On a Detection Scheme for Weak Deterministic Signals in Non-Additive Noise

(비가산성 잡음에서의 약한 확정적 신호의 검파방식에 관하여)

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要 約

비가산성 잡음을 포함하는 일반화된 관측모델에서의 확정적 신호(deterministic signal)를 위한 검파기를 구하였다. 이 논문에 사용된 관측모델로는 순가산성 잡음, 적산성잡음(multiplicative noise) 및신호의존성 잡음(signal-dependent noise)을 나타낼 수 있으며, 확정적 신호와 확률적 신호(random signal)를 다룰 수 있다. 이 일반화된 모델에서 알려진 신호(known signal)를 위한 국소최적 검파기(locally optimum detector)를 도출하여 그 성능을 해석하였다. 이 국소최적 검파기는 순가산성 잡음모델에서의 국소최적 검파기를 일반화한 것임을 보였다. 끝으로 일반화된 관측모델에서의 여러 검파기의 성능을 비교하였다.

Abstract

A parametric detection scheme for deterministic signals is obtained in a generalized observation model which contains non-additive noise. The model employed in this paper includes several special cases such as those describing purely-additive noise, multiplicative noise, and signal-dependent noise and allows the consideration of deterministic and random signals. Locally optimum detectors for known deterministic signals in the model are derived and analyzed for performance. It is shown that the locally optimum detectors are interesting generalizations of those for the purely-additive noise model. Performance of the locally optimum detectors designed for the generalized observation model is compared to that of other common detectors.

I. Introduction

The purely-additive noise (PAN) model has been one of the most commonly-used models [e.g., 1,2] in many areas of signal processing. A

reason for the popularity of the PAN model is that it is relatively easy to treat analytically and to derive for it explicit and physically-appealing structures for signal processors in various practical applications. Among the other reasons for the popularity of the PAN model is the fact that the contributions of other (non-additive) processes may be assumed to be negligible in many situations so that the PAN model usually produces quite acceptable results. However, if the PAN

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model does not yield an appropriate and reasonable approximation for the mechanism generating the observations, then the penalty which results from using the incorrect model (the PAN model) may be significant.

Non-additive noise (NAN) models are necessary in several different types of interesting situations. For example, multiplicative and signal-dependent noise terms cannot be ignored in certain communications, acoustics, and image processing applications [e.g., 3-6]: the effects of system and channel nonlinearities acting on additive signal and noise may generate NAN components in addition to PAN noise components. More basically as a natural extension of the PAN models in signal processing theory, it is of intellectual interest to study more general noisy signal (observation) models which include the PAN models as special cases.

In this paper we will concentrate our attention on the locally optimum (LO) detection of weak known signals in observations governed by a generalized noisy signal model which accommodates multiplicative and signal-dependent noise terms in addition to PAN. LO detectors, which are optimum in detecting signals in the local case of weak signals (SNR \rightarrow 0), have bases in the generalized Neyman Pearson lemma [7,8] of statistical hypothesis testing. They have been of considerable interest because detection of weak signals is more difficult than detection of signals which are comparable in strength to the noise process.

II. A Non-Additive Noise Model

The multiple-input PAN model for observations \boldsymbol{X}_{ji} in the signal detection problem can be described by

$$X_{ii} = \theta Q_i + W_{ii}, j = 1, 2, \dots, L, i = 1, 2, \dots, n$$
 (1)

Here L is the number of input channels from which observations (noisy signals) are taken, i.e., L is the array size; n is the size of the sample (a set of observations) collected at each input channel; Q_i is the common signal component at each channel at the i-th sampling instant; θ is the signal amplitude; and W_{ji} is the PAN component in the j-th channel at the i-th sampling instant. The PAN components W_{ji} is normally taken to be independent and identically distributed (i.i.d.)

random variables with common probability density function (pdf) f_W and mean zero and variance σ_W^2 , with the signal and noise being statistically independent.

Let us now consider a more flexible model capable of yielding more complicated but realistic descriptions in a broader range of situations. Instead of the purely-additive signal and noise model of (1), consider the model describing the observation X_{ij} , for i=1,2,...,n and j=1,2,...,L, by

$$X_{ii} = \alpha (\tau) e_i + \beta (\tau) S_i + \gamma (\tau)^{1-d}$$

$$[\alpha (\tau) e_i + \beta (\tau) S_i]^d N_{ii} + W_{ii}$$
(2)

The assumptions of our PAN model (1) are retained and the signal term θQ_i in (1) has been more generally broken up into deterministic (e;) and zero-mean random (Si) components with respective amplitudes $\alpha(\tau)$ and $\beta(\tau)$. However, we have now introduced the additional noise components Nii. These are i.i.d. random variables independent of the S_i , but they are generally correlated with W_{ji} . The signal term $(\alpha(\tau)e_i + \beta(\tau)Si)$ multiplies N_{ji} when d=1 to produce an additional multiplicative noise term generally correlated with the PAN component W_{ii} . On the other hand, with d = 0 we get the additional term $\gamma(\tau)N_{ii}$ which is a signal-dependent noise term, since τ controls the signal strength through $\alpha(\tau)$ and $\beta(\tau)$. It will be assumed that $\alpha(\tau)$, $\beta(\tau)$ and $\gamma(\tau)$ are nondecreasing functions of τ near $\tau = 0$ with $\alpha(\tau) = \beta(\tau) = \gamma(\tau) = 0$ at $\tau = 0$ and that d can only be 0 or 1.

Let the variance of the N_{ji} be σ_N^2 and let f_N be the common pdf of the N_{ji} . Finally we will denote by f_{NW} the common joint pdf of the (N_{ji}, W_{ji}) , which are i.i.d. bivariate random variables for i=1,2,...,n and j = 1,2,...,L. In this paper, we will restrict ourselves only to the case $\beta(\tau)$ =0 and the e_i are not all equal to zero.

With the above assumptions, our detection problem can now be formulated as a statistical hypothesis testing problem of choosing between a null hypothesis $H:\tau=0$ and an alternative hypothesis $K:\tau>0$ describing the joint pdf

$$f(x|\tau) = \prod_{i=1}^{n} \prod_{j=1}^{L} \int f_{NW}(n_{ji}, \mathbf{x}_{ji} - \alpha(\tau)e_{i} - \gamma(\tau)^{1-d} \cdot (\alpha(\tau)e_{i})^{d}n_{ji}) dn_{ji}$$

$$(3)$$

of the observation matrix $X = \{X_{ji}\}$, where x is a realization of X.

III. Locally Optimum Detectors in the Non-Additive Noise Model

An LO detector has maximum slope for the detector power function at the origin (SNR=0) in the class of all detectors which have its size or false-alarm probability (P_{fa}); therefore, the power of an LO detector is guaranteed to be no smaller than that of other detectors at least for signal strength θ in some non-null interval $(0,\theta_{M})$ with θ_{M} depending on the detectors, if the pdf of X and its derivatives are real-valued integrable functions [9]. More specifically, let D_{α} be the class of all detectors of size α for H versus K and let $P_{d}(\theta/D)$ be the power function of a detector D. Then an LO detector D_{LO} of size α is a detector in D_{α} which satisfies.

$$\max_{\mathbf{D} \in \mathbf{D_{a}}} \frac{d^{\nu} \mathbf{P_{d}} (\mathbf{0} \mid \mathbf{D})}{d \theta^{\nu}} = \frac{d^{\nu} \mathbf{P_{d}} (\mathbf{0} \mid \mathbf{D_{L0}})}{d \theta^{\nu}} \tag{4}$$

In (4) the parameter ν is defined by the following two equations:

$$\frac{d^{i} P_{d}(0|D)}{d\theta^{i}} = 0, \quad i = 1, 2, \dots, \nu - 1$$
 (5)

for all D in D_{α} and

$$\frac{\mathrm{d}^{\nu} P_{d} \left(0 \mid D_{L_{0}}\right)}{\mathrm{d} \theta^{\nu}} > 0 \tag{6}$$

In the PAN model, it can be shown [e.g. 9] that the LO detector test statistic is

$$T_{to+}(X) = \sum_{i=1}^{n} g_{i} \sum_{j=1}^{L} g_{1}(X_{ji})$$
 (7)

where

$$g_{\iota}(\mathbf{x}) = -\frac{f'_{\mathbf{w}}(\mathbf{x})}{f_{\mathbf{w}}(\mathbf{x})} \tag{8}$$

is usually called the LO nonlinearity. In (7) the subscript LO+ represents LO detector in the PAN model.

1. Reparametrization

To obtain the test statistics of the LO detectors in the generalized observation model (2), we first reparametrize the generalized observation model (2) in the following way.

- (1) When d = 1, let $\theta = \alpha(\tau)$ and define $a(\theta) = \theta$.
- (2) When d = 0, we first find positive δ and ϵ , and finite, positive p and q such that

$$\lim_{\tau \to 0^+} \frac{\alpha(\tau)}{\delta \tau^p} = 1 \tag{9}$$

and

$$\lim_{\tau \to 0^+} \frac{\gamma(\tau)}{\varepsilon \tau^q} = 1 \tag{10}$$

For d = 0 we now reparametrize, with $\triangle \equiv q/p$, the generalized observation model (2) as follows;

- (A) If $\triangle \geqslant 1$ or if $1/2 < \triangle < 1$ and $E\{N|W\} \equiv 0$, we define $a(\theta) = \theta$, $c(\theta) = \gamma(\tau)$ with $\theta = \alpha(\tau)$.
- (B) If $\triangle \le 1/2$ or if $1/2 < \triangle < 1$ and $E\{\{N\}W\}\}$ does not vanish identically, we define $c(\theta) = \theta$, $a(\theta) = \alpha(\theta)$ with $\theta = \gamma(\tau)$.

It is noteworthy that the above reparametrization does not change the structure of the LO detectors; it provides us with convenience in deriving the test statistics of the LO detectors. After the reparametrization, we have

$$X_{n} = a(\theta) e_{i} + c(\theta)^{1-d} (a(\theta) e_{i})^{d} N_{n} + W_{n}$$
 (11)

as our model in this paper. In (11), $a(\theta)=\theta$ if d=1, and at least one of $a(\theta)$ and $c(\theta)$ is 0 if d=0.

2. Locally Optimum Detector Test Statistic

The joint pdf $f(\mathbf{x}|\tau)$ of (3) is now parametrized in terms of $\theta = \alpha(\tau)$ or $\theta = \gamma(\tau)$ according to the values of d, \triangle and $E\{N/W\}$, and we denote this explicitly by writing the pdf as $\phi(\mathbf{x}|\tau)$. From the generalized Neyman Pearson lemma [7,8] the LO detector test statistic for $\theta = 0$ vs. $\theta > 0$ is obtained as the ratio

$$T_{Lo}(\mathbf{x}) = \frac{\frac{\mathrm{d}^{\nu} \phi(\mathbf{x} \mid \theta)}{\mathrm{d} \theta^{\nu}} |_{\theta = 0}}{\phi(\mathbf{x} \mid 0)}$$
(12)

where ν is the first non-zero derivative and was defined in (5) and (6). In [11] it is shown that this yields for the LO detector test statistic for most cases of interest the result

$$T_{Lo}(\mathbf{X}) = \sum_{i=1}^{n} \sum_{j=1}^{L} \left[\mathbf{a}'(0) \, \mathbf{e}_{i} \, \mathbf{g}_{1} \, (\mathbf{X}_{ji}) + \mathbf{A}_{i} \, \mathbf{g}_{2} \, (\mathbf{X}_{ji}) \right]$$
 (13)

where

$$\mathbf{g}_{2}(\mathbf{x}) = -\frac{\left[f_{\mathbf{W}}(\mathbf{x}) \mathbf{E} \left\{ N \mid \mathbf{W} = \mathbf{x} \right\} \right]'}{f_{\mathbf{w}}(\mathbf{x})} \tag{14}$$

and

$$A_{i} = \begin{cases} c'(0) & \text{if } d = 0\\ a'(0) e_{i} & \text{if } d = 1 \end{cases}$$
 (15)

The test statistic of (13) is correct for LO detection in the generalized observation model when d=1 (multiplicative noise), and also when d=0 (signal-dependent noise) and $E\{N|W\}\equiv 0$, it can be shown that (13) is still correct if $\triangle > 1/2$. When d=0, $E\{N|W\}\equiv 0$ and $\triangle = 1/2$, the LO detector test statistic becomes

$$T_{Lo}(\mathbf{X}) = \sum_{i=1}^{n} \sum_{j=1}^{L} (\mathbf{a}'(0) e_{i} \mathbf{g}_{1}(\mathbf{X}_{j_{i}}) + \mathbf{h}_{3}(\mathbf{X}_{j_{i}}))$$
 (16)

with

$$h_{3}(x) = \frac{\left[f_{W}(x) E\{N^{2} | W = x\}\right]''}{f_{W}(x)}$$
(17)

When d=0, $E\{N|W\}\equiv 0$ and $\triangle < 1/2$, we obtain

$$T_{Lo}(\mathbf{X}) = \sum_{i=1}^{n} \sum_{j=1}^{L} h_3(X_{ji})$$
 (18)

for the LO detector test statistic. A schematic diagram of the structure for the LO detector using the test statistic of (13) is shown in Figure 1.

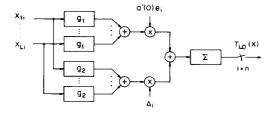


Fig.1. A structure for the locally optimum detector with the test statistic of (13).

3. Observations and Comments

(a) If d=1 (the multiplicative noise case), the test statistic always depends on the PAN noise through the LO characteristic g_1 , and the multiplicative noise term also has an effect on the LO detector test statistic through g_2 , unless g_2 is identically zero. The LO detector test statistic for d=1 is exactly the same as that which would have been obtained in the PAN model with $g_1(x)$ replaced by $g_1(x) + g_2(x)$.

- (b) If d=0 and $\triangle > 1$ or if d=0, $\triangle > 1/2$ and $E\{N|W\}\equiv 0$ (the signal-dependent noise level is low compared to the PAN noise level), only the first term in (13) that contains g_1 under the summations remains in the test statistic. This implies that in such circumstances the test statistic depends only on the known signal term $\alpha(\tau)e_i$ and that the signal-dependent noise term $\gamma(\tau)N_{ji}$ has no effect on the LO detector test statistic.
- (c) When d=0, $\triangle=1$ and $E\{N|W\}$ does not vanish identically or when d=0, $\triangle=1/2$ and $E\{N|W\}\equiv 0$ (the signal-dependent noise is comparable in strength to the PAN noise), the terms containing g_1 and g_2 in (13) or those containing g_1 and g_2 in (16) exist in the test statistic.
- (d) If d=0 and $\triangle < 1/2$ or if d=0, $\triangle < 1$ and $E\{N|W\}$ does not vanish identically (the signal-dependent noise level is high compared to the PAN noise level), the test statistic contains only g_2 in (13) or h_3 in (17). This implies that under this condition the known signal term has no effect on the test statistic; only the dependence of the two noise processes N and W has an effect on the test statistic.

4. Examples of the Locally Optimum Detector Test Statistics

Let f_{NW} be a bivariate Gaussian pdf with $E\{N\}=E\{W\}=0$, $\sigma_W^2=1$, $\sigma_N^2=s^2$ and r being the correlation coefficient between N and W. In this case, we obtain when d=1

$$T_{LO}(\mathbf{X}) = \sum_{i=1}^{n} e_i \sum_{j=1}^{L} \left[X_{ji} + rs(X_{ji}^2 - 1) \right]$$
 (19)

and when d=0 and $\triangle=1$ it is

$$T_{LO}(\mathbf{X}) = \sum_{i=1}^{n} \sum_{i=1}^{L} (e_i X_{ji} + \varepsilon \delta^{-1} \operatorname{rs} (X_{ji}^2 - 1))$$
 (20)

When d=0, $\triangle=1/2$ and r=0 we obtain

$$T_{LO}(\mathbf{X}) = \sum_{i=1}^{n} \sum_{t=1}^{L} \left[2 \delta \varepsilon^{-2} e_{t} X_{it} + s^{2} (X_{it}^{2} - 1) \right]$$
 (21)

as the LO detector test statistic.

In the second terms in (19)-(21), the squarelaw characteristic is quite interesting in this known-signal detection problem. This is because the square-law characteristic generally arises in zero-mean random-signal detection in PAN. This square-law term is a consequence of the randomness of the noise N, which can also be regarded as a random signal component with strength $\alpha(\tau)$ if d=1 and $\gamma(\tau)$ if d=0 in the NAN model. The fundamental difference, however, is that while the square-law characteristic in the LO detector for random-signal detection in the PAN model and that in (21) is a result of the second derivative operation on the noise pdf, the square-law characteristic in (19) and (20) is a consequence of the correlation between the two noise processes N and W; that is, $f_{\mathbf{W}}(\mathbf{x})$ is multiplied by $\mathbf{E}\{\mathbf{N}|\mathbf{W}=\mathbf{x}\}=\mathbf{rsx}$, differentiated once and then divided by $f_{\mathbf{W}}(\mathbf{x})$ to give $g_{\mathbf{Z}}(\mathbf{x})$, during which operation the square term is produced.

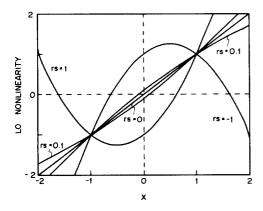


Fig.2. Locally optimum nonlinearities for bivariate gaussian noise probability density functions.

Figure 2 shows plots of the LO nonlinearity $g_{LO}(x)=x+a_0$ (x^2-1) for various values of a_0 which can be interpreted as rs, $\epsilon rs/\delta$ or $\epsilon^2 s^2/2\delta$ with $e_i=1$, i=1,2,...,n. From Figure 2, the following observations can be made:

(a) When r > 0 (the two noise processes N and W are positively correlated), observations with large absolute values (|X| > 1) are taken to be more indicative of signal presence in the NAN model than they would have been in the PAN model. This effect is more pronounced as the product rs becomes larger. This is because of the third term $c(\theta)^{1-d}[a(\theta)e_i]^dN_{ji}$ in the NAN model (11); under the alternative hypothesis K, this term makes the absolute value of an observation larger in the NAN model than in the PAN model,

because N and W are positively correlated. The reverse effect is observed for r < 0.

(b) On the other hand, observations with small absolute values (|X| < 1) are weighted more in favor of the null hypothesis H in the NAN model than they would have been weighted in the PAN model, for positively correlated N and W. This effect is again more pronounced as the product rs becomes larger. Again, the reason for this is that under the alternative hypothesis K the third term in the NAN model (11) tends to make the absolute value of an observation in the NAN model larger if r > 0 and smaller if r < 0 than it would be in the PAN model.

IV. Asymptotic Performance of the Locally Optimum Detectors

In this section, let us consider the performance of several detectors. We will particularly consider the LO+ detector (LO detector in the PAN model, whose test statistic is (7)), the linear correlator array (LCA) detector (which is optimum for detecting a known signal in Gaussian noise in the PAN model) and the sign correlator array (SCA) detector (which is optimum for detecting a known signal in noise with a double-exponential pdf in the PAN model) as well as the LO detector.

In the comparison of the asymptotic performance of two detectors, the asymptotic relative efficiency (ARE) [9,12] is generally employed. The ARE of a detector D_1 with respect to another detector D_2 is defined by

$$ARE_{1,2} = \lim_{l \to \infty} \frac{n_{2,l}}{n_{1,l}}$$
 (22)

where $n_{1,l}$ and $n_{2,l}$ are the sample sizes (numbers of observations) required by the detectors D_1 and D_2 , respectively, to attain any fixed power for any specified false-alarm probability when the signal strength $\theta = \theta_l$. If the expected value, its derivatives and the variance of a detector under the null hypothesis and those under the alternative hypothesis are respectively the same for $n \to \infty$ [12], the ARE_{1,2} can be expressed as the ratio of efficacies ξ_i ,

$$ARE_{1,2} = \frac{\xi_1}{\xi_2} \tag{23}$$

If the results for the efficacies of detectors [11] are used, we can obtain the AREs of the LO detector with respect to the LCA, SCA and LO+detectors operating on observations modeled by (2). For example, if we take the pdf corresponding to the bivariate t-distribution [13]

$$f_{xy}(x,y) = \frac{1}{2\pi s (1-r^2)^{1/2}} \left[1 + \frac{1}{k(1-r^2)} + \frac{1}{k(1-r^2)} \cdot \left(\frac{x^2}{s^2} - \frac{2rxy}{s} + y^2\right)\right]^{-\frac{(K+2)}{2}}$$
(24)

for f_{NW}, we have

$$ARE_{Lo,LCA} = \frac{k (k+1+2kr^2s^2)}{(k-2) (k+3)}$$
 (25)

$$ARE_{Lo,SCA} = \frac{k(k+1+2kr^2s^2)B^2(\frac{1}{2},\frac{k}{2})}{4(k+3)}$$
 (26)

and

$$ARE_{Lo,Lo} = 1 + \frac{2kr^2s^2}{(k+1)}$$
 (27)

where k is a decay parameter of the pdf and B(x,y) is the beta function. These AREs are shown in Figure 3. It is assumed that $\epsilon=\delta^{\triangle}$ when d=0 and $e_i=1$, i=1,2,...,n in (25)-(27).

of course, for $k \to \infty$ we get from these results the AREs for the bivariate Gaussian pdf for f_{NW} , which are

$$ARE_{LO,LCA} = 1 + 2r^2 s^2 \tag{28}$$

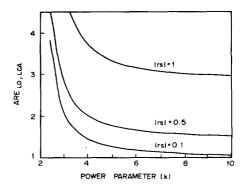
$$ARE_{Lo,sca} = \frac{\pi (1 + 2r^2 s^2)}{2}$$
 (29)

and

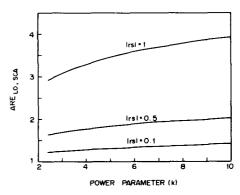
$$ARE_{L0,L0^{4}} = 1 + 2r^{2}s^{2}$$
 (30)

respectively.

It is observed from Equations (25)-(30) that as the magnitude of the product rs becomes large, the effect of the non-additive (signal-dependent or multiplicative) noise on the asymptotic performance of a detector also becomes significant. It is therefore clear that considerable performance improvements can be obtained by taking signal-dependent or multiplicative noise into account when a detector is to be operated under the NAN model.



(a) ARE_{LO,LCA}



(b) ARE_{LO,SCA}

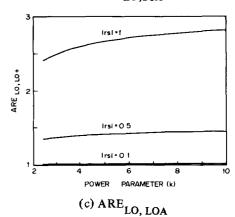


Fig.3. AREs for bivariate t-distribution.

V. Summary

In this paper a non-additive noise (NAN) model has been used for problems of detecting signals from observations corrupted by various types of noise, as an interesting generalization of the purely-additive noise (PAN) model which has been widely used in many areas of signal processing. The observation model used in this paper is a natural generalization of the purely-additive noise model to accommodate the effects of multiplicative or signal-dependent noise in addition to purely-additive noise. Locally optimum (LO) detectors which were derived for this non-additive noise model for detecting known signals were shown to be generalizations of those for the purely-additive noise model. Comparisons of performance of the locally optimum detectors and other detectors were made. It has been shown that the locally optimum detectors can significantly outperform other simpler detectors such as the linear correlator array (LCA) detector and the sign correlator array (SCA) detector in signal-dependent or multiplicative noise.

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