Learning Effects on a Joint Buyer/manufacturer Inventory Model

안전재고의 경제적 품질률 결정에 관한 연구
-철도차량부품을 중심으로-

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요 약

Joint inventory 방법을 다룬 기존의 연구는 생산비용이 일정하다는 조건만을 고려하였다. 본 논문은 기존의 연구에다 새로운 변수(learning curve ratio and learning retension)를 제조업자 측면에서 고려하여 보다 확장된 모델을 다룬다. Joint inventory 모델은 첫째 단일구매자와 둘째 학습곡선비율과 learning retention의 정도에 있어서 그 범위를 결합시키는데 이용되기 위해 개발되어 졌다. 구매자와 제조업자를 위한 로트 사이즈를 결정하기 위하여 증분비용접근방법(Incremental Cost Approach. ICA)을 쓴다.

총결합비용은 기존모델보다 현저하게 적은데 그 이유는 학습과 learning retention 효과로 인한 제조업자의 생산비 절감과 재고유지비용의 감소 때문이다.

학습과 learning retention이 현격한 경우, 총결합비용은 제조업자와 구매자의 개별적인 최적정책에서의 비용합(合)보다 적다.

소개된 모델의 효과를 보이기 위해 수치예제를 이용하였다.

1. Introduction

In several industrial market situations, the manufacturer's production policy is influenced by the buyer's order. In the case of a single buyer, the order for an item (or product) may be received at regular time intervals. The order quantity is assumed to be deterministic. The buyer and manufacturer can minimize their total variable costs without considering the cost incurred by the other.

In such a situation, the buyers place orders based on their deconomic order quantity (EOQ) and the manufacturer pursues an economic production quantity (EPQ). Instead of determining buyer and manufacturer policies independently, if both parties decide to cooperate (not necessarily within the same organization) and determine a joint economic ordering policy, both parties could possibly achieve considerable savings.

In common manufacturing production procedures, the time (or cost) to produce a unit of the product decreases over time as the volume of production increases due to the learning effects. Also a loss of learning occurs during breaks between successive production runs of the product.

Most work to date in the determination of lot size for joint inventory model assume a constant production cost and production rate (i.e., they have not incorporated the learning effects). The production time of manufacturing is decreased due to the learning effects and this decrease in production time also effects the inventory holding cost.

The objective of this paper is to develop a joint buyer/manufacturer inventory model - which is extended to include the production cost under a range of the learning curve ratio and the levels of learning retention caused by breaks in production.

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Recent studies of the joint inventory model to a single buyer have dealt with the following two catagories.

* Manufacturer's Production Quantity
  lot—for—lot of buyer's order quantity [3,4,6,7,13,15] integer multiple of buyer's order quantity [9,17]

* Manufacturer's Inventory Holding Cost
  infinite production rate [9,15,17]
  finite production rate [3]

The specifics of our analysis differ from previous works in a number of ways. First, the manufacturer's production quantity is equal to the net requirements in an integral number of consecutive planning periods.

Second, two cases are examined which incorporate the manufacturer's inventory holding cost. In the first case the manufacturer's inventory holding cost is considered during the production period when the manufacturer's production rate is similar to the demand rate of the buyer. In the second case the manufacturer's inventory holding cost is ignored during the production time when the manufacturer has an infinite production rate. The general model incorporating the inventory holding cost presented in this paper encompasses work of earlier researchers when certain restrictions of the previous works are applied.

Section 2 is devoted to reviewing earlier works on related problems. In Section 3 the joint inventory model for single buyer's case is developed. The algorithms for the joint inventory model were programmed in Fortran 77 and run on an IBM PC. The comparison with existing theory is illustrated with an example. The conclusion to this paper is presented in Section 4.

2. Current Literature Eeview

2.1 Learning Curve

The concept of the learning curve is described as follows' [16, p.376]

"Every time production of a product doubles, the new cumulative average cost (labor hours) declines by a fixed percent of the previous cumulative average. This constant is expressed as a % and is called the learning curve ratio (LCR), and identifies the learning achieved"

Mathematically the learning curve may be expressed as following [11]:

$$Y_X = Y_1 X^{-b} \qquad (2.1)$$

where

Y_X : Cumulative average production time for first X units.
Y_1 : Production time required for the first unit.
b : *Coefficient dependent on learning curve ratio.* $= \dfrac{\text{Log of learning curve ratio}}{\text{Log of 2}}$
X : Xth unit in the series being produced.

2.2 Effects of Production Breaks on Learning

In production the forgetting of learning occurs during periods of production breaks. A break in production of a minimum of several days will have an adverse effects on average production time. This decrease in production time is caused by a lack of learning retention between successive lots.

Hoffman [10] and Cochran [5] use the concept of the displacement of the number of product units to represent the loss of learning retention.

In incorporating this concept with the production lot size, Adler and Nanda [1,2] apply the concept of production breaks for lot size to cases of single and multiple products.

According to Adler and Nanda [1, p.116], there is a loss of learning retention between lot j−1 and j. The average production time per unit for j^{th} lot of q_j is

$$Y_{q_j}=\frac{Y_1[(q_j+\alpha\sum_{i=1}^{j-1}q_i)^{1-b}-(\alpha\sum_{i=1}^{j-1}q_i)^{1-b}]}{q_j} \quad\cdots\cdots (2.2)$$

where

α : Percentage of learning retention after production breake.

q_i : Number of units produced between two adjacent production breaks.

$\alpha\sum_{i=1}^{j-1}q_i$: Equivalent units of learning at start of jth lot.

Taking the average production time per unit, the total production time for q_j is

$$t_j=Y_1[(q_j+\alpha\sum_{i=1}^{j-1}q_i)^{1-b}-(\alpha\sum_{i=1}^{j-1}q_i)^{1-b}] \quad\cdots\cdots (2.3)$$

Since $t_j=q_j\ Y_{q_j}$

2.3 Effects of Learning on Production Lot Size

Incorporation of the learning curve into production lot size has been studied by several authors. Fisk & Ballou [8], Keachie & Fontana [12], Sparadlin & Pierce [19] and Smunt & Morton [18] show that successive production lots may not be of the same size and outline a dynamic programming (DP) for determining the production lot size. Adler and Nanda [1,2] analyze the effects of learning on various production lot size models. They studied the more general situation which has the same production time and the same production quantity for each cycle for single and multiple product cases. All of the above works have been under the assumption of constant (or time−invariant) demand.

The conclusion of these works is that learning is a significant factor in the determination of an optimal production lot size. Significant cost savings, relative to the classical production model (EPQ), occur whenever the demand rate is high relative to the production rate.

2.4 Incremental Cost Approach (ICA) Concept

Assuming each period requirements are greater than zero, the number of setups required to implement this feasible production policy are n (number of period). In order to reduce the number of setups to n−1, the demand of one of the periods, except the first period should be produced in the immediate preceding period having a setup. Although there exist n−1 such proposals, the proposal which yields the minimum total variable cost is the one required.

The incremental cost for each period is calculated as follows:

Incremental cost=savings of setup cost+production savings+extra inventory holding cost

If the minimum incremental cost obtained is negative, the maximum possible cost reduction is obtained by reducing the setup from n to n−1 by incorporating the corresponding proposed production. Accordingly,

Total varible cost of proposed production
=Total variable cost of existing production+Incremental Cost.

or

$$TCM(n-1)=TCM(n)+IC_i \quad\cdots\cdots (2.4)$$

The next step is to reduce the problem of determining feasible production policy for a planning horizon of n−1 periods. Similarly, minimum cost production policies n−2, n−3, ⋯⋯1 a setup can be determined from following the same logic. The proposed production is acceptable, and the iterations continue as long as the incremental cost associated with increase in the lot size is negative. If the minimum of the incremental cost obtained while trying to reduce the present number of setups by one is positive, then the present feasible production lot size is best. If it is zero, there exists an alternative best solution.

2.5 Joint Inventory Model

The literature search of the joint inventory model may be grouped under the following categories:

* Manufacturer's Production Quantity

There are two types of manufacturer's production quantity in order to meet the demand of the buyer. Most researchers [3,4,6,7,13,15] consider as a major assumption that the manufacturer will respond to the buyer's order by adapting the first one, lot—for—lot production policy. Goyal [9] introduced the joint cost of the buyer and manufacturer with a joint optimal policy resulting in the second, the manufacturer's production quantity being a multiple of the buyer's order quantity. Lee & Rosenblatt [14] studied both cases and generalized the manufacturer's production quantity category.

The integer multiple of the buyer's EOQ is shown in Figure 2.1. Since the buyer's order quantity is q^*, the manufacturer is faced with a stream of q^* units at fixed intervals of q^*/D years apart. The assumption is that the manufacturer's production quantity is some integer multiple of the buyer's EOQ ($IM \times q^*$). As shown in Figure 2.1, the inventory levels during the period become $(IM-1)xq^*$, $(IM-2)xq^*$, ..., q^*, and o.

The previous integer multiple assumption assumes that a manufacturer's production quantity is constant over an infinite planning horizons. Successive production lot sizes of the manufacturer may not be of the same size when 1) the production time for each period is variable due to learning and learning retention effects or, 2) the manufacturer is faced with a number of buyers whose order quantities are not the same.

* Manufacturer's Inventory Holding Cost

Most researchers assume an infinite production rate so that the production time of the manufacturer is negligible and the manufacturer's inventory holding cost is eliminated. As shown in Figure 2.1, the manufacturer either places the order of $IM * q^*$ from his external vendor and ships an amount of q^* to the buyer or the manufacturer has an infinite production capacity compared to the buyer's demand so that the inventory holding cost of the manufacturer is not considered during the production time.

The only researcher who considered the real manufacturer and inventory holding cost to be effected by the production lead time case is Banerjee [3,4]. He considered the effects of finite production in computing the inventory holding cost.

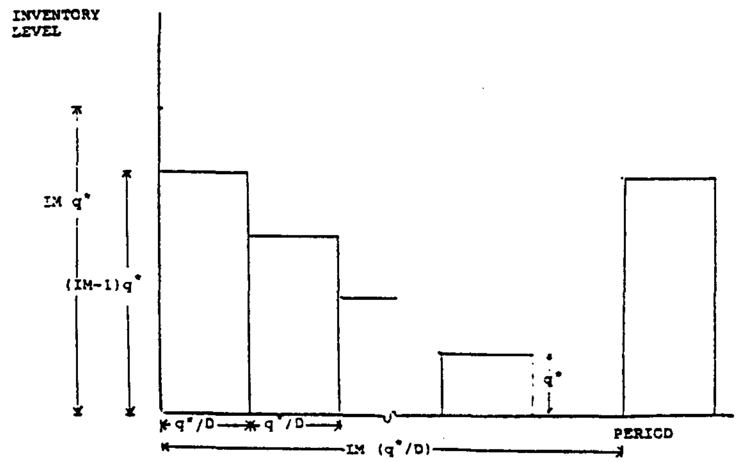

Fig.2.1 Manufacturer's Inventory Level with Integer Multiple of Buyer's Order Ruantity

When the production cost is decreased due to the learning effects, the holding cost during the production period is also a significant factor in determining the production lot size.

From this point of view, the general model incorporating the manufacturer's production rate is needed in the inventory holding cost category.

In this paper four theories listed below which have already developed in previous research are utilized to determine the lot size for the joint inventory model.

* cumulative average formula of learning curve
* concept of breaks for single product
* incremental cost approach

3. A Joint Inventory Model

The manufacturer may suggest that the buyer place a larger order size for future reorders. In this paper we assume that the increase of the order quantity means that the demand of some of the periods should be ordered in its immediate preceding period. This whole demand is received in the preceding period. This whole demand is received in the preceding period. A graphical portrayal of the inventory level with period for the buyer is illustrated in Figure 3.1. The increase of the order quantity, compared to EOQ, will create the extra inventory as shown by the large triangle versus the two small broken line triangles of Figure 3.1. However the buyer has a smaller ordering cost.

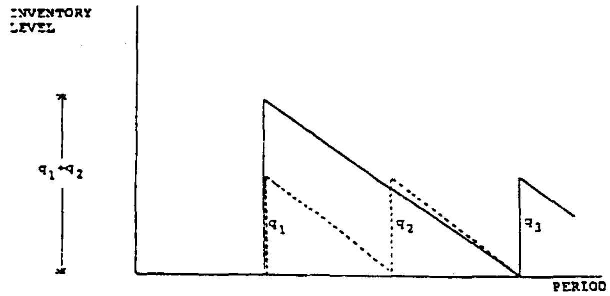

Fig.3.1 Buyer's Order Increase Policy for Single Buyer's Case

From the manufacturer's point of view, it is to his advantage to increase the buyer's order quantity. The manufacturer's production cycle times may change. This may reduce the cost of setup and create a reduction in holdin inventories. Additionally the manufacturer has the advantage of production cost with learning.

The manufacturer produces the same quantity to respond to the increased order quantity which is decided on by the cooperation of the parties.

All requirements (q_1+q_2) are shipped to the buyer at the end of period 1 and the only holding inventories for the requirements of periods 1 and 2 are incurred during the production time as shown in Figure 3.2.

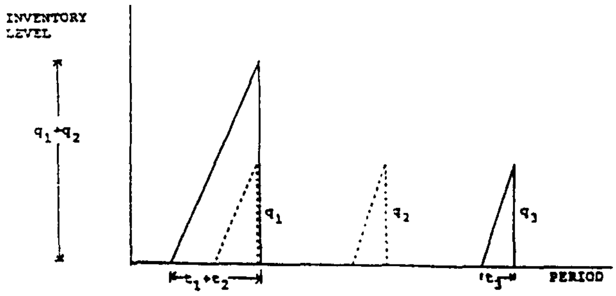

Fig.3.2 Manufacturer's Lot Sizing Policy for Single Buyer's CaseTable

The objective function for the joint inventory model is the minimization of the sum of the buyer's ordering and inventory holding costs and the manufacturer's setup, inventory holding, and production costs. The ICA is utilized to determine the joint lot size.

3.1 Assumptions

* The buyer's order quantity is determined by an EOQ policy with no shortages allowed.
* The only actual production time is the manufacturer lead time and the lead time for each buyer is negligible. We ignore the time it takes to send a buyer's order and the time it takes to deliver the completed lot to the buyer.
* The manufacturer's inventory holding cost is not only applicable during the build up of the lot but also carried over from one period to another period.
* The learning retention due to production breaks occurs as a percentage of the sum of previously produced lots. We don't consider the number of breaks between the production runs. This assumption is also used by other researchers [1,2,8,18] in the treatment of production lot size.

3.2 Single Buyer's Case

3.2.1 Cost Function

Lot−for−Lot policy

* Buyer's Total Variable Cost

The ordering cost is

$$n\ OCB \quad (3.1)$$

where OCB : Ordering cost of buyer per period.

n : Number of periods.

The inventory holding cost is

$$n(1/2\, q^2/D)\ HCB \quad (3.2)$$

where q : Order quantity of buyer.

D : Demand of buyer per year.

HCB : Holding cost of buyer per unit per year.

The total variable cost is

$$TCB(n)=n\ [OCB+(1/2\ q^2/D)\ HCB] \quad (3.3)$$

* Manufacturer's Total Variable Cost

The total variable cost of lot−for−lot policy is

$$TCM(n)=n\ SCM+\sum_{j=1}^{n} (1/2\ q\ t_j,\ HCM+K\ t_j] \quad (3.4)$$

* Joint Total Cost

The joint total cost is the sum of the buyer's (Eq. 3.3) and manufacturer's (Eq. 3.4) total variable cost.

$$JTC(n)=TCB(n)+TCM(n)=n\ [OCB+(1/2\ q^2/D)\ HCB]+n\ SCM+\sum_{j=1}^{n} (1/2\ q\ t_j\ HCM+K\ t_j) \cdots \quad (3.5)$$

3.2.2 Formulation of General Equations

To apply the algorithm the following general equations are needed:

Let

C_1 : Number of combined EOQ in immediate preceding period having a setup.

C_2 : Number of combined EOQ in current period.

Γ : $j-C_2$

ϵ : $j+C_2-1$

θ : C_1+C_2

Φ : $j-C_1-1$

δ : $j-C_1+1$

A_1 : $\alpha \sum_{i=1}^{\Phi} q_j$

A_2 : $\alpha \sum_{i=1}^{j-1} q_i$

$EHCB_j$: Extra inventory holding cost of buyer with increased order in period j.

$EHCM_j$: Extra inventory holding cost of manufacturer with proposed production in period j.

$$EHCB_j=[1/2\ \theta^2\ (q^2/D)-1/2\ C_1^2\ (q^2/D)-1/2\ C_2^2\ (q^2/D)]\ HCB$$
$$=[1/2\ (q^2/D)\ (\theta^2-C_1^2-C_2^2)]\ HCB \quad \cdots\cdots (3.6)$$

$$EHCM_j=[1/2\ \theta \sum_{k=\Gamma}^{\epsilon} t_k-1/2\ C_1 \sum_{k=j}^{j-1} t_k]\ q * HCM \quad \cdots\cdots (3.7)$$

$$t'_k=Y_1[(\sum_{i=\Gamma}^{k} q_i+A_1)^{1-b}-(\sum_{i=\Gamma}^{k} q_i-q_k+A_1 0^{1-b}] \quad \cdots\cdots (3.8)$$
where $k=j,\ j+1,\ \cdots\cdots \epsilon$

$$t'_k=Y_1[(\sum_{i=j}^{k} q_i+A_2)^{1-b}-(\sum_{i=j}^{j} q_i-q_k+A_2)^{1-b}] \quad \cdots\cdots (3.9)$$
where $k=j,\ j+1,\ \cdots\cdots \epsilon$

$$PS_j=K \sum_{k=j}^{\epsilon} (t_k-t'_k) \quad \cdots\cdots (3.10)$$

3.2.3 Algorithm

STEP 1. Take the demand schedule as lot-for-lot policy and assign the combined EOQ number of 1 in each period.

STEP 2. Compute the total variable cost of the buyer (TCB(n)) with number of setups n using Eq. 3.3

STEP 3. Compute the production time in each period using Eq. 2.3 and compute the total variable cost (TCM(n)) of manufacturer for lot-for-lot policy using Eq. 3.4.

STEP 4. Compute the joint total cost for lot-for-lot policy by adding both total variable costs

$$JUC(n)=TCB(n)+TCM(n)$$

STEP 5. The joint incremental cost for the first period and for the periods with joint lot size equal to zero is assigned a large positive value L.

STEP 6. Compute the extra inventory holding cost of the buyer ($EHCB_j$) using Eq. 3.6 and the incremental cost of the buyer (ICB_j) is

$$ICB_j=-OCB+EHCB_j$$

STEP 7. For the incremental cost of manufacturer in period j, compute as follow:

* proposed production times with Eq. 3.8
* existing production times with Eq. 3.9
* extra inventory holding cost ($EHCM_j$) with Eq. 3.7
* production saving (PS_j) with Eq. 3.10 $ICM_j=-SCM+PS_j+EHCM_j$

STEP 8. Compute the joint incremental cost (JIC_j)

$$JIC_j=ICB_j+ICM_j$$

Repeat steps 6 through 8 for period 2 through period n for the first iteration. From the second iteration onward those steps are repeated for the preceding and succeeding periods of the period identified with minimum joint incremental cost in the previous iteration.

STEP 9. Find the lowest joint incremental cost.

STEP 10. If all joint incremental cost are greater or equal to zero, then step the algorithm. This is best joint lot size for a single buyer's case, otherwise continue.

STEP 11. Combine the joint lot size, add the number of combined EOQ and compute the joint totol cost.

$$JTC(n-1)=JTC(n)+JIC_j$$

go to STEP 5.

3.2.4 Example

The same example which was presented and solved by Adler & Nanda [1] is used for the single buyer's case.

Total Requirements: R=1000 units

Cost per Unit Hour: K=$5/hour (1 year=2080 hours)

Coefficient of LCR: $b=.152=\frac{\text{Log of } 90\%}{\text{Log of } 2}$

Learning Retention: 10%

Time Required of the First Unit: $Y_1=1.87$ hours

Setup Cost of Manufacturer: SCM=$50/setup

Inventory Holding Cost: HCM=$10/year/unit

=$.0048/hr/unit

Buyer's EOQ: $q^*=100$ units

The solution procedure is explained based on the algorithm and is shown in Table 3.1.

STEP 1. The requirements for lot-for-lot policy is $q_j=100$ units and the combined EOQ number is 1 for all periods.

STEP 2. The total variable cost of buyer with number of setups to using Eq. 3.3 is

$TCB(10)=10\ [50+(1/2\ 100^2/1000)*10]=1000.$

STEP 3. Applying Eq. 2.3 to compute the production time for requirements in each period gives the following

$t'_1=1.87*(100)^{.848}=92.86$

$t'_2=1.87*[(100)+.1*100)^{.848}-(.1*100)^{.848}]$

$=87.50$

Similarly, $t'_3=84.67$, $t'_4=82.55$, $t'_5=80.83$, $t'_6=79.38$, $t'_7=78.12$, $t'_8=77.01$, $t'_9=76.01$, $t'_{10}=75.11$

Total variable cost with 10 setups using Eq. 3.4 is

$TCM(10)=10*50+\sum_{j=1}^{10}[1/2\ 100*t'_j*.0048)+5*t'_j]$

$=4766$

STEP 4. Joint total cost for lot-for-lot policy is

$JTC(10)=TCB(10)+TCM(10)$

$=1000+4766=5766$

STEP 5. $IC_1=L$

STEP 6. Incremental cost of buyer for period 2,

- Extra holding cost of buyer using Eq. 3.6 is

  $EHCB_2=[1/2*(100^2/1000)\ (2^2-1-1)]*10$

  $=100$

  where, $C_1=1$, $C_2=1$, $0=2$

- Incremental cost is

$ICB_2 = -50 + 100$

$= +50$

STEP 7. Incremental cost of manufacturer for period 2.

* proposed production time using Eq. 3.8 is

$t_2 = 1.87[(\sum_{i=1}^{2} q_i)^{.848} - (\sum_{i=1}^{2} q_i - q_2)^{.848}]$

$= 74.29$

* existing production time using Eq. 3.9 is

$t'_2 = 1.87[(100+10)^{.848} - (100-100+10)^{.848}]$

$= 87.50$

* production saving using Eq. 3.10 is

$PS_2 = 5 * (74.29 - 87.50)$

$= -66.06$

where $j=i=2$, $C_1=1$, $C_2=1$, $\Phi=0$, $\Gamma=1$, $\delta=2$, $\epsilon=2$, $A_1=0$, $A_2=10$, $T_1=208$ hrs, $T_2=416$ hrs

* extra inventory holding cost of manufacturer using Eq. 3.7 is

$EHCM_2 = [1/2 * 2 * (92.86 + 74.29) - 1/2 * 92.86 - 1/2 * 87.50] * 100 * .0048 = 36.95$

where $j=i=2$, $C_1=1$, $C_2=1$, $\theta=2$, $\Gamma=1$, $\epsilon=2$.

* Incremental cost of manufacturer is

$ICM_2 = -50 - 66.06 + 36.95$

$= -79$

STEP 8. Joint incremental cost is

$JIC_2 = ICB_2 + ICM_2$

$= 50 - 79$

$= -29$

Similarly, $JIC_3=-20$, $JIC_4=-13$, $JIC_5=-8$, $JIC_6=-4$, $JIC_7=-1$, $JIC_8=+1$, $JIC_3=+4$ $JIC_{10}=+5$.

STEP 9. The minimum JIC identified is JIC_2

$= -29$

STEP 10. There exist negative values of joint incremental cost.

STEP 11. Combine the joint lot size ($Q_1 = 100 + 100 = 200$), the combined EOQ number is 2,0,1,1,1,1,1,1,1,1 for each period and compute the joint total cost

$JTC(9) = JTC(10) + JIC_2$

$= 5766 - 29$

$= 5737$

STEP 5. $JIC_2 = L$

STEP 6. Incremental cost of buyer for period 3.

* Extra holding cost of buyer using Eq. 3.6 is

$EHCH_3 = [1/2 * (100^2/1000)\ (3^2 - 2^2 - 1)] * 10$

$= 200$

where, $C_1=2$, $C_2=1$, $\theta=3$

* Incremental cost is

$ICB_2 = -50 + 200$

$= +150$

STEP 7. Incremental cost of manufaacturer for period 3.

* proposed production time using Eq. 3.8 is

$t_3 = 1.87[(\sum_{i=1}^{3} q_1)^{.848} - (\sum_{i=1}^{3} q_1 - q_3)^{.848}] = 68.59$

* existing production time using Eq. 3.9 is

$T'_3 = 1.87[(100+20)^{.848} - (100-100+20)^{.848}]$

$= -80.40$

* production saving using Eq. 3.10 is

$PS_3 = 5 * (68.59 - 84.67)$

$= -80.40$

where $j=i=3$, $C_1=2$, $C_2=1$, $\Phi=0$, $\Gamma=1$, $\delta=2$, $\epsilon \doteq 3$, $A_1=0$, $A_2=20$, $T_3=614$ hrs, $T_1=208$ hrs

* extra inventory holding cost of manufacturer using Eq. 3.7 is

$EHCM_3 = [1/2 * 3 * \sum_{k=1}^{3} t_k - 1/2 * \sum_{k=1}^{2} t'_k - 1/2\ t'_3] * 100 * .0048$

$= 69.19$

where $j=i=3$, $C_1=2$, $C_2=1$, $\theta=3$, $\Gamma=1$, $\epsilon=3$

* Incremental cost of jmanufacture is

$ICM_3 = -50 - 80.40 + 69.19$

$= -61$

STEP 8. Joint incremental cost is

$JIC_3 = ICB_3 + ICM_3$

$= 150 - 61$

$= +89$

Table 3.1 shows that from the second iteration onward the joint incremental cost remains unchanged except for the preceding and succeeding periods of the period identified with the minimum joint incremental cost obtained from the previous iteration.

STEP 9. The minimum JIC identified is $JIC_4 = -13$

STEP 10. There exist negative values of joint incremental cost.

STEP 11. Combine the joint lot size ($Q_3=200$), the combined EOQ number is 2,0,2,0,1,1,1,1,1,1,1 for period and compute the joint total cost

$JTC(8) = JTC(9) + JIC_4$

$= 5737 - 13 = 5724$

Report the algorithm until the minimum value of the joint incremental cost is positive. The best solution is obtained in the 4^{th} iteration. The joint lot size is 200 units every other period for the first 3 lots and others are 100 units every period with a joint total cost of \$5720 (\$4570 for manufacturer and \$1150 for buyer).

3.3 Comparison with Existing Theory

Consider the case of the joint inventory model for the single buyer's case with lot–for–lot policy. An item produced to order by a manufacturer. A single buyer periodically orders and buys a batch of this item from the manufacturer, who is the buyer's sole source for this item

The following parameters are given: requirements (R)=1000 units/year, production rate (P_r)=3200 units/year, setup cost of manufacturer (SCM)=\$400/setup, ordering cost of buyer (OCB)=\$100/order, inventory holding cost of manufacturer (HCM)=\$5/unit/year, and inventory holding cost of buyer (HCB)=\$4/unit/year.

This is an existing example which is presented and solved by Banerjee (3,4) and is compared with the results of the work done in this paper. The planning length was extended from 1000 units to 2000 units to better illustrate the joint total cost difference. Although the value of the production rate assigned above

ITERATION	PERIOD	1	2	3	4	5	6	7	8	9	10	TOTAL V. COST
	Q_j	100	100	100	100	100	100	100	100	100	100	
1	ICMj	L	- 79	- 70	- 63	- 58	- 54	- 51	- 49	- 46	- 45	TCM: 4766
	ICBj	L	+ 50	+ 50	+ 50	+ 50	+ 50	+ 50	+ 50	+ 50	+ 50	TCB: 1000
	JICj	L	- 29*	- 20	- 13	- 8	- 4	- 1	+ 1	+ 4	+ 5	JTC: 5766
	Q_j	200	0	100	100	100	100	100	100	100	100	
2	ICMj	L	L	- 61	- 63	- 58	- 54	- 51	- 49	- 46	- 45	TCM: 4687
	ICBj	L	L	+150	+ 50	+ 50	+ 50	+ 50	+ 50	+ 50	+ 50	TCB: 1050
	JICj	L	L	+ 89	- 13*	- 8	- 4	- 1	+ 1	+ 4	+ 5	JTC: 5737
	Q_j	200	0	200	0	100	100	100	100	100	100	
3	ICMj	L	L	- 36	L	- 54	- 54	- 51	- 49	- 46	- 45	TCM: 4624
	ICBj	L	L	+350	L	+150	+ 50	+ 50	+ 50	+ 50	+ 50	TCB: 1100
	JICj	L	L	+314	L	+ 96	- 4*	- 1	+ 1	+ 4	+ 5	JTC: 5724
	Q_j	200	0	200	0	200	0	100	100	100	100	
4	ICMj	L	L	- 36	L	- 32	L	- 44	- 49	- 46	- 45	TCM: 4570
	ICBj	L	L	+350	L	+350	L	+150	+ 50	+ 50	+ 50	TCB: 1150
	JICj	L	L	+314	L	+318	L	+106	+ 1	+ 4	+ 5	JTC: 5720

NOTE: R = 1000 UNITS/YEAR, K =$5/HOUR, LCR = 90%, α = 10%, Y_1 = 1.87 HRS/UNIT, SCM = $50/SETUP, HCM = $10/UNIT/YEAR, Y_1 = 1.87 HRS/UNIT, BUYER EOQ = 100 UNITS.

TCM : TOTAL VARIABLE COST OF MANUFACTURER
TCB : TOTAL VARIABLE COST OF BUYER
JTC : JOINT TOTAL COST
TOTAL V. COST : TOTAL VARIABLE COST

3.1 Joint Lot Size and Joint Jncremental Cost for Single Buyer's Case

becomes an upper limit on the unit production time whenever learning is assumed to occur, we assume the reciprocal for the production rate ($1/P_r$) as being the production time for the first unit in order to apply and compare our model with the existing example.

Problem 1 in Table 3.2 is Banerjee's example. The learning curve ratio (95%) and learning retention (0%, 60%) are applied on the manufacturer's side in problems 2 and 3. In problem 4 the learning curve ratio decrease (75%) and a high learning retention (80%) are applied. The joint total cost is decreased significantly from $15400 (problem 1) to $10854 (problem 3) even with a high learning curve ratio. When the learning is increased (75%) in problem 4, the saving of joint total cost is much more significant.

The cost difference (see Table 3.2) with learning effects is only $40 higher than that of the sum of individual total variable costs (compared with Banerjee's $1000 higher cost difference). When retention of learning is highf and the learning curve ratio is low, as shown in problems 3 and 4 in Table 3.2, the joint total cost is smaller than the sum of individual total costs. One of the reasons for the high joint total cost with the existing joint iventory model is that it does not consider the savings of inventory holding cost. This is because the manufacturer responds to the buyer's demand with a lot—for—lot assumption. Another reason is that it does not consider the advantage to production cost under learning effects.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
PROBLEM		LCR	α	INDIVIDUAL LOT SIZE	TOTAL COST	JOINT LOT SIZE	I.COST	J.COST	SAVING	COST DIFFERENCE
1	M	0%	.0	800	12400*	400	14650*	12900*	1750	
	B			200	2000		2000	2500	- 500	
	T				14400		16650	15400	1250	+ 1000
2	M	95%	.0	800(2),400	9456	400(5)	11195	8996	2200	
	B			200(10)	2000		2000	2500	- 500	
	T				11456		13195	11496	1700	+ 40
3	M	95%	.6	400(5)	8864	400(5)	10243	8354	1889	
	B			200(10)	2000		2000	2500	- 500	
	T				10864		12243	10854	1389	- 10
4	M	75%	.8	400(5)	3265	400(5)	4482	2487	1995	
	B			200(10)	2000		2000	2500	- 500	
	T				5265		6482	4987	1495	- 278

NOTE M : MANUFACTURER
B : BUYER
T : TOTAL
I.COST : INITIAL TOTAL VARIABLE COST
J.COST : JOINT TOTAL COST
SAVING : I.COST - J.COST
COST DIFFERENCE : T of Col (9) - T of Col (6)
* TOTAL VARIABLE COST OF MANUFACTURER INCLUDED THE PRODUCTION COST
($5.2/UNIT, Y_1=.65HRS, K=$8/HR)

Table 3.2 Comparison Table with Existing Joint Inventory Model

4. Conclusion

Under learning and learning retention effects in the production environment, the joint lot size decreases and results in lower joint total cost. The most important joint total cost savings realized by the manufacturer are in the production and inventory holding components when significant learning and learning retention are expected.

Under significant learning and learning retention, the joint total cost with a joint inventory model is lower than the sum of the total variable costs incurred by the buyer and the manufacturer when both have separate optimizing policies.

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