Design of a Discrete Variable Structure Tracking Controller with Adaptive Feedforward Gains

(적응 순방향 이득을 갖는 이산가변 구조추종 제어기의 설계)

李 晟 準*, 李 康 熊*, 崔 桂 根*

(Sung Jun Lee, Kang Woong Lee and Keh Kun Choi)

要 約

본 논문에서는, 이산 가변 구조시스템(discrete variable structure systems)에서 스위칭 평면 상에서의 준 슬라이딩 모드(quasi- sliding mode)의 존재를 보장하고 동시에 도달위상(reaching phase) 문제를 해결하는 조건을 제시하였다. 또한 적응 제어 이론을 이용하여 이산가변 구조 추종 제어기를 설계하였다. 제안된 제어기는 가변 구조 제어기의 특징인, 외란이나 시스템 변수의 변동에 강한 특성을 가지고 있으며 기존의 적응 제어기에 비해 계산량이 적은 장점을 가지고 있다.

Abstract

In this paper conditions are derived, which ensure the existence of a quasi-sliding mode on the control switching hyperplane in discrete variable structure control systems and also remove the reaching phase problem observed in continuous-time variable structure systems. In addition, a discrete variable structure tracking controller which has adaptive properties is devised based on these results. This controller has useful properties, such as small sensitivity to the variation of plant parameters and to disturbances and its performing speed is fast compared to that of other adaptive controllers.

I. Introduction

Variable structure systems (VSS), widely studied by numerous Soviet authors, are characterized by a discontinuous control action which changes system structure on reaching a set of switching hyperplanes. This discontinuous control law results in the sliding mode, main feature of VSS. During the sliding mode, the system has properties of invariance to plant parameters and disturbances. With these excellent robustness properties, the theory of VSS has many applications in non-linear systems and plants with imprecise knowledge of their parameters [3] - [5] and recently it is applied to the design of a model reference adaptive control system [6] - [9] by Young et.al..

However, ideal sliding mode requires infinite frequency switching in control input but due to the certain non - idealities, such as delay in control input, system acquires only quasi-sliding mode at the expense of chattering problems.

In this paper, the theory of VSS is applied to

*正會員, 서울大學校 電子工學科

(Dept. of Elec. Eng., Seoul Nat'l Univ.)

接受日字 : 1987年 5月 30日

discrete system which has inherent delay in its feedback control loop. This delay nature of discrete system is related to the quasi-sliding mode as in the continuous case with control delay. We determine the conditions for the existence of the quasi-sliding mode improving the reaching problem and design the discrete variable tracking controller based on these conditions and adaptive control theory.

II. Discrete Variable Structure Systems

1. System definition

Let the continuous VSS be described by the controlled plant

$$\dot{X}(t) = A_c X(t) + b_c u(t) \tag{1}$$

$$X(t) = [x_1(t),\dots,x_n(t)]^T$$

$$Ac = \begin{bmatrix} 0 & I_{n-1} \\ -a_{c1} & -a_{c2}, \dots, -a_{cn} \end{bmatrix}, \; b_c = [0,0,\dots,b_{cn}]^T$$

with the control input and the hyperplane which defines sliding mode

$$u(t) = -F_c^T X(t) \;;\; F_c = [f_{c1},\dots,f_{cn}]^T$$
$$sc(t) = c^T X(t) \;;\; c = [c_1,\dots,c_n]^T \tag{2}$$

According to the result given by Itkis [1], the above VSS can be stabilizable and its quasi-sliding mode exists when the control input is sampled by the interval T, i.e.

$$u(t) = u(kT) \;;\; kT \leqslant t < (k+1)T \tag{3}$$

provided that

$$T < (1+a)\ \ln\left[(1+a+K)K^{-1}\right]$$
$$K = \left[\sum_{i=1}^{n-1} f_{ci}^2\right]^{1/2} \tag{4}$$
$$a = \left[\sum_{i=1}^{n-1} |a_{ci}|^2\right]^{1/2}$$

This means that the sliding mode is replaced by the quasi-sliding mode when there is limited delay in the control input and the resulting system is still controllable by the variable structure control law.

Now, for the system (1), corresponding discrete-time system obtained by sampling its input and output with the discretization period T is

$$X((k+1)T) = AX(kT) + bu(kT)$$
$$X(kT) = [x_1(kT),\dots,x_n(kT)]$$
$$A = \mathcal{L}^{-1}(sI-A_c)^{-1}\Big|_{t=T} = \Phi(T) \tag{5}$$
$$b = \int_0^T \Phi(T-\tau) b_c d\tau$$

If we define the discrete variable structure control law and the discrete hyperplane as

$$u(kT) = -F^T X(kT) \;;\; F = [f_1,\dots,f_n]^T \tag{6}$$

$$sd(kT) = c^T X(kT) \;;\; c = [c_1,\dots,c_n]^T \tag{7}$$

then we can say that the suggested DVSS may have quasi-sliding mode and can be controllable by the variable structure control law if its discretization period is in the limit described by eq(4). In what follows, kT is denoted simply by k.

2. Quasi-sliding conditions in DVSS

The sliding conditions in continuous VSS are given by the following inequalities [1], [2].

$$\lim_{sc\to o^+} \dot{sc} \leqslant 0, \qquad \lim_{sc\to o^-} \dot{sc} \geqslant 0 \tag{8}$$

It may be interpreted that the representative point (RP) of the system in the phase space, having once hit the hyperplane (sc(t)=0), cannot leave a small neighborhood of the hyperplane. In our DVSS, we can apply the same physical meaning to the system (5) so as to derive the quasi-sliding conditions in DVSS.

Consider the discrete hyperplane $sd(k) = 0$ in the phase space (see Fig.1). The direction vector of the system trajectory is $H = X(k+1)-X(k)$ and the following inequalities must be satisfied between the direction vector of the RP and the gradient vector of the hyperplane sd=0, according to our previous discussion.

$$[\text{grad}\ sd(k)]^T H = [c_1,\dots,c_n]\ [X(k+1)-X(k)]$$
$$= sd(k+1) - sd(k) \leqslant 0 \;;\; sd(k) >$$
$$\geqslant 0 \;;\; sd(k) < 0 \tag{9}$$

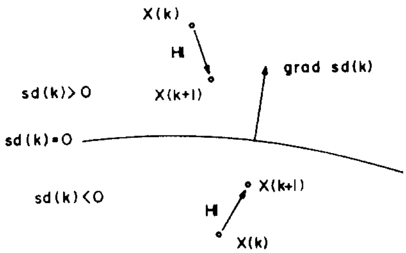


Fig.1. Grad sd(k) and H in phase space.

The above inequalities are identical to the results obtained by Milosavljevic using 'heuristic analogy' to the continuous case (in eq(8)) [10]. But regarding the stepwise nature of the discrete system, there are still another possibilities in DVSS.

Consider the following inequlity.

$$sd(k+1)\, sd(k) \leqslant 0 \tag{10}$$

This condition makes the RP pass the hyperplane in a single step -not asymptotically-regardless of its previous position in the phase space and in a neiborhood of the hyperplane, condition (10) converges to conditions (9). This inequality may provide more efficient way of removing the so-called "reaching-phase problem" observed in the continuous case, while this has shortcomings of causing relative large chattering.

Unfortunately both conditions are not the necessary and sufficient conditions for the existence of the quasi-slding mode in our DVSS but the inequalities (9) are necessary and the condition (10) is a useful criterion in determining the control gains.

3. Determining the control gains

We consider the design of DVSS for regulating the ouput of the system (5). First, we apply the conditions (9) to the system (5), then we have

$$\begin{aligned} sd(k+1)-sd(k) &= c^T X(k+1) - c^T X(k) \\ &= c^T (A-I) X(k) - c^T b F^T X(k) \leqslant 0 \; ; \quad sd(k) > 0 \\ &\geqslant 0 \; ; \; sd(k) < 0 \end{aligned} \tag{11}$$

where $u(k) = -F^T X(k) = -\sum_{i=1}^{n} f_i\, x_i(k)$

$$sd(k) = c^T X(k) = \sum_{i=1}^{n} c_i x_i(k)$$

They can be equivalently expressed for each control gain fi as

$$\begin{aligned} h_i\, x_i(k) - g\, f_i\, x_i(k) &\leqslant 0 \; ; \; sd(k) > 0 \\ &\geqslant 0 \; ; \; sd(k) < 0 \\ c^T (A-I) = [h_1, \ldots, h_n] &\quad ; \quad (i=1,\ldots,n) \\ c^T b = g & \end{aligned} \tag{12}$$

and the resulting conditions for each fi are

$$\begin{aligned} f_i = f_i^+ &\geqslant f_{ui} \; ; \; sd(k) x_i(k) > 0 \\ f_i^- &\leqslant f_{ui} \; ; \; sd(k) x_i(k) < 0 \\ f_{ui} &= h_i / g \; ; \; (i=1,\ldots,n) \end{aligned} \tag{13}$$

In order to avoid the complexity of the controller which may be caused by taking two values of f_i^+ and f_i^-, we simplify conditions (13) as follows;

$$\begin{aligned} f_i^+ &= |f_i| \; ; \; sd(k) x_i(k) > 0 \\ f_i^- &= -|f_i| \; ; \; sd(k) x_i(k) < 0 \; ; \; (i=1,\ldots,n) \end{aligned} \tag{14}$$

where $|f_i| \geqslant |f_{ui}| = |h_i/g|$

Now, if we follow the same procedure to the condition (10), we have for each fi

$$\begin{aligned} f_i^+ &= |f_i| \; ; \; sd(k) x_i(k) > 0 \\ f_i^- &= -|f_i| \; ; \; sd(k) x_i(k) < 0 \; ; \; (i=1,\ldots,n) \end{aligned} \tag{15}$$

where $|f_i| \geqslant |f_{oi}| = |j_i/g|$

$$c^T A = [j_1, \ldots, j_n]$$

Then the appropriate control gains may be obtained from the two inequalities as the function of sd, i.e. if the RP is located far apart from the hyperplane sd=0 then the conditions (15) make it reach the hyperplane fast and in a neighborhood of the hyperplane, the conditions (14) make the RP converge to the hyperplane rather smoothly. For that purpose, we take the control gains as follows

$$|f_i| = |f_{ui}| + (|f_{oi}| - |f_{ui}|)(1-\exp(-|sd(k)|/K) \quad ;(i=1,\dots,n) \tag{16}$$

where K is a constant.

Ⅲ. Adaptive Discrete Variable Structure Controller for Tracking Control

In this section we propose the controller for the plant (5) to track reference input. Since the state variable X(k) of the plant (5) consists of x_1 as a plant output and $x_2, \dots, x_n$ as successive derivatives of it in continuous-time sense, the reference input has the form $X^* = [x^*, 0, \dots, 0]^T$.

First, we construct the error system with its state variable E(k)=X*-X(k)as

$$\begin{aligned} E(k+1) &= X^* - X(k+1) \\ &= (I-A)X^* + AE(k) - bu(k) \end{aligned} \tag{17}$$

and the hyperplane

$$\begin{aligned} sd(k) &= c^T E(k) \\ &= [c_1, \dots, c_n]\ [e_1, \dots, e_n] \end{aligned} \tag{18}$$

The control input has the form

$$u(k) = F_E^T E(k) + F^{*T} X^* \tag{19}$$

However, for this tracking controller, the control input must have constant feedforward component in the steady state ($E(k) \approx 0$). In continuous-time case, the switching type control gain for the feedforward term is equivalently replaced by a constant (or nearly constant) gain due to its infinite (or very high) switching frequency [1], [2], [6], [8], [9] but, for discrete-time case, we cannot expect the same effect with the switching control gain as in (19) because of its finite siwtching frequency limited by the sampling time and the resulting control action in the steady state is very fluctuating.

Therefore we suggest new control law with the term of the feedforward control gain which is adaptively adjusted but not switched.

Consider the floowing control law.

$$u(k) = F_E^T E(k) + \hat{p}^T X^* \tag{20}$$

where E_E is normal switching gain matrix and $\hat{p}$ is an adjusted gain.

We may think that the F_E^T E(k) term corresponds to the AE(k) term in eq (17) and the $\hat{p}^T X$ term to (I-A)X. If we separate the eq (17), (18) into these two components as

$$\begin{aligned} E(k+1) &= [(I-A)X^* - b\hat{p}^T X^*] + [AE(k) - b F_E^T E(k)] \\ &= E_X(k+1) + E_E(k+1) \\ wd(k) &= c^T E(k) = c^T E_X(k) + c^T E_E(k) \\ &= sd_X(k) + sd_E(k) \end{aligned} \tag{21}$$

then we can apply the discrete variable structure control law to the subsystem represented as $E_E(K+1)$ and $sd_E(k)$.

$$\begin{aligned} & f_{Ei}^+ = |f_{Ei}| \ ;\ ad_E(k)ei(k) > 0 \\ & f_{Ei}^- = -|f_{Ei}| ;\ sd_E(k)ei(k) < 0 \\ & |f_{Ei}| \geqslant |f_{Eui}| = |h_i/g| \quad ;(i=1,\dots,n) \\ & (c^T b)^{-1}\ [c^T A] = [h_1, \dots, h_n]/g \\ & [f_{Eu1}, \dots, f_{Eun}] \end{aligned} \tag{22}$$

For the subsystem $E_X(k+1)$ and $sd_X(k)$, we apply the adaptive theory and perform the recursive estimation procedure to the control gain $\hat{p}$.

$$\hat{p}(k+1) = f(\hat{p}(k), sd_X(k), k) \tag{23}$$

However control laws (22) and (23) cannot be directly applied to the system because $sd_E(k)$ and $sd_X(k)$ are not obtained separately from sd(k). If the estimated parameter $\hat{p}(K)$ nearly converges to its true value $(c^T b)^{-1}\ [c^T(I-A)]$, $sd_X(k)$ approaches to zero and the following equality is satisfied.

$$\text{sign}\,[sd(k)] = \text{sign}\,[sd_E(k)] \tag{24}$$

Therefore eq (22) can be replaced by

$$f^{+}_{Ei} = \left|f_{Ei}\right| ;\ sd(k)e_i(k) < 0 \quad (25)$$

$$f^{-}_{Ei} = -\left|f_{Ei}\right| ;\ sd(k)\,e_i(k) < 0 \quad ;(i=1,\ldots,n)$$

Alternatively, if the subsystem is already in the quasi-sliding mode, then $sd_E(k)$ approaches to zero and

$$sd(k) \cong sd_X(k) \quad (26)$$

is safisfied. Therefore eq (23) can be replaced by

$$\hat{p}(k+1) = f(\hat{p}(k), sd(k), k) \quad (27)$$

The resulting DVSS is shown in Fig.2.

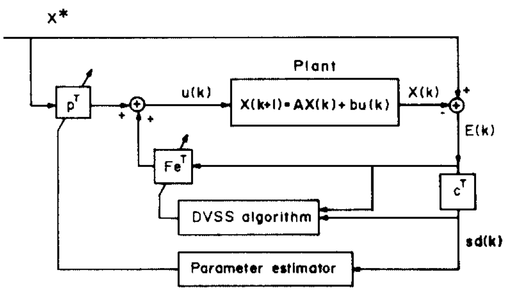


Fig.2. System block diagram.

It may appear that eq (24) and eq (26) have somewhat contradictory assumptions but the validity of proposed algorithm is proved as follows.

First, we assume that the condition (26) is violated in any initial state or in ko-th step, then the algorithm (27) estimates $\hat{p}$ (k0) which tries to nullify sd(k0+1) instead of $sd_X(k0+1)$.

$$\hat{p}(k0)^T X^* = (c^T b)^{-1}\ [c^T(I-A)X^* + sd_E(k0)] \quad (28)$$

Then from eq (21), the sd(k0+1) becomes

$$sd(k0+1) = c^T(I-A)X^* - c^T b\hat{p}(k0)^T X^* + sd_E(k0+1)$$
$$= sd_E(k0+1) - sd_E(k0) \quad (29)$$

and from eq (25), which is equivalently expressed as the condition $[sd_E(k+1)-sd_E(k)]\,sd(k) \leqslant 0$ (it is evident if eq (22) is compared with eq (25)), the following inequality is satisfied.

$$sd(k0+2)\ sd(k0+1) \leqslant 0 \quad (30)$$

We already know that this inequality means the existence of the quasi-sliding mode in the hyperplane sd(k0+1) =0 because the RP is located in a small neighborhood of sd(k0+1)=0 by the algorithm (27).

Therefore we can reach the following conclusions. If the condition (26) is violated, algorithm (25) and (27) make sd(k) zero simutaneously, the one by adjusting control parameters and the other inducing the quasi-sliding mode.

Once the system enters the quasi-sliding mode, then E(k) decays and the condition (26) is satisfied and it again satisfies the condition (24).

Since the estimation algorithm (27) operates correctly after the condition (26) is satisfied, the projection algorithm which is relatively slow but simple and gives some flexibilities against the variation of the system parameters is preferable. This fact meets the characteristics of the VSS, parameter invariance.

Since the reference input has the form [x*, 0.... 0], the vector $\hat{p}(k)$ needs only first element to be estimated and others are not used in the algorithm. This aspect informs us that the propsed algorithm has very high performing speed compared to other adaptive algorithms.

IV. Computer Simulation Results

The proposed tracking controller was adopted to the plant

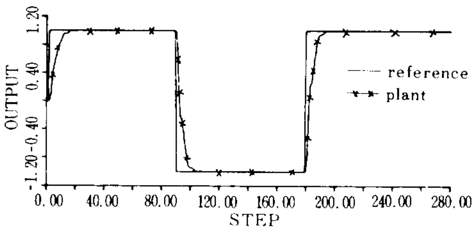
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9979 & 0.04639 \\ -.09278 & 0.8584 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.1189 \\ 4.639 \end{bmatrix} u(k)$$

and the simulation result for the pulselike reference input which varies +1, -1 alternatively was shown in Fig.3 with the control gains

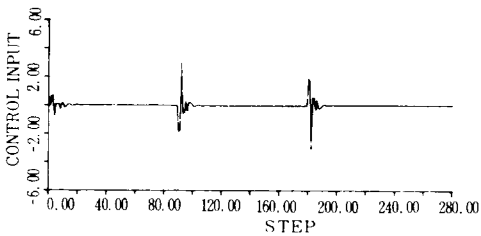
$$f_{E1}(k) = [.04+(1-.04)\,(1-\exp(-\left|sd(k)/10\right|))]\ \mathrm{sign}\,(sd(k)e_1(k))$$

$$f_{E2}(k) = [.03+(.2-.03)\,(1-\exp(-\left|sd(k)/10\right|))]\ \mathrm{sign}\,(sd(k)e_2(k))$$

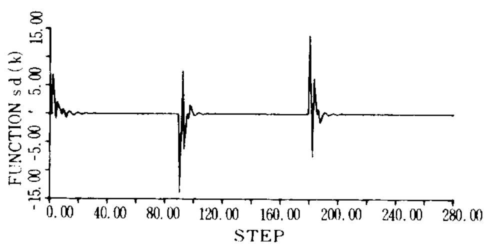
where f_{E1} and f_{E2} are obtained form eq (16) choosing f_{u1}=0.04, f_{o1}=1, f_{u2}=0.03, f_{o2}=0.2



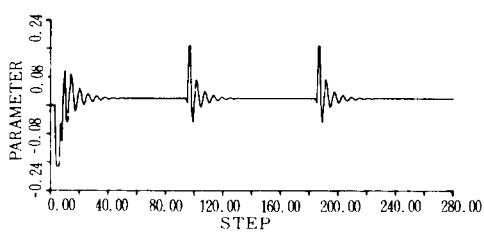
(a) output



(b) control input



(c) sd(k)



(d) parameter

Fig.3. Pulse response of the system.

and the estimation algorithm

$$\hat{P}(k+1) = \hat{P}(k) + \mu sd(k)$$

where μ is the coefficient which determines estimation speed and we choose $\mu = 0.2$

The parameter of the controller varies in relatively large amount at the change of reference input because the relationship between the controller structure and the estimation algorithm is weak, i.e. the correct estimation is achieved after the condition (26) is satisfied as previously disscused. But the estimation mechanism fills the role of it. In the steady state, it gives a constant feedforward component to the control input and in the transient, it makes the DVSS mechanism generate the quasi-sliding mode in sd(k) with its 'over-estimated parameter'. Therefore the resultant output response is satisfactory.

Fig.4 is the response of the system under the parameter variation caused by 10% random variation of the discretization period T. Fig.5 is the response of the system under the output disturbance represented in Fig 5 (a). These two figures show that the proposed DVSS has small sensitivity to the variation of plant parameters and disturbances, which is one of the main features of the VSS.

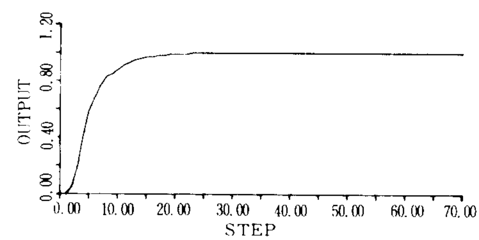
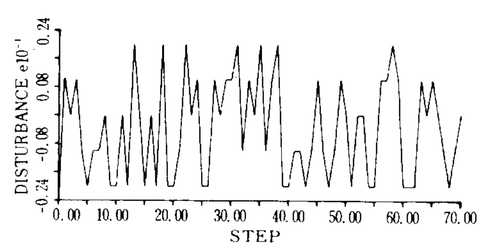
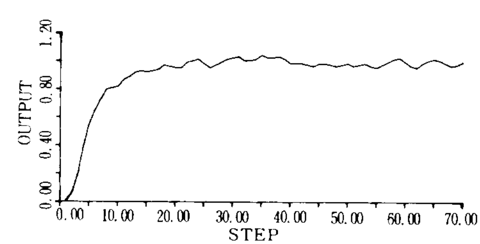


Fig.4. Step response of the system with parameter variation.



(a) disturbance



(b) output

Fig.5. Step response of the system with disturbance.

V. Conclusions

We applied the theory of variable structure systems in discrete case. Some conditions were derived, which ensure the existence of a quasi-sliding mode in discrete variable structure systems and improve the speed of reaching the hyperplane. Though these conditions are not sufficient, they provide good criterion for designing the discrete variable controllers. Based on these results and adaptive control theory, a kind of tracking controller was devised, which properties were demonstrated by computer simulations. Proposed controller has many useful properties which are the main features of continuous variable structure controllers, such as small sensitivity to the variation of plant parameters and to disturbances, and its algorithm performing speed is remarkably fast due to its simplicity compared to other adaptive controllers.

References

[1] U.Itkis, "Control systems of variable structure," John Wiley & Son, Newyork, 1976.

[2] V.I. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. on Automat. Contr.*, vol. AC-22, no. 2, pp. 212-222, Apr. 1977.

[3] J.J. Slotine, "Sliding controller design for non-linear systems," *Int. J. Contr.* vol. 40, no. 2, pp. 412-434, 1984.

[4] J.J. Slotine and S.S. Sastry, "Tracking control of non-linear systems using sliding surfaces with application to robot manipulators," *Int. J. Contr.* vol. 38, no. 2, pp. 465-492, 1983.

[5] W.C. Chan and Y.Y. Hsu, "Optimal variable structure controller for D.C. motor speed control," *IEE Proc.* vol. 131, Pt. D, no. 6, pp. 233-237, Nov. 1984.

[6] K.K.D. Young, "Asymptotic stability of model reference systems with variable structure control," *IEEE Trans. on Automat. Contr.*, vol. AC-22, pp. 279-281, 1977.

[7] F.M. Devaud and J.Y. Caron, "Asymptotic stability of model reference systems with bang-bang control," *IEEE Trans. on Automalt. Contr.*, vol. AC-20, pp. 694-696, 1975

[8] K.K.D. Young, "Design of variable structure model-following control systems," *IEEE Trans. on Automat. Contr.* vol. AC-23, pp. 1079-1083, 1978.

[9] A.S.I. Zinober, O.M.E. El-Ghezawi and S.A. Billings, "Multivariable-structure adaptive model-following control systems," *IEE Proc.* vol. 129, Pt. D, no. 1, pp. 6-12, Jan. 1982.

[10] C. Milosavljevic, "General conditions for the existence of a quasi-sliding mode on the switching hyperplane in discrete variable structure systems," Avtomatika i Telemekhanika, no. 3, pp. 36-44, Mar. 1985.