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論 文
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선형시스템을 위한 개선된 수렴속도를 갖는 기준모델 적응제어기 – SIGNAL SYNTHESIS METHOD –

Model Reference Adaptive Control for Linear System with Improved Convergence Rate – SIGNAL SYNTHESIS METHOD –

林 桂 榮*
(Kye – Young Lim)

요 약

잡음에 의하여 교란되고 계수의 NOMINAL값만을 알고 있는 선형시스템에 대한 적응제어기가 SIGNAL SYNTHESIS방법에 의하여 설계된다. 이 제어기는 LYAPUNOV DIRECT METHOD에 기초하며, 기준모델의 추종오차를 줄이고 수렴속도를 향상시키기 위하여 HAMILTON-JACOBI-BELLMAN식에 의하여 간접-보조최적해를 구한다. 시변계수의 영향과 PLANT의 교란에 대응하는 적절한 보상이 이루어지며, 모든 설계를 통하여 미지의 계수에 대한 IDENTIFICATION을 요하지 않는다.

Abstract-Adaptive controllers for linear system whose nominal values of coefficients only are known, that is corrupted by disturbance, are designed by signal synthesis model reference adaptive control (MRAC). This design is stemmed from the Lyapunov direct method. To reduce the model following error and to improve the conrergence rate of the design, an indirect suboptimal control law is de rived using the Hamilton Jacobi Bellman equation. Proper compensation for the effects of time varying coefficients and plant disturbance are suggested. In the design procedure no complete identification of unknown coefficients are required.

1. Introduction

Model reference adaptive control (MRAC) has developed, and has extensively used by several re searchers in conjunction with various applications. There are a number of ways, as indicated in the list of references, [2),3),5) 20),22),23)] that MRAC can be set for an application. Some of these schemes have been actually developed from stability point of view. In any event the stability analyses of these designs must thoroughly be reviewed. The Lyapunov direct method and the Popov hyperstability method are perhaps the most widely used approaches to analyze the stability issues of an MRAC design. Since the MRAC method have extensively used as an analytical tool to design various controllers from the stability point of view and based on the Lyapunov direct method, therefore it will be concerned with the design aspect of the controller. That design will become stable in the sense of Lyapunov. The interesting feature of the applications of the

* 正 會 員 : 金星産電(株) 研究所 責任研究員
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Lyapunov method in MRAC design is that it also enables us to have a measure of convergence rate of the adaptive scheme for analysis [1], although this task is not trivial. Such design will find many interesting applications [5]. The main contribution of this paper is to solve, although indirectly, for an optimal measure of the convergence rate of the adaptive schemes that are designed based on the Lyapunov direct method. These controllers have developed with applications of adaptive control theory to robot manipulator systems in [9)-11)]. The results are, however, general enough to be used in a number of other dynamical systems.

Before presenting the results, two different methods of parameter adaptation and signal synthesis adaptation are stated. In parameter adaptation method, feedforward and / or feedback gain matrices are adjusted so as to reduce the generalized error between the plant and the corresponding reference model. This method, in general, assures asymptotic stability [8),15)], but this method requires perfect model matching for asymptotic stability [13)]; direct adjustability and matchability of parameters [15)]. On the other hand, the signal synthesis method does not require the above two stated conditions. This method does not, however, assure asymptotic stability but it is stable in the sense of bounded error. In this paper, signal synthesis method is studied to improve the system performance. Compensation against the effects of time-varying coefficients and system uncertainties are suggested.

The organization of this paper is as follows. In Section 2, the problem statement is presented. In Section 3,4 and 5 signal synthesis MRAC based on the Lyapunov direct method is developed. Conclusions are deferred to Section 6.

2. Problem Statement

Consider a plant which has unknown time-varying coefficients as follows.

$$P : \dot{x}_p(t) = A_p(t)\, x_p(t) + B_p(t)\, u(t) + v(t), \qquad (1)$$

where $A_p(t) \in R^{n \times n}$, $B_p(t) \in R^{n \times r}$ are time-varying unknown coefficient matrices, for $n \geq r$; $x_p(t) \in R^n$ is directly measurable state vector; $u(t) \in R^r$ is the adaptive control input vector to be adjusted by certain adaptive mechanism described in the sequel; and $v(t) \in R^n$ is uncertainty vector representing unknown additive environmental disturbace such that

$$\| v(t) \| \leq \| v(t) \|_{max} \triangleq \zeta_v, \qquad (1a)$$

where $\| \cdot \|$ represents Euclidean norm, and subscript max is maximum value of the norm.

The reference model for the above plant is described by

$$M : \dot{x}_m(t) = A_m(t)\; x_m(t) + B_m(t)\; w(t), \qquad (2)$$

where $A_m \in R^{n \times n}$, $B_m \in R^{n \times r}$ are constant matrices such that the pair (A_m, B_m) is completely controllable, and A_m is hurwitzian matrix; $x_m(t) \in R^n$ is the state vector; and $w(t) \in R^r$ is the reference input vector such that

$$\| w(t) \| \leq \| w(t) \|_{max} \triangleq \zeta_w. \qquad (2a)$$

The objective of this study is to design adaptive controller to force the state of the plant (1) to follow that of the reference model(2). As a consequence of this design it is assumed that

$$\| x_p(t) \| \leq \| x_p(t) \|_{max} \triangleq \zeta_{xp}. \qquad (3)$$

Furthermore, this design will result in fast-converging error between the above two states. These problems are addressed in signal synthesis method which is stemming from the Lyapunov direct method.

3. Stable Adaptive Law

Consider $A_p(t)=A_n(t)+\triangle A(t)$, $B_p(t)=B_n(t)+\triangle B(t)$, $K_n(t)=B^{\dagger}_n(t)\,(A_m-A_n(t))$, $H_n(t)=B^{\dagger}_n(t)\; B_m$. Here $\triangle A$ and $\triangle B$ represents deviation from the nominal values of A_p and B_p. The superscript † represents the left Penrose pseudo-inverse of $B_n(t)$ which exists if $B^T_n(t)\; B_n(t)$ is a nonsingular matrix [8)], and superscript T denotes the transpose of a matrix. In the following, t in $x_p(t)$, $A_p(t)$, $B_p(t)$, $u(t)$, $v(t)$, $x_m(t)$, $w(t)$, $K_n(t)$ and $H_n(t)$ will be dropped for convenience.

Using a control law

$$u = K_n x_p + H_n w + u_s, \qquad (4)$$

in plant (1) yields

$$\dot{x}_p = A_m x_p + B_m w + B_n (u_s - h), \tag{5}$$

where u_s is control input that will be designed subsequently and $h = B_n^\dagger [\triangle A + \triangle B K_n] x_p + \triangle B H_n w + \triangle B u_s + v]$. The differential equation representing state error $e \triangleq x_m - x_p$ is

$$\dot{e} = A_m e - B_n (u_s + h). \tag{6}$$

To design a stable adaptive controller, following lemma that uses a proportional control law is established.

Lemma 1 : The system of differential equation (6), for $u_s = u_l + u_p$ such that

$$u_l = S^{-1} B_n^T P e, \tag{7a}$$

$$u_p = (\gamma_1 + \gamma_2 / \| B_n^T P e \|) B_n^T P e, \tag{7b}$$

is stable if $\beta^{-1} \triangleq 1 - \| B_n^+ \Delta B \|_{max} > 0$, where

$$\gamma_1 = \beta \| B_n^+ \Delta B \|_{max} \| S^{-1} \|, \text{ and} \tag{7c}$$

$$\gamma_2 = \beta \{ \| B_n^+ (\Delta A + \Delta B K_n) \|_{max} \zeta_{xp} + \| B_n^+ \Delta B H_n \|_{max} \zeta_w + \| B_n^+ \|_{max} \| v \|_{max} \} \tag{7d}$$

Here $0 < P = P^T \in R^{n \times n}$ is the solution of the following Riccati matrix equation.

$$-\dot{P} = A_m^T P + P A_m - 2 P B_n S^{-1} B_n^T P + Q, \tag{8}$$

where $0 < Q = Q^T \in R^{n \times n}$ and $0 < S = S^T \in R^{r \times r}$.

Proof : Defining a positive function V_1 as the Lyapunov function

$$V_1 = e^T P e, \tag{9}$$

derivative of (9) along (6) becomes

$$\dot{V}_1 = e^T (\dot{P} + A_m^T P + P A_m) e - 2 e^T P B_n u_l - 2 e^T P B_n (u_p + h). \tag{10}$$

Substituting (7a) into (10) yields

$$\dot{V}_1 = -e^T Q e + \dot{V}_{a1}, \tag{11}$$

Where $\dot{V}_{a1} = -z e^T P B_n (u_p + h)$. From (5), the upper bound of h can be estimated as follows.

$$\| h \|_{max} \leq \rho \triangleq \{ \| B_n^+ (\Delta A + \Delta B K_n) \|_{max} \zeta_{xp} + \| B_n^+ \Delta B H_n \|_{max} \zeta_w + \| B_n^+ \Delta B \|_{max} \| S^{-1} \| \, \| B_n^T P e \| + \| B_n^+ \Delta B \|_{max} \| u_p \| + \| B_n^+ \|_{max} \| v \|_{max} \}. \tag{12}$$

To achieve a

$$\underset{u_p}{\text{Min}} \; \underset{h}{\text{Max}} \; \dot{V}_1 < 0, \text{ or} \tag{13a}$$

$$\underset{u_p}{\text{Min}} \; \underset{h}{\text{Max}} \; \dot{V}_{a1} < 0, \tag{13b}$$

u_p is chosen

$$u_p = \frac{B_n^T P e}{\| B_n^T P e \|} \rho. \tag{14}$$

Rearranging the right-hand side of (12) and the fact that $1 - \| B_n^\dagger \triangle B \|_{max} > 0$, ρ becomes

$$\rho = \gamma_2 + \beta \| B_n^+ \Delta B \|_{max} \| S^{-1} \| \, \| B_n^T P e \|. \tag{15}$$

Now, $\dot{V}_{a1}$ becomes

$$\dot{V}_{a1} = -2 e^T P B_n \left(\frac{B_n^T P e}{\| B_n^T P e \|} \rho + h \right) \leq 2 \| B_n^T P e \| \, (\| h \|_{max} - \rho) \leq 0.$$

Thus $\dot{V}_1 \leq -e^T Q e$. Q. E. D.

From the above lemma, a sufficient condition of $\beta > 0$ is

$$\| \Delta B \|_{max} < 1 / \| B_n^+ \|_{max}.$$

and it is seen that u_l is the optimal solution minimizing the cost function,

$$\int_0^\infty (e^T Q(t) e + \frac{1}{2} u_l^T S(t) u_l) dt, \tag{16}$$

for the linear differential equation

$$\dot{e} = A_m e - B_n u_l. \tag{17}$$

Since u_l in Lemma 1 is chosen as the linear optimal control input of system (6) with zero disturbances (*i.e.*, $\triangle A = 0$, $\triangle B = 0$ and $v = 0$), thus the $u_s = u_l + u_p$ is a near optimal solution for (6) with small uncertainty vector h.

To improve the performance of this adaptive system next lemma is developed. In this lemma an integral control law u_z is introduced. This u_z will result in smaller error with improved transient behavior than that yielded from Lemma 1.

Lemma 2 : The system of differential equation (6), for $u_s = u_l + u_p + u_z$ such that

$$u_l = S^{-1} B_n^T P e, \tag{18a}$$

$$\bar{u}_p = (\gamma_1 + \bar{\gamma}_2 / \| B_n^T P e \|) B_n^T P e, \text{ and} \tag{18b}$$

$$\dot{u}_z = -m(t) u_z + 2 U^{-1} B_n^T P e, \tag{18c}$$

with (8) is stable if $\beta^{-1} \triangleq 1 - \| B_n^+ \Delta B \|_{max} > 0$, and

$$m(t) > -\lambda_{min}(Q) \| e \|^2 / \{ \lambda_{min}(U) \| u_z \|^2 \}, \tag{19}$$

for $0 < U = U^T \in R^{r \times r}$ and $\lambda_{min}(\cdot)$ represents the minimum eigenvalue of$(\cdot)$. Here γ_1 and γ_2 are the same as (7c) and (7d), and $\overline{\gamma_2}=\gamma_2+\beta \| B^{\dagger}_n \triangle B \|_{max} \| u_z \|$.

Proof : Defining a new positive definite function V_2 as the Lyapunov function

$$V_2 = e^T P e + \frac{1}{2} u_z^T U u_z, \tag{20}$$

then the derivative of (20) along(6) and (18) satisfies the following inequality.

$$\dot{V}_2 \leqq -e^T Q e + u_z^T (\dot{U} u_z - 2B_n^T P e), \text{ or}$$
$$\dot{V}_2 \leqq -e^T Q e - m(t) u_z^T U u_z$$
$$\leqq -\lambda_{min}(Q) \|e\|^2 - m(t) \lambda_{min}(U) \|u_z\|^2 < 0.$$

Q. E. D.

In the Lyapunov synthesis, convergence speed can be compared by a positive value. $\eta = -\dot{V} / V$[4]. In this regard, design A(correspondingly, η_A) has faster convergrence rate than design B (correspondingly, η_B) if $\eta_A > \eta_B$. From Lemma 2, it is observed that a sufficient condition of $-\dot{V}_1 / V_1 \leqq -\dot{V}_2 / V_2$ is

$$m(t) \geqq \lambda_{max}(Q) \lambda_{max}(U) / 2 \{\lambda_{min}(P) \lambda_{min}(U)\}. \tag{21}$$

Since a large value of m leads to a small value of $\| u_z \|^2$ (*cf.*, (18 c)), thus m alone which satisfies the above sufficient condition may not be effective enough for the improvement of system performance. A similar application for constraint $m>0$ is reported in [9] and [10] for an auxiliary input. The next problem is to find the proper m(t) that maximize $\eta(t)$ of the system. The direct solution of this maximization is, however, a difficult problem because $\eta(t)$ contains differential equation (6) which can not be solved '*a priori*'. Thus in the following, an indirect optimization scheme is presented that will result in some answer to the just stated problem.

4. An Indirect – Suboptimal Control Law

To imporve the transient response of the system it is desired to maximize η_2 with respect to m. This direct optimization problem is very difficult. Instead, the lower bound of η_2 is maximized with respect to m, *i.e.*, a sufficient condition which will result in an "optimal" m(t) corresponding to the largest η_2 is maximized. Due to the nature of this optimization we may call such a scheme as "indirect – suboptimal" solution.

Consider lower bound of $\eta_2 = -\dot{V}_2 / V_2$ as follows.

$$\eta_2 \geqq \frac{\lambda_{min}(Q) \|e\|^2 + m(t) \lambda_{min}(U) \|u_z\|^2}{\lambda_{max}(P) \|e\|^2 + \frac{1}{2} \lambda_{max}(U) \|u_z\|^2}$$
$$\triangleq g_2(m, u_z, e), \tag{22}$$

where $\lambda_{max}(\cdot)$ represents the maximum eigenvalue of$(\cdot)$. As mentioned before, an alternative way to maximize η_2, is to maximize at time $t=t^\circ$ the lower bound of η_2, namely, $g_2(m, u_z, e)$. Recall that u_z is a function of m and error e in(18c), it is found that $g_2(m,u_z,e)$ is sensitive for m(t) which is directly adjustable in controller. To establish a criterion to maximize $g_2(m, u_z, e)$, the following fact is introduced.

Fact 1 : For given x(t), four positive function $f_1(x)$, $f_2(x)$, $f_1^\circ(x)$ and $f_2^\circ(x)$, and $a>0$, $b>0$, if

$$f_2^\circ(x)/f_1^\circ(x) \geqq f_2(x)/f_1(x) \text{ and } f_1^\circ(x) \geqq f_1(x),$$
$$\text{for } f_2^\circ(x)/f_1^\circ(x) \geqq a/b, \tag{23}$$

then

$$\frac{ax^2 + f_2(x)}{bx^2 + f_1(x)} \leqq \frac{ax^2 + f_2^\circ(x)}{bx^2 + f_1^\circ(x)}. \tag{24}$$

Let $m = f_2(x) / f_1(x)$ and $m^\circ = f^\circ_2(x) / f_1^\circ(x)$. To achieve the condition of $g_2^\circ(m^\circ, u_z^\circ, e) \geqq g_2(m, u_z, e)$ for a given state e, following maximizaton criteion is chosen

$$J = \frac{1}{2} \int_0^\infty \{\alpha \|u_z\|^2 + \beta m^2\} dt,$$
$$\text{for } m \geqq 0, \ \alpha > 0, \ \beta > 0, \tag{25}$$

subject to the constraint equation

$$\dot{u}_z = -m u_z + 2U^{-1} B_n^T P e \triangleq -m u_z + f. \tag{26}$$

The above maximization procedure is proposed by treating f as one entity. A set of indirect – suboptimal solution of $g_2(m, u_z, e)$, namely G_2°, is defined as follows.

$$G_2^\circ = \{m^\circ \mid \text{the solution of Max J, for } m^\circ > \zeta_m\}, \tag{27}$$

where $\zeta_m = \lambda_{min}(Q) \lambda_{max}(U) / 2 \{\lambda_{max}(P) \lambda_{min}(U)\}$.

To maximize the criterion (25) and subject to (26) several different methods such as Pontryagin's maximum principle, Gradient method, Hamilton – Jacobi – Bellman (HJB) equation exist [21]. In this paper the HJB is used to obtain a solution as follows.

The "optimal" m that solves Max J is

$$m^{\circ} = \begin{cases} -m, & \text{for } m < 0 \\ 0, & \text{for } m \geq 0, \end{cases} \quad (28a)$$

where

$$m \simeq m_1 + m_2, \quad (28b)$$

for

$$m_1 = -\frac{\alpha}{2\beta} \| u_z \|^4 (f^T u_z)^{-1}, \text{ and}$$

$$m_2 = \frac{\alpha^2}{8\beta^2} \| u_z \|^{10} (f^T u_z)^{-3}.$$

A sufficient condition for maximization (28a) is obtained from the second derivative of Hamiltonian as follows.

$$| m_1 | + | m_2 | \leq \sqrt{\alpha/\beta} \| u_z \|. \quad (29)$$

Derivation of (28) is carried out by using the well known procedure in [21].

Even though the above indirect−suboptimal control law does not minimize the state error directly, but it reduces the norm of both state error e and the integral control input u_Z. The indirect−suboptimal control law is now summarized in the following algorithm.

Algorithm 1 : The system of differential equation (6) with $u_S = u_l + \bar{u}_P + u_Z$ is stable. Here u_l is given by (18a), $\bar{u}_P$ is given by (18b) and u_Z is as follows.

$$\dot{u}_z = -m^{\circ} u_z + 2U^{-1} B_n^T P e, \quad \text{with } u_z(0) = 0. \quad (30)$$

Equation (30) is the modified (18c) using m° as generated from (27) and subject to (29).

5. Numerical Example

In the following a numerical example is introduced to demonstrate the usefulness and applications of above results. Consider plant (1) whose exact value of coefficients are unknown to controller with v=0, and model (2) as follows.

$$P : \dot{x}_p = -x_p + 0.5u, \quad (1a)$$

$$M : \dot{x}_m = -20x_m + 2w. \quad (2a)$$

The feedback gains are chosen as $K_n = -30$, $H_n = 3$, and input w is chosen such that $x_m = \sin(t)$. With these gains, the plant (1) acts like

$$\dot{x}_p = -16x_p + 1.5w + 0.5u_s. \quad (1b)$$

The weighting matrices are chosen as follows. S=20,

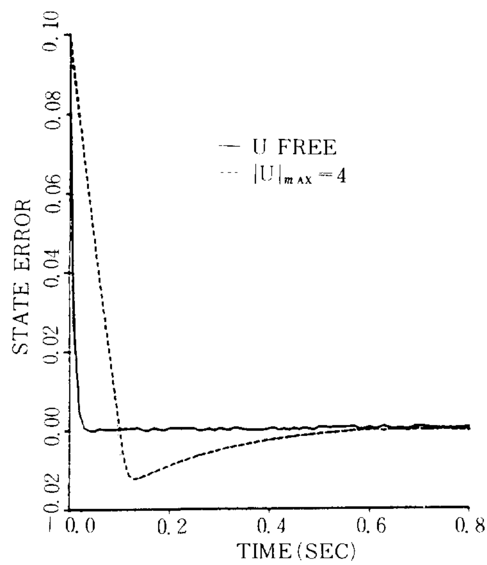

Fig.1. State error with m=0.

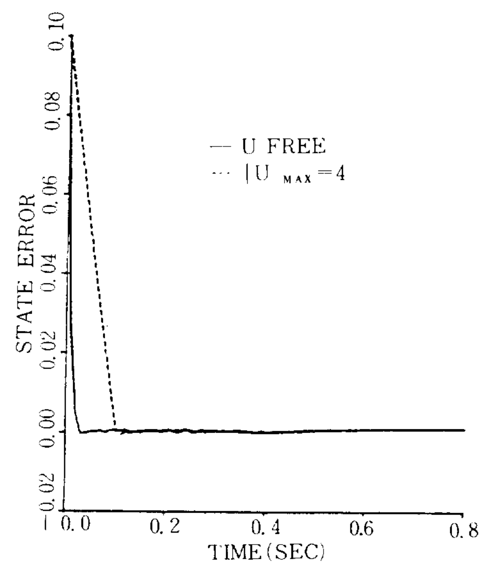

Fig.2. State error with indirect-suboptimal m° (cf., (27)).

P=20 with γ_1=0.05 and γ_2=1. The simulation results shown in Fig. 1 and Fig. 2 demonstrate the improvement (specially in the sense of overshoot) of application of the developed adaptive controller with m≠0 relative to that cases with m=0.

6. Conclusions

A plant with known nominal coefficients and additive uncertainty vector is considered in this paper. For this system adaptive controllers are designed so that the plant state follows the state of corresponding model. These controllers are designed based on the Lyapunov direct method and the resulting control schemes are developed by signal synthesis method. Simulation result shows fast reducing model following error. The integral input with indirect-suboptimal solution reduces the norm of these state error substantially. This method(direct adaptation) does not require the complete identification of unknown coefficients, thus the designed controller is fast and can be used in the real-time.

In the design procedure, delay of adjustable system has not been considered, but present information is used to control the unknown plant. The controllers for the corresponding discrete systems may be designed similarly. In the above simulation numerical constraint on the input vector have improved, although the issue of design with input constraint is not discussed theoretically in this paper. These issues and the applications of this controller in design for mechanical systems are subject of future research.

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