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# 무한 차원 시스템을 위한 유한 차원 보상기의 설계

# Finite Dimensional Compensator Design for a Class of Infinite Dimensional Systems

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## 요 약

이 논문은 무한 차원 시스템을 위한 유한 차원 보상기의 설계 절차에 관한 연구이다.

이 설계 절차는 상태 공간을 그 상태에 따라 분할한 후 유한 차원의 불안정 상태를 위한 추정기와 상태 궤환 제어기를 구성하는데 그 기초를 두고 있다.

잔여 상태의 영향을 고려하여 유한 차원 추정기의 운동식을 수정함으로써 궤환선 시스템이 안정하게 됨을 고찰하였다.

**Abstract**-This paper is concerned with a design procedure for constructing finite dimensional compensators for a class of infinite dimensional systems. Basically, this procedure consists of decomposing the state space on the basis of its modes and utilizing well-known finite dimensional algorithms for constructing observers and state feedback control laws. The finite dimensional observer dynamics is modified to account for the effect of the residual modes so as to achieve the stability of the closed loop system asymptotically.


## 1. Introduction

The control of infinite dimensional systems described by linear partial differential equations or functional differential equations presents some challenging features that are absent in the finite dimensional situations. Although the stabilization by state feedback is an interesting problem in itself, one can never observe the whole state in infinite dimensional systems and hence it is necessary to investigate the stabilization by output feedback. The theory of linear quadratic regulators and observers for infinite dimensional systems provides a procedure for constructing a controller, which is in general infinite dimensional itself. However, the implementation of controllers of infinite order is not feasible in practice, and hence the problem of stabilizing infinite dimensional systems by dynamic finite dimensional controllers has gained intensive interest in recent years.

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In constructing controllers for infinite dimensional systems, the most popular approach used to consist of replacing the infinite dimensional system by finite dimensional reduced order model and applying standard finite dimensional techniques to obtain a controller for this reduced order model. However, it has been shown that under certain circumstances, the interaction of the controller with the unmodelled dynamics of the system destabilizes the closed loop system as a whole.[5] Some results on the existence of finite dimensional compensators have been established in the absence of this unfavorable interaction,[6),7),8)] but this assumption is severely restrictive and unrealistic. Also, some existence results have been discussed based on the closeness of the reduced order model and the actual system.

In[8)], Schumacher presented a design procedure for constructing finite dimensoional compensators for a class of infinite dimensional systems under some assumptions including finitely many unstable modes and completeness of eigenvectors. This approach has been extended to include the systems with unbounded input and output operators by Curtain[9),10)] and Curtain and Salamon.[11)] Some results on finite dimensional compensators for some class of infinite dimensional systems have been derived recently using frequency domain methods.[12),13),14)]

In the present paper, we propose another approach to the design of finite dimensional compensators. The idea underlying this approach is to modify the finite dimensional observer to take into account the effect of the residual modes of the system. The basic idea is explained in the next section. In Section 3, we set up the problem in a rigorous way. The main result which establishes the existence of finite dimensional compensator is given in Section 4. A simple example of a class of infinite dimensional systems for which the results in Section 4 can be applied is provided in Section 5. Some final remarks follow in Section 6.

## 2. Preliminaries

In view of the complicated technicalities and the assumptions involved in establishing rigorous results, it seems useful to review th previous approaches and to motivate our approach to the compensator design. For Banach spaces X and Y, we denote by L(X, Y) the space of bounded linear operators from X to Y and write L(X) for L(X, X), We denote by $|\cdot|$ norms of vectors and operators.

We consider an infinite dimensional system of the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

on a Banach space X with finite dimensional spaces $U=R^m$, $Y=R^p$, where A is the infinitesimal generator of a strongly continuous semigroup S(.) of linear bounded operators on X, $B \in L(U,X)$ and $C \in L(X,Y)$.
We assume that with the decomposition of the state space $X=X_n \oplus X_r$ where the subspace $X_n$ is finite dimensional, the system (2.1) is also decomposed correspodingly with respect to this decomposition

$$\begin{aligned} \begin{pmatrix} \dot{x}_n(t) \\ \dot{x}_r(t) \end{pmatrix} &= \begin{pmatrix} A_n & 0 \\ 0 & A_r \end{pmatrix} \begin{pmatrix} x_n(t) \\ x_r(t) \end{pmatrix} + \begin{pmatrix} B_n \\ B_r \end{pmatrix} u(t) \\ y(t) &= (C_n \quad C_r) \begin{pmatrix} x_n(t) \\ x_r(t) \end{pmatrix} \end{aligned} \tag{2.2}$$

on $X_n \oplus X_r$, where $A_n$ is a bounded operator on $X_n$, and $A_r$ generates a strongly continuous semigroup $S_r(t)$ on $X_r$ which is exponentially stable.

Earlier results on the existence of finite dimensional compensators have been established on the basis of zero spillover assumption, that is, $B_r=O$ or $C_r=O$ [6),7),9)]. A design procedure which avoids the zero spillover assumption has been proposed by Schumacher [8)] and further pursued in [9),10)]. This approach basically consists of finding an infinite dimensional observer and state feedback to form an infinite dimensional compensator which stabilizes the system (2.1) and then finding a finite dimensional one which is close to the stabilizing infinite dimensional one using a certain parametrization of compensators. In other words, they first find exponentially stable pairs $A+BF$ and $A+GC$ and then approximate G by $\tilde{G}$ using generalized eigenvectors of $A+BF$ to show the existence of a finite dimensional subspace V such that Image $\tilde{G} \subset V$ and $(A+BF)\,V \subset V$. In addition to usual spectral decomposition and spectrum determined growth assumptions, their construction necessitates the assumption that the generalized eigenvectors of $A+BF$ is complete in X for some $F \in L(X,U)$ such that $A+BF$ is exponentially stable.

With the decomposition in (2.2), it is well known in infinite dimensional system theory that the pair (A,B) is exponentially stabilizable if $A_n - B_nF$ is stable for some $F \in L(X_n, U)$. In our approach, we first construct a finite dimensional observer for estimating state $x_n(t)$ in $X_n$ and an $F \in L(X_n, U)$ such that $A_n - B_nF$ is stable

$$\dot{z}_n(t) = A_n z_n(t) + G(y(t) - \hat{y}(t)), \quad z_n(t) \in X_n, \qquad u(t) = Fz_n(t), \tag{2.3}$$

where $\hat{y}(t)$ is to be determined from $z_n(t)$ later. With control u(.) in (2.3), we have

$$x_r(t) = S_r(t)x_{or} + \int_0^t S_r(t-r)B_r u(r)dr, \tag{2.4}$$

where $x_0 = x_{0n} \oplus x_{0r}$, $x_{0n} \in X_n$, $x_{0r} \in X_r$

If $S_r(t)$ is such that

$$|S_r(t)| \leq ae^{\alpha t}, \quad a \geq 1, \quad \alpha < 0,$$

then we have from (2.4) (2.4)

$$x_r(t) \to -A_r^{-1}B_r u(t) \quad \text{as} \quad \alpha \to -\infty$$

for $t > 0$.

Hence, for sufficiently small $\alpha$, we choose

$$\hat{y}(t) = C_n z_n(t) - C_r A_r^{-1} B_r u(t) = (C_n - C_r A_r^{-1} B_r F) z_n(t). \tag{2.5}$$

We will show this reasoning leads to the existence of finite dimensional compensators under some assumptions.

### 3. Preliminaries and Assumptions

We consider systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \qquad y(t) = Cx(t) \tag{3.1}$$

on a Banach space X with finite dimensional spaces $U = R^m$, $Y = R^p$, where A is the infinitesimal generator of a strongly continuous semigroup T(.) of linear bounded operator on X, $B \in L(U,X)$ and $C \in L(X,Y)$. Throughout this paper, we consider solutions in the mild sense, namely, as solutions of the associated integral equation

$$X(t) = T(t)X_0 + \int_0^t T(t-r)Bu(r)dr. \tag{3.2}$$

As a measure of stability for a semigroup S(.) on X, we use the growth constant $\omega$, which determines the exponential decay rate of $|S(t)|$ and is obtained by the formula

$$\omega := \lim_{t \to \infty} (1/t)\log|S(t)|.$$

The semigroup is said to be exponentially stable if its growth constant is negative. We suppose that a desired minimum degree of stability has been specified a priori by a growth constant $\rho_s < 0$. A semigroup is called simply stable if its growth constant is smaller than $\rho_s$. We will also say that the pair(A,B) is stabilizable if there exists an $F \in L(X, U)$ such that semigroup generated by $A + BF$ is stable. A pair (C,A) will be called detectable if the pair$(A^*, C^*)$ is stabilizable.

A key to the stabilizability is the following decomposition of the state space based on the modes.

(AO) Spectrum decomposition assumption : The spectrum $\sigma(A)$ of the operator A contains a bounded part $\sigma_n(A)$ seperated from the rest $\sigma_r(A) := \sigma(A) - \sigma_n(A)$ in such a way that a rectifiable, simple closed curve T can be drawn so as to enclose on open set containing $\sigma_n(A)$ in its interior and $\sigma_r(A)$ in its exterior.

Under (AO), we obtain a natural state space decomposition

$$X = X_n \oplus X_r : X_n = \Pi X, \quad X_r = (I - \Pi)X \tag{3.3}$$

$$\Pi := (1/2\pi i)\int_T (sI - A)^{-1} ds \quad \in L(X) \tag{3.4}$$

$$A = \begin{pmatrix} A_n & 0 \\ 0 & A_r \end{pmatrix}, \quad T(t) = \begin{pmatrix} T_n(t) & 0 \\ 0 & T_r(t) \end{pmatrix} \tag{3.5}$$

$$B = \begin{pmatrix} B_n \\ B_n \end{pmatrix}, \quad C = (C_n, \Pi C_r)$$

where $A_n = A|X_n$ is bounded, $A_r = A|X_r$ is the infinitesimal generator of the semigroup $T_r(t) = T(t)|X_r$ on $X_r$, $T_n(t) = T(t)|X_n$, $B_n = \Pi B$, $B_r = (I - \Pi)B$, $C_n = C$ and $C_r = C(I - \Pi)$. Furthermore, $\Pi$ and $(I - \Pi)$ commute with A and T(t).

We need the spectrum decomposition assumption in the following form.

(A1) There exists a sequence $\{\rho_n\}$, $\rho_s \geq \rho_1 > \rho_2 > \cdots$, $\rho_n \to -\infty$, such that the spectrum decomposition assumption (A0) is satisfied with $\sigma_n(A)$ for each $n \geq 1$, where

$$\sigma_n(A) := \sigma(A) \cap \{s : R_e\, s \geq \rho_n\}$$

This again yields the decomposition of the state space

$$X = X_n \oplus X_{rn} : X_n = \Pi_n X, \quad X_{rn} = (I - \Pi_n) X \quad (3.6)$$

with the projection $\Pi_n$ corresponding to $\sigma_n(A)$. Correspondingly, we write

$$A = \begin{pmatrix} A_n & 0 \\ 0 & A_{rn} \end{pmatrix}, \quad T(t) = \begin{pmatrix} T_n(t) & 0 \\ 0 & T_{rn}(t) \end{pmatrix}$$
$$B = \begin{pmatrix} B_n \\ B_{rn} \end{pmatrix}, \quad C = (C_n \ \ C_{rn}). \quad (3.7)$$

As in the finite dimensional situations, we need assumptions on the stabilizability of the pair (A, B) and detectability of the pair (C,A). In the present context, these can be expressed in the following way. In the following, we write $A_u$, $B_u$, $C_u$, $\Pi_u$ for $A_1$, $B_1$, $C_1$, $\Pi_1$ and $X_1$.

(A2) The subspace $X_n$ is finite dimensional for each $n \geq 1$.

(A3) The pair $(A_u, B_u)$ is stabilizable and the pair $(C_u, A_u)$ is detectable.

We note that $A_u$, $B_u$ and $C_u$ are finite dimensional operators, and hence we can rely on the familiar concepts of stabilizability and detectability in finite dimensions.

With respect to the above decomposition, we need the following technical assumptions.

(A4) There exists a constant $c_\Pi$ such that $|\Pi_n| \leq c_\Pi$, $n \geq 1$.

(A5) $|T_{rn}(t)| \leq M_n e^{\sigma_n t}$ for a sequence $\{\delta_n\}$, $\rho_s \geq \delta_1 > \delta_2 > \cdots$, such that $M_n / \delta_n \to 0$ as $n \to \infty$.

For easy reference, we quote the following two lemmas which will be used repeatedly.

LEMMA 3.1.[8] Suppose that $A_1$ and $A_2$ are generators of semigroups on the Banach spaces $X_1$ and $X_2$, respectively, with growth constants $\omega_1$ and $\omega_2$.

Suppose that $A_3 \in L(X_1, X_2)$. Then the operator on $X_1 \oplus X_2$ defined by

$$\begin{pmatrix} A_1 & 0 \\ A_3 & A_2 \end{pmatrix}$$

generates a semigroup with growth constant equal to max $(\omega_1, \omega_2)$.

LEMMA 3.2[3] Let A be the infinitesimal generator of a semigroup T(.) on the Banach space X satisfying $|T(t)| \leq Me^{\alpha t}$. Let $B \in L(X)$ and let S(.) be the semigroup generated by A + B. Then,

$$|S(t)| \leq Me^{(\alpha - M|B|)t}$$

## 4. Existence Result

With the preliminaries in Section 3, we proceed to prove the existence result.

To the system(3.1), we add the compensator of finite dimension

$$\dot{z}_n(t) = A_n z_n(t) + B_n u(t) + G[y(t) - \hat{y}(t)],$$
$$z_n(t) \in X_n,$$
$$u(t) = F z_n(t) \quad (4.1)$$

where $\hat{y}(t)$ is to be determined.

We define

$$z_{rn}(t) = -A_{rn}^{-1} B_{rn} u(t) = -A_{rn}^{-1} B_{rn} F z_n(t) \quad (4.2)$$

and write

$$e_n(t) = x_n(t) - z_n(t),$$
$$e_r(t) = x_{rn}(t) - z_{rn}(t) \quad (4.3)$$

As motivated in Section 2, we choose

$$\hat{y}(t) = C_n z_n(t) + C_r z_r(t) = C_n z_n(t) - C_{rn} A_{rn}^{-1} B_{rn} F z_n(t) \quad (4.4)$$

Then, we obtain

$$y(t) - \hat{y}(t) = Cx(t) - [C_n z_n(t) - C_{rn} z_{rn}(t)] = C_n e_n(t) + C_{rn} e_{rn}(t) \quad (4.5)$$

From equations (3.7) and (4.1), we can show

$$\dot{e}_n(t) = A_n x_n(t) - G[y(t) - \hat{y}(t)] = (A_n - GC_n)\, e_n(t) + GC_{rn} e_{rn}(t) \quad (4.6)$$

Also, we obtain

$$\dot{e}_{rn}(t) = A_{rn}[x_{rn}(t) - z_{rn}(t)] + A_{rn}^{-1} B_{rn} F\{A_n z_n(t) + B_n u(t) + G[y(t) - \hat{y}(t)]\} \quad (4.7)$$
$$= (A_{rn} + A_{rn}^{-1} B_{rn} F G C_{rn})\, e_{rn}(t) + A_{rn}^{-1} B_{rn} F (A_n + B_n F)\, x_n(t) - A_{rn}^{-1} B_{rn} F (A_n + B_n F - GC_n)\, e_n(t)$$

With control u(.) in (4.1), we have

$$x_n(t) = (A_n + B_nF)\, x_n(t) - B_nFe_n(t) \tag{4.8}$$

From (4.6), (4,7) and (4.8), we obtain the following equation

$$\begin{pmatrix} \dot{e}_n(t) \\ \dot{x}_n(t) \\ \dot{e}_{rn}(t) \end{pmatrix} = A_e \begin{pmatrix} e_n(t) \\ x_n(t) \\ e_{rn}(t) \end{pmatrix}$$

$$A_e = \begin{pmatrix} A_n - GC_n & 0 & -GC_{rn} \\ -B_nF & A_n + B_nF & 0 \\ -A_{rn}^{-1}B_{rn}F(A_n + B_nF - GC_n) & A_{rn}^{-1}B_{rn}F(A_n + B_nF) & A_{rn} + A_{rn}^{-1}B_{rn}FGC_{rn} \end{pmatrix} \tag{4.9}$$

We also note that from the relation

$$\begin{pmatrix} e_n(t) \\ x_n(t) \\ e_{rn}(t) \end{pmatrix} = \begin{pmatrix} -I & I & 0 \\ 0 & I & 0 \\ A_{rn}^{-1}B_{rn}F & 0 & I \end{pmatrix} \begin{pmatrix} z_n(t) \\ x_n(t) \\ x_{rn}(t) \end{pmatrix} \tag{4.10}$$

we obtain

$$\begin{pmatrix} z_n(t) \\ x_n(t) \\ x_{rn}(t) \end{pmatrix} = \begin{pmatrix} -I & I & 0 \\ 0 & I & 0 \\ A_{rn}^{-1}B_{rn}F & A_{rn}^{-1}B_{rn}F & I \end{pmatrix} \begin{pmatrix} e_n(t) \\ x_n(t) \\ x_{rn}(t) \end{pmatrix} \tag{4.11}$$

For n > 1, we have the decomposition of the spectrum $\sigma_n(A)$ into $\sigma_u(A)$ into $\sigma_u(A) \cap \{\sigma_n(A) - \sigma_u(A)\}$. With respect to this decomposition of $\sigma_n(A)$, we have the decomposition of the space $X_n$

$$X_n = X_u \oplus X_{sn}$$

Correspondingly, we write

$$A_n = \begin{pmatrix} A_u & 0 \\ 0 & A_{sn} \end{pmatrix}, \quad B_n = \begin{pmatrix} B_u \\ B_{sn} \end{pmatrix}, \quad C_n = (C_u \quad C_{sn}) \tag{4.12}$$

With the assumptions and notations before, we can show the following result on the existence of finite dimensional compensators for zero spillover case.

PROPOSITION 4.1. Under the hypotheses(A1), (A2) and (A3), suppose that $B_{rn}=O$ or $C_{rn}=O$ for some n>1. and that $T(t) \mid X_{sn} \oplus X_{rn}$ is stable. Then, there exists a finite dimensional compensator for the system(3.1).

PROOF. For case where $B_{rn}=O$, the one-by-two lower left block of $A_e$ becomes a zero block. When $C_{rn}=O$, the two-by-one right upper block of $A_e$ becomes a zero block. By the assumption (A3), there exist $F_u \epsilon L(X_u, U)$ and $G_u \epsilon L(Y, X_u)$ such that $A_u - B_uF_u$ and $A_u - G_uC_u$ are stable. Choose $F \epsilon L(X_u \oplus X_{sn}, U)$ and $G \epsilon L(Y, X_u \oplus X_{sn})$ as

$$F = (F_u \quad 0), \quad G = \begin{pmatrix} G_u \\ 0 \end{pmatrix} \tag{4.13}$$

Then, the two-by-two left upper block becomes stable and $A_e$ generates a stable semigroup. Hence, we have

$$|(e_n(t)\ x_n(t)\ x_{rn}(t))| \leq ce^{\rho t} |(e_n(0),\ x_n(0)\ x_{rn}(0))|$$

for some c > 1, $\rho < \rho_s$.

From the relation (4.11), we conclude that

$$|(z_n(t),\ x_n(t),\ x_{rn}(t))| \leq c_1 e^{\rho t} |(z_n(0),\ x_n(0),\ x_{rn}(0))|$$

for some $c_1 > 1$. / / /

Now, we prove our main existence result.

THEOREM 4.1. Consider the system (3.1) under the assumptions (A1) − (A5). Then, there exists a compensator of the form (4.1) which has a finite order and has the growth constant less that $\rho_s$.

PROOF. By assumption (A3), there exits $F_u \epsilon L(X_u, U)$ and $G_u \epsilon L(Y, X_u)$ so that $A_u + B_uF_u$ and $A_u - G_uC_u$ are stable with growth constants $\rho_f, \rho_g$ less than $\rho_s$, respectively. Choose $F\ \epsilon L(X_n, U)$ and $G_u \epsilon L(Y, X_n)$ as in (4.13) for n > 1. With this choice of F and G, we can see

$$FG = \begin{pmatrix} F_uG_u & 0 \\ 0 & 0 \end{pmatrix}$$

The assumption (A5) implies that $|A_{rn}^{-1}| < M_n / |\delta_n|$, and hence we have the estimate

$$|A_{rn}^{-1}B_{rn}FGC_{rn}| \leq (M_n / |\delta_n|)\, |B|\, |F_uG_u|\, |C|\, c_{\text{II}} (1 + c_{\text{II}})^2$$

which goes to zero as $n \to \infty$.
Therefore, there exists a sufficiently large N so that for all $n > N$, the semigroup generated by $A_{rn} - A_{rn}^{-1}B_{rn}FGC_{rn}$ has the growth constnat $\rho_r$ smaller than $\rho_s$.
For $n > N$, we can show

$$A_n - GC_n = \begin{pmatrix} A_u - G_uC_u & -G_uC_u \\ 0 & A_{sn} \end{pmatrix},$$

$$A_n + B_nF = \begin{pmatrix} A_u + B_uF_u & 0 \\ B_{sn}F_u & A_{sn} \end{pmatrix} \qquad (4.14)$$

We let

$$A_{e1} = \begin{pmatrix} A_n - GC_n & 0 \\ -B_nF & A_n + B_nF \end{pmatrix} \qquad (4.15)$$

Then, the growth constant of $A_{e1}$, denoted by $\rho_{e1}$, is equal to the maximum of $\rho_f$, $\rho_g$ and the growth constant of $A_{sn}$, which is smaller than $\rho_s$. From (4.14), (4.15) and the assumption (A4), we can see that the semigroup $T_{e1}(t)$ generated by $A_{e1}$ satisfies

$$|T_{e1}(t)| \leqq M_1 e^{\rho_{e1} t}$$

for some $M_1$ which can be chosen independent of $n > N$.
Let $A_{e2}$ be the block operator obtained by equating the one-by-two left lower block of $A_e$ to be a zero block. Then, by Lemma 3.1, the semigroup $T_{e2}(t)$ generated by $A_{e2}$ the has growth constant $\rho_{e2}$, $\rho_{e2} = \max\{\rho_r, \rho_{e1}\}$ which is smaller than $\rho_s$. Also, from (A4) and (A5), we can see

$$|T_{er}(t)| \leqq M_2 e^{\rho_{e2} t}$$

for $M_2 > 1$ which also can be chosen independent of $n > N$.
From (4.14), we have

$$F(A_n + B_nF) = F_uA_u + F_uB_uF_u$$
$$FGC_n = (F_{sn}G_{sn}C_{sn},\ F_{sn}G_{sn}C_u) \qquad (4.16)$$

Hence, it follows from (A4), (A5) and (4.15) that the norm of one-by-two lower left block of $A_e$ can be made arbitrarily small. Since $A_{e2}$ generates a semigroup with the growth constant $\rho_{e2}$ less than $\rho_s$, it follows from Lemma 3.2 that for suffieciently large $n > N$, the growth constant for $A_e$ is smaller than $\rho_s$, and hence stable. From the relation (4.11), we conclude that the closed loop system with compensator (4.1) is stable. ///

REMARK 4.1. With respect to the decomposition $X_n = X_u \oplus X_{sn}$, we can write the equation (4.1) for the observer as

$$\begin{pmatrix} \dot{z}_u(t) \\ \dot{z}_{sn}(t) \end{pmatrix} = \begin{pmatrix} A_u - G_uC_u + G_uC_{rn}^{-1}B_{rn}F_u & -G_uC_{sn} \\ 0 & A_{sn} \end{pmatrix} \begin{pmatrix} z_u(t) \\ z_{sn}(t) \end{pmatrix} + \begin{pmatrix} B_u \\ B_{sn} \end{pmatrix} u(t) + \begin{pmatrix} G_u \\ 0 \end{pmatrix} y(t)$$

Hence, we can see that $z_{sn}(t)$ satisfies the same dynamic relation as $x_{sn}(t)$ and the dynamic relation for $z_u(t)$ has been corrected to account for the effect of the residual modes $x_{rn}(t)$ by an additional input term from $z_{sn}(t)$ and the modification of the system matrix

## 5. An Example

The conditions (A1)−(A5) are fairly general and are satisfied by many infinite dimensional systems of interest. As for a specific example, we consider the system discussed in [4]. Let X be a real, seperable Hilbert space with inner product $<.,.>$ and norm $|\,.\,|$. We consider

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} b_iu_i(t) \qquad (5.1)$$
$$y(t) = [\langle c_1, x(t)\rangle,\ \langle c_2, x(t)\rangle \cdots \langle c_p, x(t)\rangle]^T$$

Where $b_i \in X$, $1 \le i \le m$, $c_i \in X$, $1 \le i \le p$, and the generator A of a semigroup T(t) is a self-adjoint operator with compact resolvent. In this case, the spectrum $\sigma(A)$ is just point spectrum $\sigma_p(A)$ with real eigenvalues $\{\mu_i, i \ge 1\}$, $\omega_0 \ge \mu_1 > \mu_2 > \ldots$, for some finite real $\omega_0$, and the corresponding eigenmanifolds are finite dimensional. Furthermore, the eigenvalues are isolated with no accumulation point, $|\mu_i| \to \infty$. Hence, there are at most finitely many eigenvalues $\{\mu_j, j = 1, 2 \cdots, J\}$ which are greater than or equal to the desired growth constant $\rho_s$, each with finite dimensional eigenspace. A fundamental fact in the present case is that the eigenvectors $\{\phi_{jk}$, $k = 1, 2, \ldots, r_j$, $k \ge 1\}$ are complete in X, where $r_j$ is the multiplicity of the eigenvalue $\mu_j$. The condition (A1) is satisfied by taking $\delta_n = \mu_{J+n-1}$, $n \ge 1$. Then, we have

$$x = x_n + x_{rn} = \sum_{j=1}^{J+n-1} \sum_{k=1}^{r_j} \langle x, \phi_{jk} \rangle \phi_{jk} + \sum_{j=J+n}^{r_j} \langle x, \phi_{jk} \rangle \phi_{jk}$$

where $X_n$ = span of $\{\phi_{jk}, k = 1, 2, ..., r_j, j = 1, 2, ..., J+n-1\}$ is $(r_1+r_2+...+r_{J+n-1})$ --dimensional. We also have

$$Ax = \sum_{j=1}^{\infty} u_j \sum_{k=1}^{r_j} \langle x, \phi_{jk} \rangle \phi_{jk}$$

$$= \left\{ x \in X : \sum_{j=1} u_j \sum_{k=1}^{r_j} |\langle x, \phi_{jk} \rangle|^2 < \infty \right\} \quad \text{and}$$

$$A_n X_n = \sum_{j=1}^{J+n-1} \mu_j \sum_{k=1}^{r_j} \langle x_n, \phi_{jk} \rangle \phi_{jk}, \quad x_n \in X_n$$

$$A_{rn} x_{rn} = \sum_{j=J+n}^{\infty} \mu_j \sum_{k=1}^{r_j} \langle x_{rn}, \phi_{jk} \rangle \phi_{jk}, \quad x_{rn} \in X_{rn}$$

The semigroup $T(t)$ on X generated by A is

$$T(t)x = \sum_{j=1}^{\infty} e^{\mu_j t} \sum_{k=1}^{r_j} \langle x, \phi_{jk} \rangle \phi_{jk}$$

$$T(t)x_n = \sum_{j=1}^{J+n-1} e^{\mu_j t} \sum_{k=1}^{r_j} \langle x_n, \phi_{jk} \rangle \phi_{jk}, \quad x_n \in X_n,$$

$$T_{rn}(t)x_{rn} = \sum_{j=J+n}^{\infty} e^{\mu_j t} \sum_{k=1}^{r_j} \langle x_{rn}, \phi_{jk} \rangle \phi_{jk}, \quad x_{rn} \in X_{rn}$$

Hence, $|T_{rn}(t)| \leq e^{\mu_{J+n} t}$, $t > 0$, for $n \geq 1$, with $1/\mu_{J+n} \to 0$, and the condition (A5) is satisfied, Since A is self-adjoint, $\Pi_n$ is an orthogonal projection operator onto the subspace $X_n$ for $n > 1$, and hence $|\Pi_n| = 1$ for $n \geq 1$. therefore, the condition (A4) is also satisfied.

It remains to check the conditions (A2) and (A3). In the present case, $X_u$ = span of $\{\phi_{jk}, k = 1, 2, ..., r_j, j = 1, 2, ..., J\}$ is N-dimensional, $N := r_1+r_2+...+r_J$. Then, the projection $(C_u, A_u, B_u)$ is similar to the triple $(C_f, A_f, B_f)$ where $A_f \in R^{N\times N}$, $B_f \in R^{N\times m}$, $C_f \in R^{p\times N}$ are defined by

$$A_f = \text{diag}\{\mu_1, \cdots, \mu_1, \mu_2, \cdots, \mu_{J-1}, \mu_J, \cdots, \mu_J\}$$

$$B_f = \begin{bmatrix} \langle \phi_{11}, b_1 \rangle \cdots \langle \phi_{11}, b_N \rangle \\ \cdots\cdots\cdots\cdots \\ \langle \phi_{1r_1}, b_1 \rangle \cdots \langle \phi_{1r_1}, b_N \rangle \\ \cdots\cdots\cdots\cdots \\ \langle \phi_{Jr_J}, b_1 \rangle \cdots \langle \phi_{Jr_J}, b_N \rangle \end{bmatrix}$$

$$C_f = \begin{bmatrix} \langle c_1, \phi_{11} \rangle \cdots \langle c_1, \phi_{1r_1} \rangle \cdots \langle c_1, \phi_{J1} \rangle \cdots \langle c_1, \phi_{Jr_J} \rangle \\ \langle c_2, \phi_{11} \rangle \cdots \langle c_2, \phi_{1r_1} \rangle \cdots \langle c_2, \phi_{J1} \rangle \cdots \langle c_2, \phi_{Jr_J} \rangle \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \langle c_p, \phi_{11} \rangle \cdots \langle c_p, \phi_{1r_1} \rangle \cdots \langle c_p, \phi_{J1} \rangle \cdots \langle c_p, \phi_{Jr_J} \rangle \end{bmatrix}$$

Hence, if the finite dimensional system $\dot{z}(t) = A_f z(t) + B_f u(t)$, $y(t) = C_f z(t)$ is stabilizable and detectable in the familiar finite dimensional sense, the conditions (A2) and (A3) are satisfied and a finite dimensional compensator for the system (5. 1) can be constructed following the procedure in Section 4.

We note that the self-adjoint operator with compact resolvent often arises in classical boundary value problems. As for a simple such example, we take on $X = L_2[0,1]$ the operator $A: X \to X$

$$Af = (d^2/dz^2)f + 5\pi^2 f,$$

with boundary condition

$$(d/dz)f(z) = 0, \quad z = 0, 1$$

where

$$D(A) = \{f \in L_2[0, 1] : (d/dz)f \text{ exists, is absolutely continuous and } (d/dz)f(z) = 0, \ z = 0, 1\}$$

In this case, the spectrum consists of the simple eigenvalues $\{\pi^2(5-j^2), j = 0,1,2,...\}$ with the corresponding eigenfunctions $\{\cos(j\pi z), j = 0,1,2,...\}$.

## 6. Conclusions

In this paper, we proposed a design procedure for constructing finite dimensional compensators for infinite dimensional systems with bounded input and output operators. The procedure is basically finite dimensional in that we construct a finite dimensional observer and correct its dynamics to account for the effect of the residual modes. The close scrutinization of the methods provides some information on how small the dimension of the compensator can be. It is, of curse, of interest to extend our results to the systems with unbounded control and sensing.

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