Journal of the Korean
OR/MS Society
Vol. 13 No. 2 December 1988

—Research Paper—

A Decision Model with Expert't Biased Information Transmission

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ABSTRACT

This study suggests an optimal decision process when decision maker is confronted with expert's biased information under the situation that the bias is caused mainly by the difference of their interest. In order to make honest transmission of expert's probabilistic information, the concept of expert use and scoring rule to provide expert with an incentive is used in this paper. And expected regret concept is introduced to evaluate the value of expert's information. A simple example is also shown.

I. INTRODUCTION

In classical Bayesian revision of probabilities, decision maker(DM) integrates empirical data or observations with his prior information. In this model, instead of assuming that DM makes observation to get more information, DM hires an expert who is asked for transmitting his assessment of subjective probabilities of possible states to DM. With the information transmitted and DM's prior probability, DM makes a decision. However, since their payoff structures differ, expert would not necessarily transmit his subjective probability accurately to DM. That is, expert biases his subjective probability motivationally.

Green[1] presented the model considering motivational bias in transferring expert's observation, but meaningful optimal decision rules could be derived only in restricted case, i.e., two possible actions and two possible observations. In team theory[2], the actions of all members are taken individually, perhaps after some communication, whereas in this paper, there exists only one DM who chooses a single action affecting his own welfare.

This study suggests a solving procedure under the situation that expert's motivational bias exists. Under the assumption that DM and expert are expected monetary value decision maker(EMVer), DM finds expert's true assessment of subjective probability after receiving expert's transmission and DM combines this probability with prior probability to derive posterior probability. In this procedure, the concept of expert use[3] and scoring rule[4,5] is used.

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2. DESCRIPTION OF MODEL

Suppose DM must choose an action among a set of actions $A=\{a_1,\cdots,a_k\}$. A possible state is one of $S=\{s_1,\cdots,s_m\}$. For simplicity, consider only two states, $S=\{s_1, s_2\}$, but it is not so restrictive since DM may consider s_1 as the state that he wishes to highlight, and s_2 as the complementary set of s_1 composing remaining states. Denote DM's prior probability as $R=(r, 1-r)'$, where t means transpose of vector, and his payoff matrix as $U=\{u_{km}\}$ where u_{km} represents DM's payoff associated with action a_k and realized state s_m.

And denote expert's payoff matrix as $V=\{v_{km}\}$, his assessment of subjective probability as $P=(p, 1-p)'$, to be transmit to DM as $Q=(q, 1-q)'$, which may or may not be P. Z is a 2xK stochastic matrix representing DM's random action plan such that Q'Z means the probability of DM's taking each action from expert's pointview.

Assume following conditions : DM and expert are EMVers, expert knows Z and thinks DM will make a decision according to q and Z, and DM knows expert's mind maximizing his own payoff, in other words, P is converted to Q using Z and V.

Elements of expert's payoff matrix V are functions of q. In detail, elements in one column of V are given as strictly increasing functions of q, and those in another column as strictly decreasing functions of q, and vice versa. This property provides expert with incentive to make honest transwission. This idea corresponds to the concept of scoring rule[5].

Further, for simplicity, assume that elements in each column are all identical, that is, $v_{11}=v_{21}=\cdots=v_{k1}$, and $v_{12}=v_{22}=\cdots=v_{k2}$. This assumption can be validated since the action taken by DM is his own decision while expert is responsible only for q which he has transmitted. For example, suppose that two states, "R" (Rainy), "NR"(Not Rainy) exist. Expert reports the probability of event "R" is 0.7. Then his role is completed. To take an umbrella or not is entirely at DM's disposal.

3. ANALYSIS OF EXPERT'S AND DM'S MODELS

Expert thinks that DM will take an action with a 1xK probability vector Q'Z. After assessing subjective probability P, expert converts P into Q in order to maximize his espected value. Then, expert's expected value becomes Q'ZVP. In other words, expert's problem can be represented as the following nonlinear optimization problem :

$$\begin{aligned} &\text{Maximize } Q'ZVP \\ &s.t. \quad 0 \leq q \leq 1 \end{aligned} \tag{1}$$

Optimal solution q^* of (1) is expert's optimal transmission of subjective probability. From DM's pointview, this problem can be solved through following two phases. In the first phase, DM finds expert's true assessment p from q^* inversely. The true probability p may becomes a unique point or an interval. Suppose all $v_{k1}(k=1,\cdots,K)$ are strictly increasing functions of q^*, and all v_{k2} are strictly decreasing functions of q^*. Then, it can be easily shown that q^* defined in (1) is monotone increasing function of p. Thus, expert's true probability can be found as a point or an interval from transmitted probability q. For simplicity. consider only interval type since unique point type can be treated as a special case of interval one.

In the second phase, consider DM's posterior probability interval that is denoted as (rp_1, rp_2) using his prior probability, likelihood function, and expert's true probability (P_1, P_2). Likelihood function can be assessed by DM and interpreted as a subjective measure of expert's credibility. If DM feels that expert is very competent, he will assign high probability that the expert will respond with a narrow distribution whose bulk encompasses true value.

To partially resolve this uncertainty, he turns on the radio and waits for the weather report. However, the DM is uncertain about the weather report p(probability of event R), and makes a subjective appraisal of the dependence between the expert's advice and actual weather. He feels that the weatherman is more likely to state high probability of R than high probability of event NR in this case. Then, he assigns the following likelihood fumction ;

$Pr\{p \mid R, d\} = 3p^2, \quad 0 \leq p \leq 1$

$Pr\{p \mid NR, d\} = 3(1-p)^2, \quad 0 \leq p \leq 1$

where d is state of DM's information. The DM's posterior probability interval (rp_1, rp_2) is calculated by Bayes formula,

$$rp_1 \wedge Pr\{R \mid p_1 d\} = \frac{Pr\{p_1 \mid R,d\} Pr\{R \mid d\}}{Pr\{p_1 \mid R,d\} Pr\{R \mid d\} + Pr\{p_1 \mid R,d\} Pr\{R \mid d\}}$$

$$rp_2 = Pr\{R \mid p_2 d\} = \frac{Pr\{p_2 \mid R,d\} Pr\{R \mid d\}}{Pr\{p_2 \mid R,d\} Pr\{R \mid d\} + Pr\{p_2 \mid R,d\} Pr\{R \mid d\}}$$

where, $Pr\{R \mid d\}$and $Pr\{NR \mid d\}$ is DM's prior probabilities.

Note that (rp_1, rp_2) can be obtained from (2) and (3) only when DM's posterior probability, rp, is a monotone function of p. If rp is not a monotone function of p. (rp_1, rp_2) must be found by nonliniar search technique, that is, the maximum and minimum should be searched in interval (p_1, p_2).

Therefore, it is necessary to investigate the condition for that rp is a montone function of p. Let $L_1(p) = Pr\{p \mid s_1, d\}$, $L_2(p) = Pr\{p \mid s_2 d\}$. Without loss of generality, suppose that L_1 and L_2 are quadratic functions of p, i. e., $L_1 = ap^2 + bp + c$ and $L_2 = a(1-p)^2 + b(1-p) + c$, and $L_2(p) = L_1(1-p)$. Then, $rp = Pr\{s_1 \mid p, d\} = r_1 L_1 / (r_1 L_1 + r_2 L_2)$ where r_1 and r_2 are r and $1-r$, respectively.

Differentiating,

$d(rp)/dp = [r_1 L_1'(r_1 L_1 + r_2 L_2) - r_1 L_1 (r_1 L_1' + r_2 L_2')] / (r_1 L_1 + r_2 L_2)^2$

where $L_1' = 2ap + b$ and $L_2' = 2ap - (2a+b)$

Let $g(p)$ = numerator of $d(rp)/dp = r_1 r_2 (L_1' L_2 - L_1 L_2')$

For rp being a nondeceasing function of p, $g(p) \geq 0$ must be satisfied for $0 \leq p \leq 1$. Since $r_1 r_2 \geq 0$ for any r, $g(p) \geq 0$ means $L_1' L_2 - L_1 L_2' \geq 0$.

$$L_1' L_2 - L_1 L_2' = (a+b)\{-2ap^2 + 2ap + b + 2c\} \quad (4)$$

Since $\int_0^1 L_i \, dp = 1$ for $i = 1, 2$, $c = 1 - (1/3)a - (1/2)b$. (5)

Substituting (5) for c in (4),

$$L_1' L_2 - L_1 L_2' = -2a(a+b)(p-1/2)^2 + (2-(1/6)a)(a+b) \quad (6)$$

Based on (6), feasible regions for rp to be a nonincreasing function of p can be plotted in Fig. 1. Any pair (a,b) in the lined region yields a nondecreasing rp function of p, and any pair (a, b) in the dotted region yields a nonincreasing rp function of p. If a pair (a, b) is neither in the lined nor in the dotted region, that is, $3 \leq a \leq 12$, then the corresponding rp is not montone function of p. In this case,(rp_1, rp_2) interval must be found by nonlinear search technique.

When DM has only prior probability, he will make a decision with UR,K×1 vector, each element of which represents expected value associated with action k. Fig. 2 shows this graphically. Solid Line 1 represents DM's maximum expected value for each r. a_1 is the best action for $0 \leq r \leq c_1$, a_2 for $c_1 \leq r \leq c_2$, and a_3 for $c_2 \leq r \leq 1$. Note that line 1 should be of piecewise linear convex form since the problem is maximization one. Broken Line 2 represents DM's expected value with perfect information on the realized state. Broken line1 repesents maximum amount that DM is willing to pay for perfect information at any fixed r.

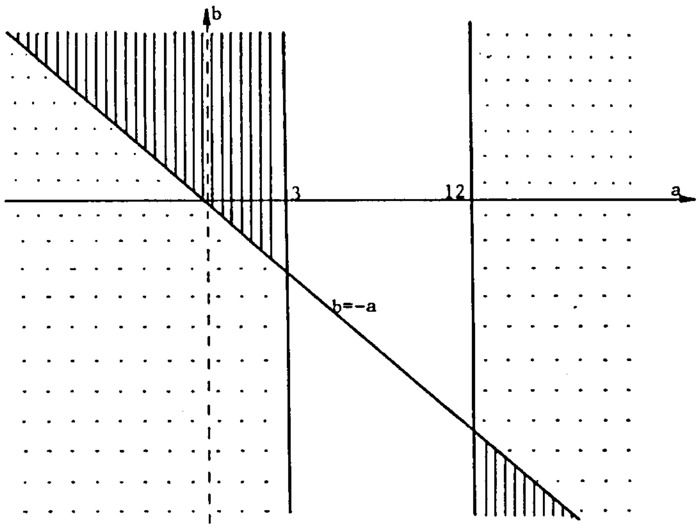

Figure 1. Feasible regions for rp

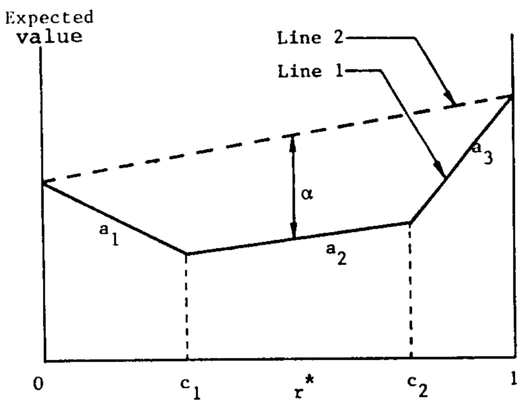

Figure 2. Decisions with prior probability

Consider the case that there are no changes of action. There are four possible types for the magnitude of r versus rp resulting in no change of action after receiving the expert's information, as follows :

types	r	rp	r vs.rp	△EU
1	−	−	〈	$-\triangle_1$
2	−	−	〉	$+\triangle_2$
3	+	+	〈	$+\triangle_3$
4	+	+	〉	$-\triangle_4$

where "+" and "−" mean the slope of expected value line at r or rp are positive and negative, respectively, and "+△" and "−△"mean the amounts of DM's expected value increased and decreased after receiving expert's information, respectively. The increase or decrease of DM's expected value, are obtained as follows :

$$\triangle EU = U_{k(rp),rp} - U_{k(r),r} \quad (7)$$

Where k(rp) and k(r) mean the action based on DM's posterior and prior probabilities, respectively, and $U_{k,p}$ is DM's expected value attainable with action k based on probability p. △EU can be interpreted as follows :

△EU does not always mean the expected value of expert's information according to decision analysis context since information has value in a decision problem only if it results in a change of action to be taken by DM. Therefore, △EU does not always mean the upper bound on the amount that DM should pay to expert since the chance of expected value is felt only in the DM's mind without any change of action, that is, positive or negative △EU represents that DM underestimated or overestimated his expected value by that quantity.

Next, consider the case that there are changes of action. There are six possible types for the magnitude of r versus rp resulting in a change of action after receiving the expert's information, as follows :

types	r	rp	r vs.rp	△EU
5	−	−	〈	$-\triangle_5$
6	−	−	〉	$+\triangle_6$
7	+	+	〈	$+\triangle_7$
8	+	+	〉	$-\triangle_8$
9	−	+	〈	$+\triangle_9$ or $-\triangle_9$
10	+	−	〉	$+\triangle_{10}$ or $-\triangle_{10}$

In these types, △EUs are obtained by using (7). Here, define "expected regret" (ER) as the difference of expected value that is incurred when DM doesn't make a proper change of action in case i. It is reasondable to use ER, not △EU, as the value of expert's information, that is, the contribution of expert's information to DM. ERs are obtained by

$$ER = u_{k(rp),rp} - u_{k(r),rp} \quad (8)$$

Consider the expected payment to expert. Expert receives v_{ki} from DM corresponding to his transmission q after nature reveals one of two states. Before the revelation of nature, DM derives expected payment(EP) to the expert by the following equation :

$$EP = v_{k1,q}{}^{rp} + v_{k2,q}{}^{(1-rp)}$$

where $v_{k1,q}$ and $v_{k2,q}$ represent the first and second column element of V at q, respectively.

In summary, whether there is a change of action after receiving expert's information or not, △EU and ER can be represented as (7) and (8), respectively, and EP as (9). From DM's point of view, hiring expert is justified only when ER is not less than EP. Then, the relationships among these quantities associated with "action change" are summarized as follows :

	Action didn't change $k(rp)=k(r))$	Action changed $k(rp)\neq k(r))$
△EU	positive or negative	positive or negative
ER	zero	strictly positive
EP	nonnegative	nonnegative

4. EXAMPLE

An investor who is an EMVer wants to buy one of three kinds of stock(X, Y and Z) with a certain amount of money. Over six-month period of interest, he thinks that the economy will either advance or stagnate, with r percents of chance it will advance. But, he feels uncertain about his prior probability r.

In order to obtain more information about r, he decides to hire an expert in economy forecasting.

Table 1. ER and EP for 8 cases

(1,000 $)

case	ER	Payment to expert		rp*	EP
		when s_1	when s_2		
1	.000	.008	.450	.001	.449
2	.000	.512	.244	.694	.430
3	.000	.032	.496	.015	.489
4	.000	.648	.136	.739	.514
5	.000	.512	.244	.125	.277
6	.946	.200	.438	.257	.377
7	1.935	.200	.438	.133	.406
8	1.983	.032	.496	.127	.437

Then, consider two states, i. e., s_1(economy advances) and s_2(ecmomy stagnates) and three actions, i. e., a_1(buy stock X), a_2(buy stock Y), and a_3(buy stock Z). Suppose that likelihood functions assessed by the investor are as follows :

$$P_r\{p \mid s_1, d\} = 3p^2 \qquad 0 \leq p \leq 1$$

$$P_r\{p \mid s_2, d\} = 3(1-p)^2 \qquad 0 \leq p \leq 1$$

And also, suppose that the following data are obtained :

Random action plan : $Z = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.4 & 0.4 \end{pmatrix}$

Payoff matrices : (1,000 $)

	investor's payoff		expert's payoff	
	s_1	s_2	s_1	s_2
a_1	8	2	$.8q^2$	$.5(1-q)^3$
a_2	5	4	$.8q^2$	$.5(1-q)^3$
a_3	3	5	$.8q^2$	$.5(1-q)^3$

This hypothesized example is solved for eight cases of r and q by computer code and the result is shown as follows :

case	r	q	p		rp	
1	.1	.1	.0859	.0938	.0010	.0012
2	.8	.8	.4219	.4375	.6805	.7076
3	.3	.2	.1563	.1641	.0145	.0162
4	.8	.9	.4531	.4609	.7331	.7452
5	.2	.8	.4219	.4375	.1175	.1314
6	.6	.5	.3203	.3281	.2499	.2635
7	.4	.5	.3203	.3281	.1290	.1372
8	.8	.2	.1563	.1641	.1206	.1335

where the estimated p has been found as an interval with tolerance 0.01

The relationship between the investor's expected value and subjective probability is depicted in Fig.3. Considering the points, 1/3, 0.4, at which the investor's action must be changed, we can easily verify that there occurs no change of action in case 1 through case 5. But, in case 6 through case 8, there must be change of action after expert's report. For example, consider case 6 as also depicted in Fig.3. The investor's prior probability was 0.6, so his best actionwas a_1. But, after the expert's report q=0.5 his posterior probability rp* is 0.2567(rp* is simply the mid-point of rp interval). Thus, his best action becomes a_3. In this case, his expected regret(ER) becomes :

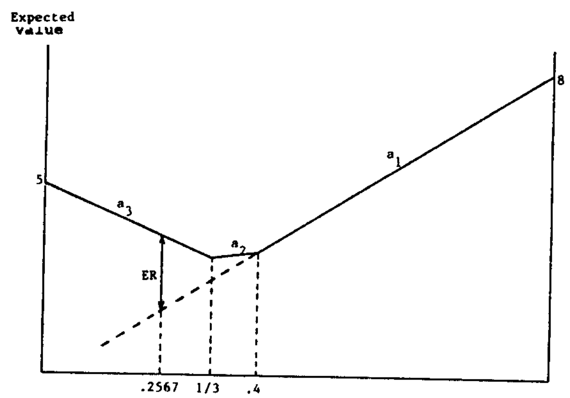

Figure 3. Analysis of case 6

$ER = u_{k=3, rp} - u_{k=1, rp} = 946.4(\$)$

Same procedure of calculation can be performed for other remaining cases. And the expected payment(EP) to the expert can be obtained using rp*, as shown in Table 1. When ER and EP in Table 1 are compared, in case 1 throught case 5, ERs are all zeros since these are no change of action, while EPs are all positive. Thus, they are not desirable situations in the investor's point of view. But, in case 6 through case 8, ER is greater than EP for all cases. Therefore, hiring expert is justified in these cases.

5. CONCLUSION

This study is concerned with expert's motivational bias that stems from expert's different viewpoint maximizing his own payoff with DM's. In this model, expert's payoff matrix V is not of given constants, rather it is determined by DM as a function of q and V is dependent only on the subjective probability that expert has transmitted and realized state, irrespective of DM's action.

Within this framework using the expert use, scoring rule, and expected regret concept, DM's optimal decision process has been proposed.

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