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무한차원 시스템을 위한 선형 이차상태 궤한 제어기의 견인성에 관한 연구

On Robustness of Linear Quadratic State Feedback Regulators for Infinite Dimensional systems

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요 약

이 논문은 시스템 내 불확실성이 존재하는 경우에 있어서, 무한차원 시스템을 위한 선형 이차상태 궤환 제어기의 견인 안정성에 관한 연구이다. 시스템과 입력 함수의 비선형 동요에 대해 점근 안정성과 지수 안정성이 유지되는 조건을 고찰 하였다. 입력 공간이 유한 차원인 경우에 대하여, 유한 차원의 불안정 상태를 위한 선형이차 상태 제어기에 기초하여 구성된 상태 궤환 제어기의 견인성이 개별적으로 논의 되었다.

Abstract - This paper is concerned with the robust stability of linear quadratic state feedback regulators for infinite dimensional systems in the presence of system uncertainties. Several robustness results ensuring the asymptoitc stability and exponential stability of the perturbed closed loop system are derived for a class of nonlinear perturbations of the system and input operators satisfying the matching condition. For the case where the input space is finite dimensional, some robust properties of the state feedback regulator designed on the basis of the linear quadratic regulator for finite dimensional unstable modes are also discussed seperately.

1. Introduction

It had long been recognized that an essential property of a controller is their robustness, that is, the ability to maintain stability and perturbations, since any model is at best an approximation of reality. In classical frequency domain techniques for single-input single-output systems, the robustness issue is naturally handled via various graphical means displaying system model in terms of its frequency response. Therefore, there have been many attempts to develop means for incorporating robustness specifications into multivariable feedback design procedures. Considerable interest has recently been devoted towards the robust properties of multivariable control systems designed on the basis of linear quadratic (LQ) techniques in the presence of perturbations in system dynamics. Through these researches, the robust stability of LQ state feedback regulators against dynamical, timevarying and nonlinear perturbations in the feedback gain has been characterized in terms of gain and phase margins

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or in terms of the minimum singular value of the return difference matrix.[8) 12)]

However, for most physical systems, a realistic model includes distributed effects. Whereas in some cases these effects can be safely ignored, there are many important physical systems for which these distributed effects must be taken into account. Great progress has been made in the system theory in the context of infinite dimensional spaces, and by now, the LQ feedback theory for infinite dimensional systems are well established extending many results in finite dimensions. This paper attempts to understand the robust properties of LQ state feedback regulators for infinite dimensional systems. Although it is tempting to expect the concepts of controllability, stability and stabilizability as natural extensions of the familiar finite dimensional concepts, there are several different concepts of controllability, stability and stabilizability in infinite dimensions, which become equivalent in finite dimensions. Futhermore, weaker concept of approximate controllability does not imply stabilizability in general.[4)] In this paper, we consider the most robust type of stability, namely, exponential stability.

In the next section, we collect some notations and preliminary results regarding evolution equations on Hilbert spaces. In Section 3, robust stability of LQ state feedback regulators for infinite dimensional systems in the presence of Lipschitzian perturbations of the system and input operators are discussed. For the case where input space is finite dimensional, we also discuss the robust properties of the state feedback controller based on the LQ regulator for finite dimensional unstable modes. Some final remarks follow in Section 4.

2. Preliminaries

We summarize some results on evolution equations and LQ state feedback regulators on the Hilbert space.[1), 2)] For Hilbert spaces X and Y, we denote by L(X, Y) the space of bounded linear operators from X to Y and write L(X) for L(X, X). We denote by $\langle .,. \rangle$ inner products in Hilbert spaces and by $|.|$ norms of vectors and operators. A bounded self-adjoint operator P in L(X) will be called nonnegative $(P \geq 0)$, positive $(P > 0)$ or strictly positive $(P >> 0)$ according to the following standard assumptions:

$$P \geq 0,\ \langle=\rangle,\ \langle Px, x\rangle \geq 0,\ x \in X,$$
$$P > 0,\ \langle=\rangle,\ \langle Px, x\rangle > 0,\ x \neq 0 \in X,$$
$$P \gg 0,\ \langle=\rangle,\ \langle Px, x\rangle \geq \beta |x|^2,\ x \neq 0 \in X \text{ for some } \beta > 0,$$

We consider a system on a infinite dimensional Hilbert space X

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(0) = x_0 : x(t) \in X,\ u(t) \in U,$$
$$y(t) = Cx(t);\ y(t) \in Y, \tag{2.1}$$

with Hilbert spaces U and Y (finite or infinite dimensional), where A is the generator of a strongly continuous semigroup S(.) of linear bounded operators on X, $B \in L(U, X)$ and $C \in L(X, Y)$. Throughout this paper, we consider solutions in the mild sense, namely, as solutions of associated integral equation

$$x(t) = S(t)x_0 + \int_0^t S(t-r)Bu(r)\,dr \tag{2.2}$$

We recall that for each semigroup S(.), a growth constant ω

$$\omega := \lim_{t \to \infty} (1/t) \log |S(t)|$$

is well-defined and determines the exponential decay rate of $|S(t)|$. We say that the semigroup S(t) (or A) is exponentially stable if its growth constant is negative. The pair (A, B) is called stabilizable if there exists a $k \in L(X, U)$ such that the semigroup generated by A-BK is exponentially stable. The pair (C, A) is called detectable if the pair (A^*, C^*) is stabilizable.

The optimal LQ regulator problem is to find the control law that minimizes the performance index

$$J = \int_0^\infty \{\langle x(t), Qx(t)\rangle + \langle u(t), Ru(t)\rangle\}\,dt \tag{2-3}$$

subject to the constraint of (2.2) where $Q > 0$ on X and $R >> 0$ on U. If (A, B) is stabilizable and $(Q^{1/2}, A)$ is detectable, then the optimal control u(t) and the optimal trajectory x(t) are given by

$$u(t)=-Kx(t);K=R^{-1}B^*P$$
$$\dot{x}(t)=(A-BK)x(t),\quad x(0)=x_0,$$

where $P\in L(X)$ is the solution of the algebraic Riccati equation (ARE)

$$\langle Px, Az\rangle+\langle Ax, Pz\rangle-\langle PBR^{-1}B^*Px, z\rangle+\langle x, Qz\rangle=0,\quad x, z\in D(A) \qquad (2.4)$$

It is well known[1), 2)] that the ARE of (2.4) has a unique nonnegative self-adjoint solution P and that the semigroup generated by $A-BK$ is exponentially stable.

It is also known[4)] that if (A^*, B^*) is approximately controllable (respectively, exactly controllable) on some interval $[0, T]$, then the operator P is injective (respectively, is an isomorphism). However, unlike in the finite dimensional case, the existence of a bounded inverse of P, which is equivalent to the surjectivity of P, is not guaranteed even for the exactly controllable pair (A^*, C^*) unless additional conditions are imposed on A. This implies that the usual Liapunov function candidate $V(x):=\langle x, Px\rangle$, $x\in X$, is not strictly positive in general.

In this paper, we consider the following nonlinear perturbations in A and B

$$\dot{x}(t)=Ax(t)+Bf(x(t))+Bg(u(t)) \qquad (2.5)$$

where $f: X\to U$ and $g: U\to U$ such that $f(0)=0$, $g(0)=0$, satisfy the Lipschitz condition.

$$|f(x)-f(y)|\leqq c_L|x-y|,\quad x, y\in X,$$
$$|g(u)-g(v)|\leqq c_L|u-v|,\quad u, v\in U \qquad (2.6)$$

for some $c_L>0$.

In the following, we derive conditions such that the parametrized feedback controller

$$u(t)=-K_\mu x(t),\quad K_\mu=\mu K_0,\ K_0=R^{-1}B^*P \quad (2.7)$$

guarantees that the perturbed closed loop system remains asymptotically stable (i.e., $x(t:x_0)\to 0$ as $t\to\infty$), or exponentially stable (i.e., $|x(t:x_0)|\leq ae^{\alpha t}|x_0|$ for some $a\geq 1$, $\alpha<0$). In (2.7), μ is a positive constant to be chosen to ensure suitable stability margin.

Of course, we consider th solution of (2.5) in the mild sense as the solution of the integral equation

$$x(t)=S(t)x_0+\int_0^t S(t-r)B[f(x(r))+g(-K_\mu x(r))]dr \qquad (2.8)$$

with control $u(t)=-K_\mu x(t)$.

We note that under the above assumptions $x(t)=0$ is an equilibrium solution for (2.9).

Since, with control $u(t)$ in (2.7),

$$|g(-K_\mu x)-g(-K_\mu y)|\leqq c_L|K_\mu||x-y|,$$

one can show the following for the equation (2.8).[3)]

PROPOSTION 2.1. For each $x_0\in X$, the equation (2.8) possesses a unique strongly continuous solution $x(t;x_0)$ on the interval $[0, T]$ for any $T>0$. Moreover,

$$|x(t;x_0)-x(t;x_1)|\leqq ae^{\alpha t}|x_0-x_1|,\quad x_0, x_1\in X, \qquad (2.9)$$

for some $a>1$ and $\alpha>0$, where $x(t;x_0)$ and $x(t:x_1)$ are the solutions of (2.8) with initial conditions x_0 and x_1, respectively. If $x_0\in D(A)$, then the mild solution $x(t;x_0)$ also satisfies the differential equation (2.5) (called the strong solution).

Specializing the results in Ichikawa[5)] to the case under consideration, we can state

PROPOSITION 2.2. Let $\{N(t), t\geq 0\}$ be a one-parameter family of nonlinear operators on X with $D(N(t))=X$, $t>0$. Suppose that for some positive continuous function $h(.)$ defined on $(0, \infty)$, $N(t)$ satisfies the following inequality

$$|N(t)x|\leqq h(t)|x|,\quad x\in X,\quad t\geqq 0. \qquad (2.10)$$

Then, the following two statements are equivalent.

(N1) $\int_0^\infty|N(t)x|^p dt\leqq M^p|x|^p$, $x\in X$, $t\geqq 0$ for some $M\geqq 0$, $p>0$

(N2) $|N(t)x|\leqq ce^{\alpha t}|x|$, $x\in X$, $t\geqq 0$ for some $c\geqq 1$, $\alpha<0$

For the solution $x(t;x_0)$ of (2.8), we define a one-parameter family of nonlinear operators $\{N(t), t\geq 0\}$ by

$$N(t)x_0=x(t;x_0),\quad x_0\in X$$

Then, (2.9) implies that the inequality (2.10) is satisfied for $\{N(t), t>0\}$. Hence, to show the exponential stability of the perturbed system (2.8), we need to show

$$\int_0^\infty |x(t;x_0)|^2 dt \leqq c|x_0|^2, \quad x_0 \in X$$

for the solution $x(t;x_0)$ of the equation (2.8).

3. Robustness results

3.1 Case where $(Q^{1/2}, A)$ is detectable

We first consider the asymptotic stability of (2.8) for the case where $(Q^{1/2}, A)$ is detectable. Then, the closed loop system (2.8) is asymptotically stable if

$$\langle u, Rg(u)\rangle + \langle Rg(u), u\rangle - \langle u, Ru\rangle(1+\beta)/\mu \geqq 0, \quad u \in U, \tag{3.1}$$

$$\beta(1-\alpha)\langle x, Qx\rangle - \langle f(x), Rf(x)\rangle \geqq 0, \quad x \in X \tag{3.2}$$

for some constants $\alpha>0$, $\beta>0$.

PROOF. Let $K := K_\mu$ and take $x_0 \in D(A)$. Then, the mild solution $x(t)$ of (2.8) is also a strong solution by Proposition 2.1 and hence, by manipulating the ARE of (2.4), we can establish

$$\begin{aligned}-(d/dt)\langle x(t), Px(t)\rangle &= (1/\mu)\{\langle -Kx(t), Rg(-Kx(t))\rangle + \langle Rg(-Kx(t)), -Kx(t)\rangle - \langle RKx(t), Kx(t)\rangle(1+\beta)/\mu\} \\ &\quad + \langle f(x(t)) - (\beta/\mu)Kx(t), R[f(X(t)) - (\beta/\mu)Kx(t)]\rangle/\beta \\ &\quad + \{\beta(1-\alpha)\langle x(t), Qx(t)\rangle - \langle f(x(t)), Rf(x(t))\rangle\}/\beta + \alpha\langle x(t), Qx(t)\rangle\end{aligned} \tag{3.3}$$

Integrating (3.3) form 0 to t, we obtain

$$\begin{aligned}\langle x_0, Px_0\rangle - \langle x(t), Px(t)\rangle &= (1/\mu)\int_0^t \{\langle -Kx(r), Rg(-Kx(r))\rangle + \langle Rg(-Kx(r)), -Kx(r)\rangle - \langle RKx(r), Kx(r)\rangle(1+\beta)/\mu\} dr \\ &\quad + \int_0^t \langle f(x(r)) - (\beta/\mu)Kx(r), R[f(x(r)) - (\beta/\mu)Kx(r)]\rangle dr/\beta + \int_0^t \{\beta(1-\alpha)\langle x(r), Qx(r)\rangle - \langle f(x(r)), Rf(x(r))\rangle\} dr/\beta \\ &\quad + \alpha\int_0^t \langle x(r), Qx(r)\rangle dr\end{aligned} \tag{3.4}$$

Since $D(A)$ is dense in X and $x(t)$ depends continuously on $x_0 \in X$, (3.4) holds for any $x_0 \in X$. Since $\langle x(t), Px(t)\rangle > 0$ for $t>0$, each term in the RHS of (3.4), which is nonnegative by hypotheses, is bounded by $\langle x_0, Px_0\rangle$. Hence, by taking limit as $t \to \infty$, we conclude

$$I_1 := \int_0^\infty \{\langle -Kx(r), Rg(-Kx(r))\rangle + \langle Rg(-Kx(r)), -Kx(r)\rangle - \langle RKx(r), Kx(r)\rangle(1+\beta)\mu\} dr < \infty \tag{3.5}$$

$$I_2 := \int_0^\infty \langle f(x(r)) - (\beta/\mu)Kx(r), R[f(x(r)) - (\beta/\mu)Kx(r)]\rangle dr < \infty \tag{3.6}$$

$$I_3 := \int_0^\infty \{\beta(1-\alpha)\langle x(r), Qx(r)\rangle - \langle f(x(r)), Rf(x(r))\rangle\} dr < \infty \tag{3.7}$$

$$I_4 := \int_0^\infty \langle x(r), Qx(r)\rangle dr < \infty \tag{3.8}$$

From (3.6) and $R>>0$, it follows tht

$$\int_0^\infty |f(x(t)) - (\beta/\mu)Kx(t)|^2 dt < \infty \tag{3.9}$$

Since

$$\int_0^\infty \langle f(x(t)), Rf(x(t))\rangle dt = -I_3 - \beta(1-\alpha)I_4,$$

(3.7) and (3.8) implies that

$$\int_0^\infty |f(x(t))|^2 dt < \infty \tag{3.10}$$

in view of the fact $R>>0$.

Using the inequality $|a+b|^2 \leqq 2|a|^2 + 2|b|^2$ and the triangle inequality, we have

$$\begin{aligned}\int_0^\infty |Kx(t)|^2 dt &\leqq 2(\mu/\beta)^2 \int_0^\infty |f(x(t)) - (\beta/\mu)Kx(t)|^2 dt + 2(\mu/\beta)^2 \int_0^\infty |f(x(t))|^2 dt < \infty\end{aligned} \tag{3.11}$$

where the last inequality follows from (3.9) and (3.10).

Now, pick $J \in L(X)$ such that the semigroup $T(.)$ generated by $A - JQ^{1/2}$ is exponentially stable, that is,

$$|T(t)| \leqq ce^{\alpha t}, \quad c \geqq 1, \quad \alpha < 0,$$

Then, we write (2.8) as

$$x(t) = T(t)x_0 + \int_0^t T(t-r)\{JQ^{1/2}x(r) + B[f(x(r)) - (\beta/\mu)Kx(r)] + (\beta/\mu)BKx(r) + Bg(-Kx(r))\}dr$$

from which it follows

$$|x(t)|^2 \leqq 2|T(t)x_0|^2 + 6\left|\int_0^t T(t-r)JQ^{1/2}x(r)dr\right|^2 + 6\left|\int_0^t T(t-r)B[f(x(r)) - (\beta/\mu)Kx(r)]dr\right|^2 + 12(\beta/\mu)^2\left|\int_0^t T(t-r)BKx(r)dr\right|^2 + 12\left|\int_0^t T(t-r)Bg(-Kx(r))dr\right|^2$$

$$\leqq 2c^2e^{2\alpha t}|x_0|^2 + 6c_1\int_0^t e^{\alpha(t-r)}\langle Qx(r), x(r)\rangle dr + 12(\beta/\mu)^2c_2\int_0^t e^{\alpha(t-r)}|f(x(r)) - (\beta/\mu)Kx(r)|^2dr + 12c_3\int_0^t e^{\alpha(t-r)}|Kx(r)|^2dr$$

where in the last inequality we used the Lipschitz condition

$$|g(-Kx)| \geqq c_L|Kx|$$

and c_i, $i=1, 2, 3$ are constants independent of t

$$c_1 = 2c^2|J|^2/|\alpha|, \quad c_2 = 2c^2|B|^2/|\alpha|,$$
$$c_3 = 2c^2|B|^2c_L^2/|\alpha|.$$

From (3.8), (3.9) nd (3.11), it follows that each term in the RHS of the last inequality in (3.12) belongs to $L_1(0, \infty)$. Hence, integrating both sides of the above inequality, we obtain

$$\int_0^\infty |x(t)|^2dt < \infty$$

Next, we observe that those terms in the RHS of the last inequality in (3.12) are all continuous and have their derivatives also in $L_1(0, \infty)$, and hence go to 0 as $t \to \infty$. Hence, we conclude that

$$x(t) \to 0 \text{ as } t \to \infty. ///$$

From Theorm 3.1, we can easily deduce the following corollaries.

COROLLARY 3.1. In addition to the hypothesis in Theorem 3.1, suppose $g(u)=u$,

$$\beta(1-\alpha)\langle x, Qx\rangle - \langle f(x), Rf(x)\rangle \geqq 0$$

for some $\alpha > 0, 0 < \beta < 2\mu - 1$

PROOF. Since $g(u) = u$, $u \in U$, (3.1) is satisfied with $0 < \beta < 2\mu - 1$.

COROL LARY 3.2. In addition to the hypotheses in Theorem 1, suppose $f(x)=0$, $x \in X$. Then the closed loop system (2.8) is asymptotically stable if

$$\langle u, Rg(u)\rangle + \langle Rg(u), u\rangle - \langle u, Ru\rangle(1+\beta)/\mu \geqq 0, \quad x \in U, \tag{3.12}$$

for some $\beta > 0$.

PROOF. Since $f(x)=0$, $x \in X$, (3.2) is satisfied for $\alpha=1$. / / /

REMARK 3.1. We can establish st ronger results if we specialize the above discussion to the case where the perturbations are linear, that is,

$$f(x) = Fx, \quad g(u) = Gu \tag{3.13}$$

for some $F \in L(X, U)$, $G \in L(U)$.

Then, the perturbed closed loop system becomes

$$\dot{x}(t) = (A + BF - BGK_\mu)x(t) \tag{3.14}$$

Since $A+BF-BGK_\mu$ is a perturbation of the generator A by a bounded operator $BF-BGK_\mu$, it generates a strongly continuous semigroup T(.) of bounded linear operators on X. With (3.13), (3.1) and (3.2) becomes

$$RG + G^*R - R(1+\beta)/\mu \geqq 0$$
$$\beta(1-\alpha)Q - F^*RF \geqq 0$$

for some $\alpha > 0$, $\beta > 0$.

Under the assumptions in Theorem 3.1 or in Corollary 3.1, 3.2, we established

$$\int_0^\infty |T(t)x_0|^2dt < \infty, \quad x \in X \tag{3.15}$$

By Datko's result on the strongly continuous semigroup of bounded linear operators,[7] (3.15) implies that

$$|T(t)x| \leqq ce^{\alpha t}|x|, \quad x \in X$$

for some $c > 1$, $\alpha < 0$, that is, x(t) in (3.14) is exponentially stable. / / /

3.2 Case where Q is strictly positive

For the case where Q is strictly positive, the inequality (3.12) can be sharpened while yielding the stronger exponential stability.

THEOREM 3.2. Suppose (A, B) is stabilizable, $Q>>0$ and $f(x)=0$, $x\in X$. Then, the closed loop system (2.8) is exponentially stable if

$$\langle u, Rg(u)\rangle + \langle Rg(u), u\rangle - \langle u, Ru\rangle/\mu \geq 0,$$
$u\in U$.

PROOF. Proceeding as in the proof of Theorem 3.1, we can establish

$$\langle x_0, Px_0\rangle-\langle x(t), Px(t)\rangle = (1/\mu)\int_0^t \{\langle -Kx(r), Rg(-Kx(r))\rangle+\langle Rg(-Kx(r)), -Kx(r)\rangle-(1/\mu)\langle RKx(r), Kx(r)\rangle\}dr+\int_0^t\langle x(r), Qx(r)\rangle dr \quad (3.16)$$

for $x_0\in X$.

It follows that

$$\begin{aligned}(d/dt)\langle x(t), Px(t)\rangle &= -\langle x(t), Qx(t)\rangle-(1/\mu)\{\langle -Kx(t), Rg(-Kx(t))\rangle-\langle Rg(-Kx(t)), -Kx(t)\rangle-(1/\mu)\langle RKx(t), Kx(t)\rangle\}\leq\\ &-\langle x(t), Qx(t)\rangle\\ &\leq -q|x(t)|^2\\ &\leq -q|P|^{-1}\langle x(t), Px(t)\rangle \end{aligned} \quad (3.17)$$

where the relation $\langle x, Qx\rangle \geq q|x|^2$, $x\in X$, for some $q>0$ was used.

From (3.17), we obtain

$$\langle x(t), Px(t)\rangle\leq e^{-pt}\langle Px_0, x_0\rangle \quad (3.18)$$

with $p := q|P|^{-1}$

Choose $J \in L(X)$ as in the proof of Theorem 3.1 and write

$$x(t)=T(t)x_0+\int_0^t T(t-r)[JQ^{1/2}x(r)+Bg(-Kx(r))]dr$$

Then, we have

$$\begin{aligned}|x(t)|^2 &\leq 2|T(t)x_0|^2+4\left|\int_0^t T(t-r)JQ^{1/2}x(r)dr\right|^2\\ &+4\left|\int_0^t T(t-r)Bg(-Kx(r))dr\right|^2\\ &\leq 2c^2e^{2\alpha t}|x_0|^2+4c_1\int_0^t e^{\alpha(t-r)}\langle x(r), Qx(r)\rangle dr\\ &+4c_2\int_0^t e^{\alpha(t-r)}|x(r)|^2dr\\ &\leq 2c^2e^{2\alpha t}|x_0|^2-4c_1\int_0^t e^{\alpha(t-r)}(d/dr)\langle x(r), Px(r)\rangle dr-(4c_2/q)\int_0^t e^{\alpha(t-r)}(d/dt)\langle x(r), Px(r)\rangle dr\\ &\leq 2c^2e^{2\alpha t}|x_0|^2+(4c_1-4c_2/q)e^{\alpha t}\langle x_0, PX_0\rangle\\ &+|\alpha|(4c_1+4c_2/q)\int_0^t e^{\alpha(t-r)}\langle x(r), Px(r)\rangle dr \end{aligned} \quad (3.19)$$

where

$$c_1=2c^2|J|^2/|\alpha|,\quad c_2=2c^2c_L^2|B|^2|K|^2/|\alpha|.$$

Using (3.18) and (3.19), we can show

$$\int_0^\infty |x(t)|^2dt\leq M|x_0|^2 \quad (3.20)$$

From Proposition 2.2 and the discussion following the proposition, we conclude that

$$|x(t)|\leq ae^{bt}|x_0|,\quad x_0\in X$$

for some $a\geq 1$, $b<0$. / / /

3.3 Case with finite dimensional input space

In many paractical situations, the input space is finite dimensional, say, $U=R^m$. If the state feedback control law is derived as above based on the stabilizable pair (A, B) and detectable pair ($Q^{1/2}$, A) (or strictly positive Q), the preceeding robustness results are still valid. In the following, we consider robust stablity of state feedback controller designed on the basis of LQ regulators for unstable modes of the system.

To deal with this case, we let $f(x)=Fx$ for some $F\in L(X)$ and write

$$x(t)=S(t)x_0+\int_0^t S(t-r)[BFx(r)+Bg(-K_\mu x(r))]dr \quad (3.21)$$

where K_μ is suitably defined later.

It is natural to assume

(A1) Spectrum decomposition assumption: The spectrum $\sigma(A)$ of the operator A contains a bounded part $\sigma_u(A)$ seperated from the rest $\sigma_s(A) := \sigma(A) - \sigma_u(A)$ in such a way that a rectifiable, simple closed curve T can be drawn so as to enclose an open set containning $\sigma_u(A)$ in its exterior.

Under (A1), we obtain a natural state space decomposition

$$X = X_u \oplus X_s : X_u = \Pi X, \ X_s = (I - \Pi) X$$

where Π is a projection on X defined by

$$\Pi = (1/2\pi i)\int_T (sI - A)^{-1} ds \in L(X)$$

Correspondingly, the following notation will be used with respect to this decomposition

$$A = \begin{pmatrix} A_u & 0 \\ 0 & A_s \end{pmatrix}, \quad B = \begin{pmatrix} B_u \\ B_s \end{pmatrix}, \quad F = (F_u \ \ F_s) \qquad (3.22)$$

For the above decomposition, we assume

(A2) X_u is finite dimensional and the semigroup $S_s(t)$ generated by A_s on X_s is exponentially stable. As in the finite dimensional case, we shall need assumptions on the stabilizability of the pair (A_u, B_u):

(A3) The pair (A_u, B_u) is stabilizable.

with respect to the decomposition $X = X_u \oplus X_s$, we write (3.21) as

$$\begin{pmatrix} \dot{x}_u(t) \\ \dot{x}_s(t) \end{pmatrix} = \begin{pmatrix} A_u & 0 \\ 0 & A_s \end{pmatrix}\begin{pmatrix} x_u(t) \\ x_s(t) \end{pmatrix} - \begin{pmatrix} B_u \\ B_s \end{pmatrix}^{(F \ uF_s)} \begin{pmatrix} x_u(t) \\ x_s(t) \end{pmatrix} + \begin{pmatrix} B_u \\ B_s \end{pmatrix}^{g(-K\mu x(t))} \qquad (3.23)$$

Next, we consider a ARE on X_u

$$A_u^* P_u + P_u A_u - P_u B_u R^{-1} B_u^* P_u + Q_u = 0 \qquad (3.24)$$

where $R \in L(U)$ is strictly positive.

We let

$$K_u = R_u^{-1} B_u^* P_u$$

and define $K_0 \in L(X_u \oplus X_s, U)$ by

$$K_0 = (K_u \ \ 0)$$

We consider the parametrized control law

$$u(t) = -K_\mu x(t), \quad K_\mu = \mu K_0 \qquad (3.25)$$

The following result characterizes a clss of infinite dimensional systems to which the robust stability of the finite dimensional regulator for the unstable modes of the system is carried over to some extent.

THEOREM 3.3. Consider the system (3.21) with control u(.) in (3.25). Suppose $B_u F = 0$ and $Q_u >> 0$ in addition to (A1), (A2) and (A3). Then, the closed loop system (3.21) is exponentially stable if $A_s + B_s F_s$ generates an exponentially stable semigroup and

$$\langle u, Rg(u)\rangle + \langle Rg(u), u\rangle - \langle u, Ru\rangle/\mu \geq 0, \quad u \in U. \qquad (3.26)$$

PROOF. Under the hypotheses, we have

$$\dot{x}_u(t) = A_u x_u(t) + B_u g(-\mu K_u x_u(t)) \qquad (3.27)$$

Hence, specializing Theorem 3.2 to the finite dimensional system (3.27) with (3.6), we find that $x_u(t)$ is exponetially stable, that is,

$$|x_u(t)| \leq c_1 e^{\sigma t} |x_{0u}|,$$

for some $c_1 \geq 1$, $\sigma < 0$, where $x_{ou} = \Pi x_o$.

Since the semigroup $S_\rho(.)$ generated by $A_s + B_s F_s$ is exonentially stable, there exists constants $c_2 > 1$, $\rho < 0$, such that

$$|S_\rho(t) x_s| \leq c_2 e^{\rho t} |x_s|, \quad x_s \in X_s$$

From (3.23), we have

$$x_s(t) = S_\rho(t) x_{0s} + \int_0^t S_\rho(t-r) B_s [F_u x_u(r) + g(-\mu K_u x_u(r))] dr$$

where $x_{0s} = (1 - \Pi) x_o$.

It follows that

$$|x_s(t)| \leq c_2 e^{\rho t} |x_{0s}| + c_3 \int_0^t e^{\rho(t-r)} e^{\sigma r} dr |x_{0u}| = c_2 e^{\rho t} |x_{0s}| + c_3 |x_{0u}| (e^{\rho t} - e^{\sigma t})/(\rho - \delta)$$

where, $c_3 = c_1 c_2 |B_s| (|F_u| + c_1 \mu |K_u|)$

Hence, the response of the system (3.21) with control u(.) in (3.25) under the assumptions is

$$\begin{aligned} |x(t)| &= |x_s(t) + x_u(t)| \\ &\leq c_2 e^{\rho t} |x_{0s}| + [c_3 (e^{\rho t} - e^{\sigma t})/(\rho - \delta) + c_1 e^{\sigma t}] |x_{0u}| \\ &\leq c_2 |I - \Pi| e^{\rho t} |x_0| + [c_3 (e^{\rho t} - e^{\sigma t})/(\rho - \delta) + c_1 e^{\sigma t}] |\Pi| |x_0| \end{aligned}$$

from which the exponential stability of x(t) in (3.21) follows. / / /

We can also show

COROLLARY 3.3. Consider system (3.21) with control u(.) in (3.25). Suppose $B_uF_s=0$, $g(u)=Gu$, $u \in U$, for some $G \in L(U)$ and $(Q_u^{1/2}, A_u)$ is stabilizable in addition to (A1), (A2) and (A3). Then, the closed loop system (3.21) is exponentially stable if $A_s+B_sF_s$ generates an exponentially stable semigroup and

$$RG+G^*R-R(1+\beta)/\mu \geq 0$$
$$\beta(1-\alpha)Q_u-F_u^*RF_u \geq 0$$

for some $\alpha>0$, $\beta>0$.

PROOF. With the discussion in Remark 3.1, we can see that under the hypotheses,

$$\dot{x}_u(t)=A_ux_u(t)+B_uF_ux_u(t)-\mu B_uGK_ux_u(t)$$

is exponentially stable. Hence, we can proceed as in the proof of Theorem 3.3 and conclude that (3.21) is exponentially stable. / / /

4. Conclusions

In this paper, the LQ state feedback regulator for infinite dimensional systems has some robustness properties in the presence of uncertainty in the system and input operators satisfying the matching condition. Also, the state feedback regulator designed on the basis ofthe LQ regulator for finite dimensional unstable modes has shown to carry robustness properties to some extent. An important aspect to investigate would be the properties of the controllers where the state is roconstructed rather than measured directly.

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