customers arrive according to the Poisson process with rate λ. We consider only the naive customers so that no customers balk and renege. Let T_V be the total vacation period, i.e., time from the system depletion until the server begins to be busy. Let T_B be the server occupation period. This occupation period is different from the busy period of Baba[1] in that we do not consider the zero-length busy periods that arise when the server finds no customers upon return from a vacation. But our occupation period enables one to obtain the busy period considering the number of vacations in the total vacation duration. Hence without loss of generality, we may call the occupation period the busy period which makes more practical sense. In the sequel, '$*$' denotes the Laplace-Stieltjes transform(LST) of the corresponding random variables with transform arguement θ. Let V and U be the random variables denoting the first and second type of vacations with distribution functions V(t) and U(t) respectively. Then we have

Pr(no customers arrive during a first type of vacation)

$$=\int_0^\infty e^{-\lambda t}dV(t)=V^*(\lambda) \qquad (2.1)$$

and

Pr(no customers arrive during a second type of vacation)

$$=\int_0^\infty e^{-\lambda t}dU(t)=U^*(\lambda) \qquad (2.2)$$

The number of vacations in the total vacation duration is a geometric random variable. Hence, we have

$$T_V=\begin{cases} V & \text{with probability } 1-V^*(\lambda) \\ V+U & \text{with probability } V^*(\lambda)[1-U^*(\lambda)] \\ V+U+V & \text{with probability } V^*(\lambda)U^*(\lambda)[1-V^*(\lambda)] \\ \vdots & \vdots \qquad\qquad \vdots \end{cases} \qquad (2.3)$$

Then the LST of T_v is easily obtained and is given by

$$T_v^*(\theta)=\frac{[1-U^*(\lambda)]V^*(\lambda)V^*(\theta)U^*(\theta)+[1-V^*(\lambda)]V^*(\theta)}{1-V^*(\lambda)U^*(\lambda)V^*(\theta)U^*(\theta)} \cdots\cdots (2.4)$$

and the mean of T_v becomes

$$E(T_v)=-(d/d\theta)T_v^*(0)=\frac{E(V)+V^*(\lambda)E(U)}{1-V^*(\lambda)U^*(\lambda)} \cdots\cdots (2.5)$$

Now, the busy period T_B is initiated by the customers arriving during the total vacation duration. Let K be this number. Then we have

$$Pr(K=k)=V(k,t)[1+V^*(\lambda)U^*(\lambda)+(V^*(\lambda))^2(U^*(\lambda))^2+...]$$

$$+U(k,t)V^*(\lambda)[1+V^*(\lambda)U^*(\lambda)+(V^*(\lambda))^2+...]$$

$$=\frac{V(k,t)+V^*(\lambda)U(k,t)}{1-V^*(\lambda)U^*(\lambda)} \cdots\cdots (2.6)$$

where

$V(k,t)=Pr(k$ customers arrive during the first type of vacation)

$$=\int_0^\infty \frac{e^{-\lambda t}(\lambda t)^k}{k!}dV(t) \cdots\cdots (2.7)$$

and

$U(k,t)=Pr(k$ customers arrive during the second type of vacation)

$$=\int_0^\infty \frac{e^{-\lambda t}(\lambda t)^k}{k!}dU(t) \cdots\cdots (2.8)$$

Then the probability generating function(PGF) of K becomes

$$K(z)=\sum_{k=1}^{\infty} z^k Pr(K=k)$$

$$=\frac{V^*(\lambda-\lambda z)+V^*(\lambda)U^*(\lambda-\lambda z)-V^*(\lambda)-V^*(\lambda)U^*(\lambda)}{1-V^*(\lambda)U^*(\lambda)} \cdots\cdots (2.9)$$

and the mean of K is given by

$$E(K)=(d/dz)K(1)=\frac{\lambda[E(V)+V^*(\lambda)E(U)]}{1-V^*(\lambda)U^*(\lambda)} \cdots\cdots (2.10)$$

The busy period T_B can be represented as a random sum:

$$T_B=B_1+B_2+.....+B_K \cdots\cdots (2.11)$$

where B_i is the sub-busy period of an ordinary M/G/1 queus with generic respresentation B. Then the LST of T_B becomes

$$T_B^*(\theta)=K(B^*(\theta))\frac{V^*(\lambda-\lambda B^*(\theta))+V^*(\lambda)U^*(\lambda-\lambda B^*(\theta))-V^*(\lambda)(1+U^*(\lambda))}{1-V^*(\lambda)U^*(\lambda)} \cdots (2.12)$$

and its mean becomes

$$E(T_B)=-(d/d\theta)T_B(0)=\frac{\lambda E(S)}{1-\lambda E(S)}\cdot\frac{E(V)+V^*(\lambda)E(U)}{1-V^*(\lambda)U^*(\lambda)} \cdots\cdots (2.13)$$

where S is the service time random variable. Finally the server idle probability is given by

$$\Pr(\text{server is idle})=\frac{E(T_V)}{E(T_B)+E(T_V)}=1-\lambda E(S)=1-\rho \cdots\cdots (2.14)$$

This server idle probability is the same as that of ordinary M/G/1 queues. It appeals to our intuition that the increase of system size due to the server vacations prolongs the busy period with the same rate as the customers found by the returning server increase the total vacation period. Actually it can be seen that the server idle probability is not affected by any combinations of vacation policy as long as the server immediately takes another vacation when he finds no customers waiting for service. This is due to the following reasoning: the mean of T_B is, from eq.(2.11), given by E(B)E(K) where $E(B)=E(S)/(1-\lambda E(S))$. But E(K) is the mean number of customers arriving during the mean total vacation duration which gives us $E(K)=\lambda\cdot E(T_V)$. Hence,

$$\begin{aligned}\Pr(\text{server is idle})&=E(T_V)/[E(B)E(K)+E(T_V)]=E(T_V)/[E(T_V)(\lambda E(B)+1]\\&=1-\lambda E(S)=1-\rho\end{aligned}$$

3. System Size Distribution

In this section, we define supplementary variables to derive system equations. The supplementary variables are the remaining service time, the remaining vacation time of the first type and the remaining vacation time of the second type. Similar derivation of the system equations with one type of vacation can be found in Baba[1]

Let N(t) be the number of customers at time t. Since we are interested in steady-state behavior, initial state of the system can be arbitrary. Define following random variables and probabilities:

$$Y=\begin{cases}0 & \text{if the server is in the system}\\1 & \text{if the server is on vacation of the first type}\\2 & \text{if the server is on vacation of the second type}\end{cases}$$

$\hat{S}$=remaining service time for the customer in service

$\hat{V}$=remaining vacation time of the first type

$\hat{U}$=remaining vacation time of the second type

$$P_n(x,t)=\Pr(N(t)=n, x<\hat{S}\leq x+d_x, Y=0)\ n=1,2,\ldots \cdots\cdots (3.1)$$

$$Q_n(x,t)=\Pr(N(t)=n, x<\hat{V}\leq +d_x\ Y=1)\ n=0,1,\ldots \cdots\cdots (3.2)$$

$$R_n(x,t)=Pr(N(t)=n, {}_x<\hat{U}\le x+d_x, Y=2)\ n=0,1,\ldots \quad (3.3)$$

Let $s(x)$, $v(x)$ and $u(x)$ be the probability density functions of service time random variable S and vacation random variables V and U respectively. Then we have

$$P_1(x-dt,\ t+dt)=p_1(x,t)\ (1-\lambda\, dt)+P_2(o,t)s(x)dt+Q_1(o,t)s(x)dt + R_1(o,t)s(x)dt+o(dt) \quad (3.4)$$

Subtracting $P_1(x,t)$ from both sides and dividing by dt as dt tends to 0 yields

$$-(\partial/\partial x)P_1(x,t)+(\partial/\partial t)P_1(x,t)=-\lambda P_1(x,t)+P_2(o,t)s(x) + Q_1(o,t)s(x)+R_1(o,t)s(x) \quad (3.5)$$

In the similar way, we have following steady-state systems of equations:

$$(d/dx)P_1(x)=\lambda P_1(x)-P_2(0)s(x)-Q_1(0)s(x)-R(0)s(x) \quad (3.6)$$

$$(d/dx)P_n(x)=\lambda P_n(x)-\lambda P_{n-1}(x)-P_{n+1}(0)s(x)-Q_n(0)s(x) - R_n(0)s(x) \qquad (n\geqq 2) \quad (3.7)$$

$$(d/dx)Q_0(x)=\lambda Q_0(x)-P_1(0)v(x)-R_0(0)v(x)$$

$$(d/dx)Q_n(x)=\lambda Q_n(x)-\lambda Q_{n-1}(x) \qquad (n\geqq 1) \quad (3.9)$$

$$(d/dx)R_0(x)=\lambda R_0(x)-Q_0(0)u(x) \quad (3.10)$$

$$(d/dx)R_n(x)=\lambda R_n(x)-\lambda R_{n-1}(x) \qquad (n\geqq 1) \quad (3.11)$$

Taking Laplace transforms on the above differential-difference equations, we have

$$\theta P_1{}^*(\theta)-P_1(0)=\lambda P_1{}^*(\theta)-P_2(0)S^*(\theta)-Q_1(0)S^*(\theta)-R_1(0)S^*(\theta) \quad (3.12)$$

$$\theta P_n{}^*(\theta)-P_n(0)=\lambda P_n{}^*(\theta)-\lambda P_{n-1}(\theta)-P_{n+1}(0)S^*(\theta) -Q_n(0)S^*(\theta)-R_n(0)S^*(\theta) \qquad (N\geqq 2 \quad (3.13)$$

$$\theta Q_0{}^*(\theta)-Q_0(0)=\lambda Q_0{}^*(\theta)-P_1(0)V^*(\theta)-R_0(0)V^*(\theta) \quad (3.14)$$

$$\theta Q_n{}^*(\theta)-Q_n(0)=\lambda Q_n{}^*(\theta)-\lambda Q_{n-1}{}^*(\theta) \qquad (n\geqq 1) \quad (3.15)$$

$$\theta R_0{}^*(\theta)-R_0(0)=\lambda R_0{}^*(\theta)-Q_0(0)U^*(\theta) \quad (3.16)$$

$$\theta R_n{}^*(\theta)-R_n(0)=\lambda R_n{}^*(\theta)-\lambda R_{n-1}{}^*(\theta) \qquad (n\geqq 1) \quad (3.17)$$

Define p(z,0). Q(z,0) and R(z, 0) as the PGF s of $P_n(0)$, $Q_n(0)$ and $R_n(0)$, respectively. Also define following double transforms:

$$P^*(z,\theta)=\sum_{n=1}^{\infty} P_n^*(\theta)z^n \quad \cdots\cdots (3.18)$$

$$P^*(z,\theta)=\sum_{n=0}^{\infty} Q_n^*(\theta)z^n \quad \cdots\cdots (3.19)$$

$$R^*(z,\theta)=\sum_{n=0}^{\infty} R_n^*(\theta)z^n \quad \cdots\cdots (3.20)$$

Then from the system equations, we have

$$(\theta-\lambda+\lambda z)P^*(z,\theta)=-S^*(\theta)[P(z,0)-P_1(0)z]/z-S^*(\theta)[P(z,0)-P_0(0)]$$

$$-S^*(\theta)[R(z,0)-R_0(0)]+P(z,0) \quad \cdots\cdots (3.21)$$

$$(\theta-\lambda+\lambda z)Q^*(z,\theta)=-[P_1(0)+R_0(0)]V^*(\theta)+Q(z,0) \quad \cdots\cdots (3.22)$$

$$(\theta-\lambda+\lambda z)R^*(z,\theta)=-[Q_0(0)\cdot U^*(\theta)+R(z,0) \quad \cdots\cdots (3.23)$$

From eq.(3.21), (3.22) and (3.23), we have, by letting $\theta=\lambda-\lambda z$,

$$P(z,0)=\frac{zS^*(\lambda-\lambda z)[P(z,0)-P_0(0)+R(z,0)-R(z,0)-R_0(0)-P_1(0)]}{z-S^*(\lambda-\lambda z)} \quad \cdots\cdots (3.24)$$

$$Q(z,0)=[P_1(0)+R_0(0)]V^*(\lambda-\lambda z) \quad \cdots\cdots (3.25)$$

$$R(z,0)=Q_0(0)U^*(\lambda-\lambda z) \quad \cdots\cdots (3.26)$$

From $R_0(0)=R(0,0)$ and $Q_0(0)=Q(0,0)$, we have

$$Q(0)=[P_1(0)+R_0(0)]V^*(\lambda) \quad \cdots\cdots (3.27)$$

$$R(0)=Q_0(0)U^*(\lambda) \quad \cdots\cdots (3.28)$$

Then, we have

$$Q_0(0)=\frac{V^*(\lambda)P_1(0)}{1-U^*(\lambda)V^*(\lambda)} \quad \cdots\cdots (3.29)$$

$$R_0(0)=\frac{U^*(\lambda)V^*(\lambda)P_1(0)}{1-U^*(\lambda)V^*(\lambda)} \quad \cdots\cdots (3.30)$$

Substituting(3.29) and (3.30) into(3.25) and (3.26) yields

$$Q(z,0)=\frac{P_1(0)V^*(\lambda-\lambda z)}{1-V^*(\lambda)U^*(\lambda)} \quad \cdots\cdots (3.31)$$

$$R(z,0)=\frac{P_1(0)V^*(\lambda)U^*(\lambda-\lambda z)}{1-U^*(\lambda)V^*(\lambda)} \quad \cdots\cdots (3.32)$$

Then from eq.(3.24)

$$P(z,0)=\frac{P_1(0)zS^*(\lambda-\lambda z)[V^*(\lambda-\lambda z)-1+V^*(\lambda)(U^*(\lambda-\lambda z)-1)]}{[1-V^*(\lambda)U^*(\lambda)][z-S^*(\lambda-\lambda z)]} \quad \cdots\cdots (3.33)$$

From eq. (3.21), (3.22) and (3.33),

$$P^*(z,\theta)=\frac{P_1(0)z[S^*(\lambda-\lambda z)-S^*(\theta)][V^*(\lambda-\lambda z)-1+V^*(\lambda)(U^*(\lambda-\lambda z)-1)]}{[1-U^*(\lambda)V^*(\lambda)][\theta-\lambda+\lambda z][z-S^*(\lambda-\lambda z)]} \quad \cdots\cdots (3.34)$$

$$Q^*(z,\theta)=\frac{P_1(0)[V^*(\lambda-\lambda z)-V^*(\theta)]}{[1-U^*(\lambda)V^*(\lambda)][\theta-\lambda+\lambda z]} \quad \cdots\cdots (3.35)$$

$$R^*(z,\theta)=\frac{P_1(0)V^*(\lambda)[U^*(\lambda-\lambda z)-U^*(\theta)]}{1-U^*(\lambda)V^*(\lambda)][\theta-\lambda+\lambda z]} \quad \cdots\cdots (3.36)$$

By applying L'hospital's rule, we have

$$P^*(1,0)=\frac{P_1(0)\lambda E(S)[E(V)+V^*(\lambda)E(U)]}{[1-U^*(\lambda)V^*(\lambda)][1-\lambda E(S)]} \quad \cdots\cdots (3.37)$$

$$Q^*(1,0)=\frac{P_1(0)E(V)}{1-U^*(\lambda)V^*(\lambda)} \quad \cdots\cdots (3.38)$$

$$R^*(1,0)=\frac{P_1(0)V^*(\lambda)E(U)}{1-U^*(\lambda)V^*(\lambda)} \quad \cdots\cdots (3.39)$$

Then, from $P^*(1,0)+Q^*(1,0)+R^*(1,0)=1$, we have

$$P_1(0)=\frac{(1-\lambda E(S))[1-U^*(\lambda)V^*(\lambda)]}{E(V)+V^*(\lambda)E(U)} \quad \cdots\cdots (3.40)$$

The mean system size becomes

$$L=(d/dz)V^*(1,0)+(d/dz)Q^*(1,0)+(d/dz)R^*(1,0)$$

$$=\lambda E(S)+\frac{\lambda^2 E(S^2)}{2[1-\lambda E(S)]}+\frac{\lambda[E(V^2)+V^*(\lambda)E(U^2)]}{2[E(V)+V^*(\lambda)E(U)]} \quad \cdots\cdots (3.41)$$

The first two terms are the mean system size of ordinary M／G／1 queue and the third term is the increment of mean system size due to the server vacation policy.

4. Waiting Time Distribution

The LST of waiting time(including the service time) of an arbitrary customer is given by

$$W^*(\theta)=\{\sum_{n=1}^{\infty} P_n^*(\lambda)[S^*(\theta)]^{n-1}+\sum_{n=0}^{\infty} Q_n^*(\theta)[S^*(\theta)]^n+\sum_{n=0}^{\infty} R_n^*(\theta)[S^*(\theta)]^n\} S^*(\theta)$$
$$=\frac{[1-\lambda E(S)]\theta S^*(\theta)}{\theta-\lambda+\lambda S^*(\theta)}\cdot\frac{[1-V^*(\theta)]+V^*(\lambda)[1-U^*(\theta)]}{[E(V)+V^*(\lambda)E(U)]\theta}$$
$$=\frac{(1-\lambda E(S))\theta S^*(\theta)}{\theta-\lambda+\lambda S^*(\theta)}\left\{\frac{E(V)}{E(V)+V^*(\lambda)E(U)}\cdot\frac{1-V^*(\theta)}{\theta E(V)}+\frac{V^*(\lambda)E(U)}{E(V)+V^*(\lambda)E(U)}\cdot\frac{1-U^*(\theta)}{\theta E(U)}\right\} \quad \cdots\cdots (4.1)$$

Fuhrmann and Cooper[2] proved that for M／G／1 system with one type of vacation, the waiting time is the sum of two independent random variables: waiting time in the ordinary M／G／1 queue and the remaining vacation time. We now illustrate that the decomposition property still works for the queues with mixed types of vacations. We further propose a method to directly obtain the LST of the waiting time by applying the decomposition property. Let us rewrite eq.(4.1) as

$$W^*(\theta)=W^*_{M/G/1}(\theta)[w_V V_R^*(\theta)+w_U U_R^*(\theta)] \quad \cdots\cdots (4.2)$$

where

$$W^*_{M/G/1}(\theta)=\frac{(1-\lambda E(S))\theta S^*(\theta)}{\theta-\lambda+\lambda S^*(\theta)} \quad \cdots\cdots (4.3)$$

$$V_R^*(\theta)=\frac{1-V^*(\theta)}{\theta E(V)} \quad \cdots\cdots (4.4)$$

$$U_R^*(\theta)=\frac{1-U^*(\theta)}{\theta E(U)} \quad \cdots\cdots (4.5)$$

$W_{M/G/1}^*(\theta)$ is the LST of the waiting time of the ordinary M／G／1 queue. $V_R(\theta)$ and U_R^* are the LST's of the remaining vacation times of the first and second type of vacations respectively. w_V and w_U are the weights that vary depending on the combination of vacations. So eq.(4.2) tells us that the waiting time is the sum of two random variable: waiting time in the ordinary M／G／1 queue and the convex combination of the two remaining vacation times. The weights can be interpreted in this way. The mean total vacation duration is given by eq.(2.5). Out of this, the portion that the vacation of first type takes is $E(V)[1-U^*(\lambda)V^*(\lambda)]$. The ratio of this portion to the whole vacation period is given by

$$w_V=\frac{E(V)}{E(V)V^*(\lambda)E(U)}$$

In the similar way, the second weight w_U is the contribution by the second type of vacations. Hence the proportion is give by

$$w_U=1-w_V=\frac{V^*(\lambda)E(U)}{E(V)+V^*(\lambda)E(U)}$$

In this way we can derive the waiting time distribution(hence the system size distribution, too) of a queueing system with any mixture of vacations without resorting to the complicated system equations. All we need to calculate is the contribution of each type of vacations to the whole vacation. For example, let us consider the system with vacation sequence {V, V, U, U, U···}. The mean total vacation period can be esaily obtained and is given by

$$E(T_V)=\frac{(1-V^*(\lambda))(1-U^*(\lambda))V^*(\lambda)E(V)+(V^*(\lambda))^2E(U)}{1-U^*(\lambda)}$$

Hence, contributions of each type of vacation to the whole vacation period become

$$w_V=\frac{E(V)(1-V^*(\lambda))(1-U^*(\lambda))V^*(\lambda)}{E(V)(1-V^*(\lambda))(1-U^*(\lambda))V^*(\lambda)+(V^*(\lambda))^2E(U)}$$

$$W_u=1-W_V.$$

Then the LST of this vacation system can be obtained from eq.(4.2)

5. T-policy Revisited

If both types of vacations are of fixed length T, then our system becomes the M/G/1 queue with T-policy introduced by Heyman[3]. Heyman proved that there exist a certain value of T that optimizes the system under a cost structure. We let the lengths of vacations be fixed values T_1 and T_2. That is, the server scans the system T_1 time units after the system becomes empty. If any customer is found, the server begins to serve. If no customers are found, server scans the system in T_2 time units and so on until any customers are found. Let h be the customer holding cost per unit time for each customer and R be the server scanning cost per scan. Then during the cycle time(i.e.T_V+T_B), the mean number of scans, N_s, is given by

$$N_s=(1-V^*(\lambda))+2V^*(\lambda)(1-U^*(\lambda))+3V^*(\lambda)U^*(\lambda)(1-V^*(\lambda))+\cdots$$
$$=[1+V^*(\lambda)]/[1-V^*(\lambda)U^*(\lambda)] \cdots\cdots (5.1)$$

Hence during a cycle time, total scanning cost becomes

$$C_s=R[1+V^*(\lambda)]/[1-V^*(\lambda)U^*(\lambda)] \cdots\cdots (5.2)$$

Then scanning cost per unit time, C_U, is given by

$$C_U=C_s/[E(T_V)+E(T_B)]$$
$$=R(1+V^*(\lambda))(1-\lambda E(S))/[T_1+V^*(\lambda)T_2] \cdots\cdots (5.3)$$

The customer holding cost becomes hL, where L is given by(3.41) with E(V), E(U), $E(V^2)$ replaced by T_1, T_2, T_1^2, and T_2^2 respectively. Then total cost per unit time is given by

$$C(T_1, T_2)=C_U=hL \cdots\cdots (5.4)$$

Partial derivatives with respect to T_1, and T_2 tell us that the system is optimized when $T_1=T_2$

=T. Then the optimal value of T is given by

$$T^* = \frac{2[1-\lambda E(S)]R}{ah} \quad \cdots\cdots (5.56)$$

which is the result of Heyman[3].

6. Summary

We have obtained the server idle probability, the system size distribution and the waiting time distribution for a mixed-vacation system. The decomposition property was shown to work for this type of vacation systems too. We have proposed a method that enables one to directly obtain the system size distribution and the waiting time. This can be accomplished by calculating the contributions of each vacation to the whole vacation period. We revisited the T-policy of Heyman and showed that under the same cost structure, the system cost is minimized when two scanning intervals are the same.

7. Acknowledgement

The author would like to thawk Korea Scieuce and Engineering foundation for funding and anonymous referees for helpful Comments.

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