<研究論文>

쉘構造物의 彈塑性解析에 관한 새로운 解析法의 硏究

A Study of New Approach on Elasto-Plastic Analysis of shell Structures

權 宅 鎭*
Kwun, Taek Jin
朴 剛 根**
Park, Kang Geun

要 旨

連續體의 解析에 있어서, 特別한 境遇를 除外하고는, 構造物의 槪略的인 擧動을 把握해야 될 境遇가 종종 있다. 이러한 要求에 副應하기 위해서 剛體要素法(Rigid Element Method)이라 불리우는 새로운 解析法이 開發되었다.

剛體要素法은 元來 坪井研究室에서 壁式프리캐스트 鐵筋콘크리트 構造物의 彈塑性解析[1]을 하기 위해서 開發된 解析法에서 着眼하여, 耐震壁과 같은 連續體에 適用[2,3]함으로서 시작된 數値解析法이다. 그 후 著者들은 圓筒쉘,[4,5] 球形쉘[6] 혹은 이들이 組合된 쉘構造物[7,8]에 適用할 수 있도록 開發·擴張하였다.

剛體要素法의 基本槪念은 連續體의 分割된 각 要素를 剛體(rigid body)라고 假定하고, 각 要素들은 要素의 剛性으로 置煥된 假想스프링으로 서로 連結되어 있다고 假定하여, 이 假想스프링의 擧動을 評價함으로서 全體構造物의 擧動을 把握하는 解析法이다. 이때 要素의 周邊에 取해진 스프링은 解析을 單純化하기 위해서 軸力, 面內剪斷力 및 面外剪斷力만을 傳達한다고 假定하고, 要素의 剛體變位(自由度)는 要素內의 任意의 한 點에서 取하며, 이 點에서의 剛體變位(rigid displacements)는 要素의 周邊에 取해진 스프링을 통하여 다른 要素로 傳達된다.

上記와 같은 剛體要素法의 槪念을 連續體의 彈性 및 彈塑性解析에 適用하면, 解析的 槪念이 單純할 뿐만 아니라 構造物 全體의 自由度數를 대폭 줄여 컴퓨터 計算時間을 節約할 수 있는 잇점이 있고, 巨視的인 모델(macroscopic modeling)과 微視的인 모델(microscopic modeling)의 中間的인 性格을 가지기 때문에 構造物의 破壞狀況에 대해서도 그 槪略을 把握할 수 있다.

本 論文에서는 剛體要素法을 보다 一般化된 解析法으로 開發·擴張하기 위해서 從前의 單層스프링시스템(single-layer spring system)으로 解析이 어려웠던 問題點들을 補完한 復層스프링시스템(double-layer spring system)을 使用함으로서 휨, 비틀림의 效果를 把握할 수 있는 理論的 槪念을 適用한 새로운 球要素, 圓筒要素 및 平面要素를 開發하고, 이러한 剛體要素들의 適合매트릭스의 誘導 및 解析的인 方法을 定式化하였다. 또 휨, 비틀림 및 剪斷力의 效果를 考慮한 四角形圓筒要素 및 菱形圓筒要素를 利用하여 圓筒쉘의 彈性 및 彈塑性解析할 수 있는 프로그램을 開發하고, 이 프로그램으로 캔틸레버로된 連續形鐵筋콘크리트 圓筒쉘의 彈性 및 彈塑性解析에 適用하여 構造物의 擧動에 관한 數値解析의 結果, 卽 內力의 分布, 龜裂의 進展, 破壞의 狀況 및 變形의 狀態 等을 把握해 보았다.

* 建築構造技術士, 工博, 成均館大學校 建築工學科 敎授
** 工碩, 慶南大學校 建築工學科 講師

1. INTRODUCTION

There are often cases that have to grasp the rough behaviors, excepting particular cases, for the analysis of continuum in structural engineering. To correspond with these requirements, it was proposed the new analytical method called the Rigid Element Method (abbreviated REM). Comparing with the other methods, the one of merits in this approach is that the theoretical concepts is simple and the most notable advantage is that the computing time can be considerably reduced, especially in non-linear structural analysis. The REM was originally applied from the idea of rigid body-spring model, and was a kind of numerical discretization techniques for analyzing the elasto-plastic analysis of wall-type precast concrete structures(1). The present authors applied the method to continuum such as shear wall(2, 3), and developed various elements, that is, the rectangle-shaped circular cylindrical element(4), the rhombus-shaped circular cylindrical element (5), the spherical element(6), etc.

In order to improve the above elements in this paper, it will be explained the theoretical basis of new elements—the spherical bending element, the cylindrical bending element and the flat bending element, in consideration of the bending, torsion and shear deformation effects, and derived the adaptation matrix of these new elements. And then it will be proposed the analytic procedure for the elasto-plastic analysis of structures by using the REM. The present theory will be applied to the elastic and the elasto-plastic analysis for the cantilevered cylindrical shell, and some numerical results of structural behavior, that is, the distribution of stresses, the pattern of cracks and the mode of fractures will be shown.

2. THE RIGID ELEMENT METHOD

2.1 New Ideas for Analysis of Structures

The REM has the fundamental concepts as follows: Each discreted element of continuum is regarded as a rigid body, and interconnected with virtual springs having the stiffness of the element. The virtual spring is assumed to transmit only the axial force, the shear force and the normal force. The degree of freedom of the element is placed in a point within the element, and the rigid displacements in this point affect other elements through the virtual springs around the element. And then the soution domain of overall structure is estimated by the behavior of elements and virtual springs.

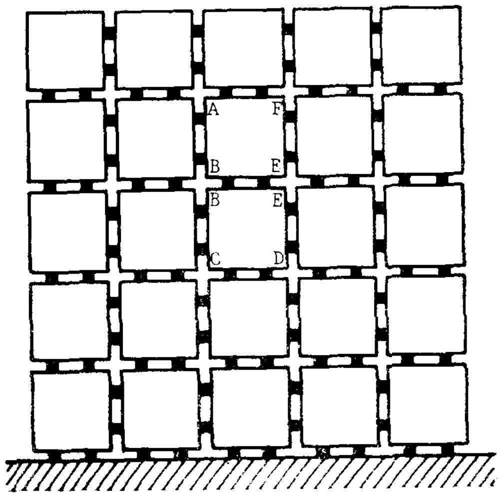
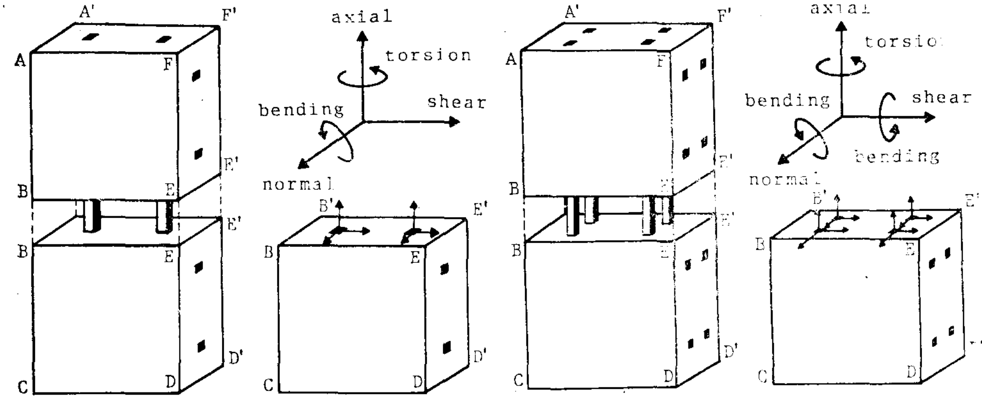


Fig. 1. Rigid body-spring system

Consider the rigid body-spring system which is idealized a set of rectangle-shaped rigid bodies as shown in Fig. 1, and draw out two adjacent elements as shown in Fig. 2 (a) or (b). The



(a) Single-layer spring system (b) Double-layer spring system

Fig. 2. Conception for rigid body-spring system

rigid body-spring system is composed of the elements which are interconnected with some springs, and the internal forces are transmitted with the springs on the contact surface of two adjacent bodies.

The mechanical properties of single-layer spring system and double-layer spring system in thickness direction are shown in Fig. 2 (a) and (b). Because the springs only transmit the axial force, the shear force and the normal force, the double-layer spring system can effectively estimate not only bending effects in thickness direction but also torsional actions, in comparison with the single-layer spring system.

With the assumption that the rotational displacement $\Theta_i (i=x, y, z)$ is very small, the displacements u, v, w in a point around an element are affected by 6 rigid displacements (D_X, D_Y, D_Z, Θ_X, Θ_Y, Θ_Z) in the center of the element, and can be written as follows:

$$\begin{aligned} u &= A_1^u D_X + A_2^u D_Y + A_3^u D_Z + A_4^u \Theta_X + A_5^u \Theta_Y + A_6^u \Theta_Z \\ v &= A_1^v D_X + A_2^v D_Y + A_3^v D_Z + A_4^v \Theta_X + A_5^v \Theta_Y + A_6^v \Theta_Z \\ w &= A_1^w D_X + A_2^w D_Y + A_3^w D_Z + A_4^w \Theta_X + A_5^w \Theta_Y + A_6^w \Theta_Z \end{aligned} \quad (1)$$

where $A_i^u, A_i^v, A_i^w (i=1\sim6)$ are the coefficients changed according to the shape of element.

From eq.(1), the adaptation equation of element can be rewritten as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} A_1^u & A_2^u & A_3^u & A_4^u & A_5^u & A_6^u \\ A_1^v & A_2^v & A_3^v & A_4^v & A_5^v & A_6^v \\ A_1^w & A_2^w & A_3^w & A_4^w & A_5^w & A_6^w \end{bmatrix} \begin{Bmatrix} D_X \\ D_Y \\ D_Z \\ \Theta_X \\ \Theta_Y \\ \Theta_Z \end{Bmatrix} \quad (2)$$

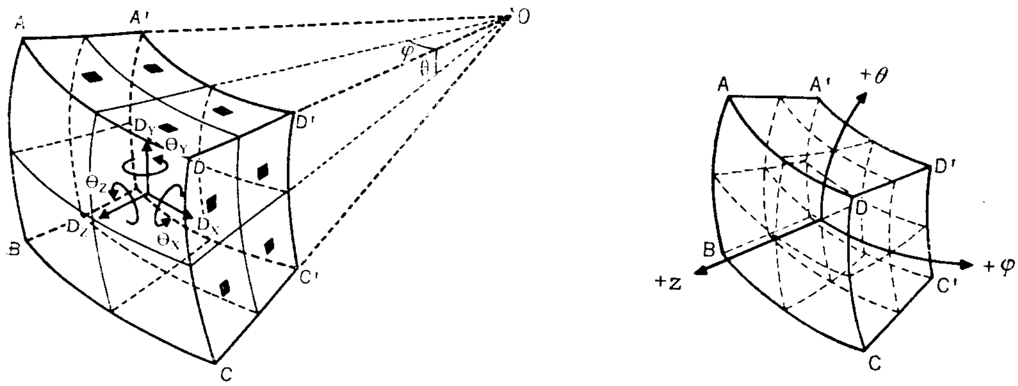
or $\{U\} = [A]\{D\}$ (3)

Where $[A]$ is called the adaptation matrix of element.

2.2 Spherical Bending Element

Considering the spherical shell which is divided to the mesh of spherical elements, the spherical bending element can be depicted as shown in Fig. 3(a).

The spherical bending element has 3 movable rigid displacements (D_X, D_Y, D_Z and 3 rotational rigid displacements (Θ_X, Θ_Y, Θ_Z) at the center of the element.



(a) Unit element (b) Coordinate system

Fig. 3. Spherical bending element

Considering the displacements of spring affected by 6 centroidal rigid displacements, the adaptation matrix of the spherical bending element can be derived as follows :

$$[A]_{SB}=\begin{bmatrix} \cos\varphi & 0 & -\sin\varphi & -(R+z)\sin\theta\sin\varphi & (R+z)\cos\theta-R\cos\varphi & -(R+z)\sin\theta\cos\varphi \\ -\sin\theta\sin\varphi & \cos\theta & -\sin\theta\cos\varphi & R\cos\theta-(R+z)\cos\varphi & R\sin\theta\sin\varphi & (R+z)\sin\varphi \\ \cos\theta\sin\varphi & \sin\theta & \cos\theta\cos\varphi & R\sin\theta & -R\cos\theta\sin\varphi & 0 \end{bmatrix} \quad (4)$$

where φ, θ, z are determined by the coordinate system for the spherical bending element as shown in Fig. 3(b).

Putting $z=0$ in eq. (4), the adaptation matrix of the spherical element not considered bending in thickness direction can be derived.

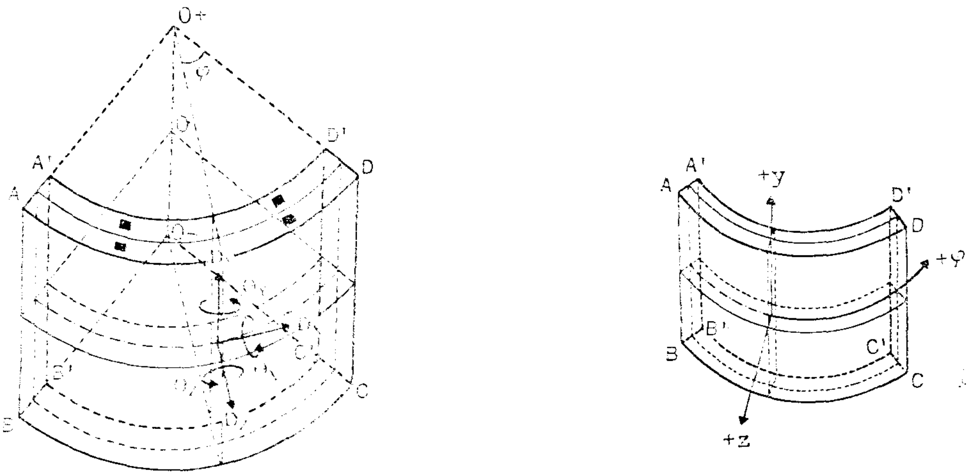
The spherical bending element can be applied to the analysis for bending, torsion and shear deformation effects of shell structures, that is, the pressure vessel, the water tank and the rcof of sports building, which compose of spherical shape.

2.3 Cylindrical Bending Element

Circular cylindrical shells can be divided to the mesh of rectangle-shaped cylindrical elements or rhombus-shaped cylindrical elements, and so forth.

In same way as the spherical bending element, the rectangle-shaped circular cylindrical bending element as shown in Fig. 4 (a) has 6 rigid displacements (D_X, D_Y, D_Z, Θ_X, Θ_Y, Θ_Z) at the center of the element.

Considering the relation between 6 centroidal rigid displacements and displacements of spring



(a) Unit element (b) Coordinate system

Fig. 4 Cylindrical bending element

in a point around the element, the adaptation matrix of the circular cylindrical bending element can be derived as follows:

$$[A]_{CB}=\begin{bmatrix} \cos\varphi & 0 & -\sin\varphi & -y\sin\varphi & (R+z)-R\cos\varphi & -y\cos\varphi \\ 0 & 1 & 0 & R-(R+z)\cos\varphi & 0 & (R+z)\sin\varphi \\ \sin\varphi & 0 & \cos\varphi & y\cos\varphi & -R\sin\varphi & -y\sin\varphi \end{bmatrix} \qquad (5)$$

where φ, y, z are determined by the coordinate system for the cicular cylindrical bending element as shown in Fig. 4(b).

Putting $z=0$ in eq. (5), the adaptation matrix of the circular cylindrical element not considered bending in thickness direction can be derived.

2.4 Flat Bending Element

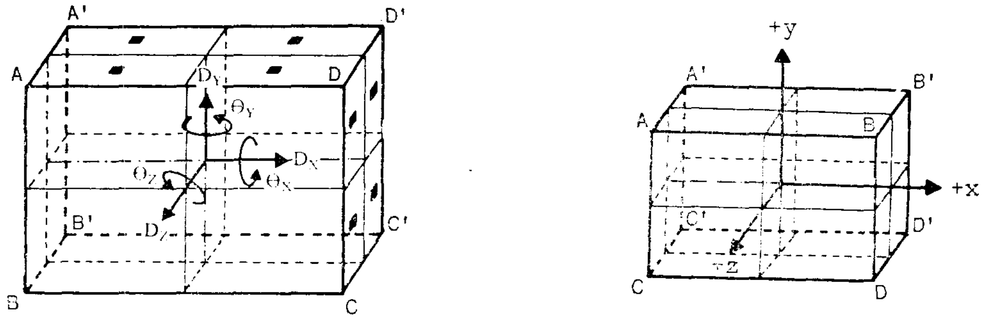
Arbitrary curved shells approximately can be divided to the mesh of flat elements.

Considering the relation between 6 centroidal rigid displacements and displacements of spring in a point around the flat bending element as shown in Fig. 5(a), the adaptation matrix of the flat bending element can be derived as follows:

$$[A]_{FB}=\begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix} \qquad (6)$$

where x, y, z are determined by the coordinate system for the flat bending element as shown in Fig. 5(b).

Applying the flat bending element to plane problems and plate problems, the degrees of freedom are given as (D_X, D_Y, Θ_Z) and (D_Z, Θ_X, Θ_Y), respectively.



(a) Unit element (b) Coordinate system

Fig. 5. Flat bending element

2.5 Analytical Method

Considering the spring-α which is connected a point 'i' of element-M with a point 'j' of element-N, the adaptation matrix of spring-α, from eq. (3), can be expressed as follows:

$$\{U_i\}_M=\{A_i\}_M\{D\}_M \tag{7}$$

$$\{U_j\}_N=\{A_j\}_N\{D\}_N \tag{8}$$

The equation that transform the displacements u, v, w into the displacements d_A, d_S, d_N corresponding in each side of the element is derived as follows:

$$\begin{Bmatrix} d_A \\ d_S \\ d_N \end{Bmatrix} = \begin{bmatrix} \cos\eta & \sin\eta & 0 \\ -\sin\eta & \cos\eta & 0 \\ 0 & 0 & \xi \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{9}$$

$$\text{or } \{d\}=[T^1]\{U\} \tag{10}$$

where η, ξ are changed according to the direction of the element side. From eq. (7), (8) and (10), the following formulas are derived.

$$\{d_i\}_M=[T_i^1]_M[A_i]_M\{D\}_M \tag{11}$$

$$\{d_j\}_N=[T_j^1]_N[A_j]_N\{D\}_N \tag{12}$$

Considering the transformation matrix $[T^2]$ between the global coordinate system for overall structure and the local coordinate system for each element and combining eq. (11) and (12), the relative displacement $\{d\}_\alpha$ of spring-α is given as follows:

$$\begin{aligned}\{d\}_\alpha &= \{d_i\}_M+\{d_j\}_N \\ &=\{[T_i^1]_M[A_i]_M[T^2]_M[I]_M+[T_j^1]_N[A_j]_N[T^2]_N[I]_N\}\{\bar{D}\}\end{aligned} \tag{13}$$

where $[I]$ is the unit matrix for each element and $\{\bar{D}\}$ is the expanded column vector of rigid displacements in the global coordinate system.

The internal force $\{f\}_\alpha$ of spring-α is written as follows:

$$\begin{Bmatrix} f_A \\ f_S \\ f_N \end{Bmatrix} = \begin{bmatrix} k_A & 0 & 0 \\ 0 & k_S & 0 \\ 0 & 0 & k_N \end{bmatrix}_\alpha \begin{Bmatrix} d_A \\ d_S \\ d_N \end{Bmatrix}_\alpha \quad (14)$$

$$\text{or } \{f\}_\alpha = [K]_\alpha \{d\}_\alpha \quad (15)$$

where $[\mathbf{K}]_\alpha$ is the stiffness matrix of spring-α.

Substituting eq. (13) into eq. (15), the internal force of spring-α rewritten as

$$\{f\}_\alpha = \{[K]_\alpha [T_i^1]_M [A_i]_M [T^2]_M [I]_M + [K]_\alpha [T_j^1]_N [A_j]_N [T^2]_N [I]_N\} \{\bar{D}\} \quad (16)$$

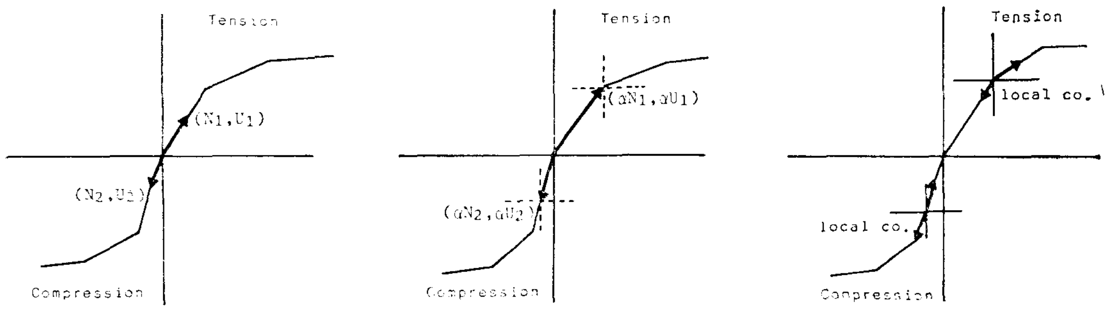
3. PROCEDURE OF ELASTO-PLASTIC ANALYSIS

For the elasto-plastic analysis of shell structures by the REM, it is assumed that the stress-strain relationship of springs is regarded as tri-linear model, and selected the elastic grade (the 1st grade) when the internal force is backward in plastic range (the 2nd grade and the 3rd grade).

The best important information required in the analysis of structures by the REM is to obtain the spring stiffness with the mechanical properties of continuum.

The spring stiffness can be obtained by the experimental testing in general, and differs according to the constitutive material of structures.

Because the reinforced concrete structure differs the tensional stiffness from the compressive stiffness, we need to distinguish the stress-strain relationship for tension and compression.



(a) Results for unit load (b) Calculation of increment load (c) Conception for local coordinate system

Fig. 6. Procedure of elasto-plastic analysis

Fig. 6 shows the procedure of the elasto-plastic analysis with distinguishing the tensional stiffness from the compressive stiffness.

Stage 1 : Because we cannot previously decide the choice of the compressive stiffness and the tensional stiffness for the axial force of spring, it is calculated with the tensional stiffness at first. If the sign of the relative displacement is negative in results, it is converted to the compressive stiffness. And this procedure is repeated until the spring stiffness is consistent with the relative displacement.

Stage 2 : From the results for the unit load in elastic range, it is chosen a spring which can be reached at a node first of all, and calculated the increment load which the spring be reached at the node.

Stage 3 : After taking the origin of the local coordinate system at the points that the internal force of each spring for the unit load is multiplied by the increment load, it is selected the each spring stiffness again.

Stage 4 : These procedures are continuously iterated, from stage 1 to stage 3, up to the destruction of structure.

4. ELASTO-PLASTIC ANALYSIS OF CANTILEVERED CYLINDRICAL SHELL

4.1 Modeling

Consider a part of the cantilevered cylindrical shell composed of the partial circular cylindrical shells (r=60 cm, l=90 cm, t=3 cm, angle=60°).

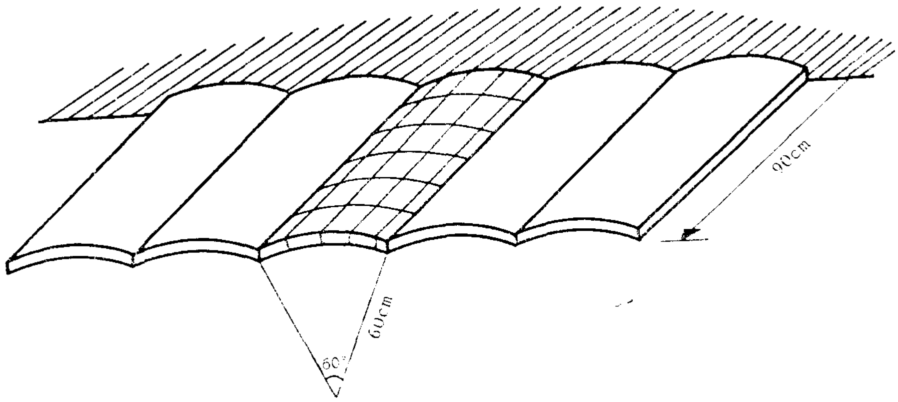


Fig. 7. Cantilevered cylindrical shell

The solution domain of this model is divided into a set of finite rectangle-shaped cylindrical elements as shown in Fig. 7.

This idealized model consists of 31 elements and 192 springs, and the degree of freedom is $31 \times 6 = 186$.

And the model is vertically acted the unit load, in consideration of each element size.

4.2 Results of Numerical Analysis

The results of the elastic and the elasto-plastic analysis of the cantilevered cylindrical shell, using the rectangle-shaped cylindrical bending elements, are as follows.

The internal force diagrams in the elastic analysis are drawn in Fig. 8. The pattern of cracks and the mode of fractures in the elasto-plastic analysis are shown in Fig. 9, where the sign conventions are ○ : the tension in the 2nd grade, ● : the tension in the 3 rd grde, □ : the compression in the 2nd grade and ■ : the compression in the 3rd grade, and the solid line represent for upper springs and the dotted line for lower springs. And the deformation state of structure

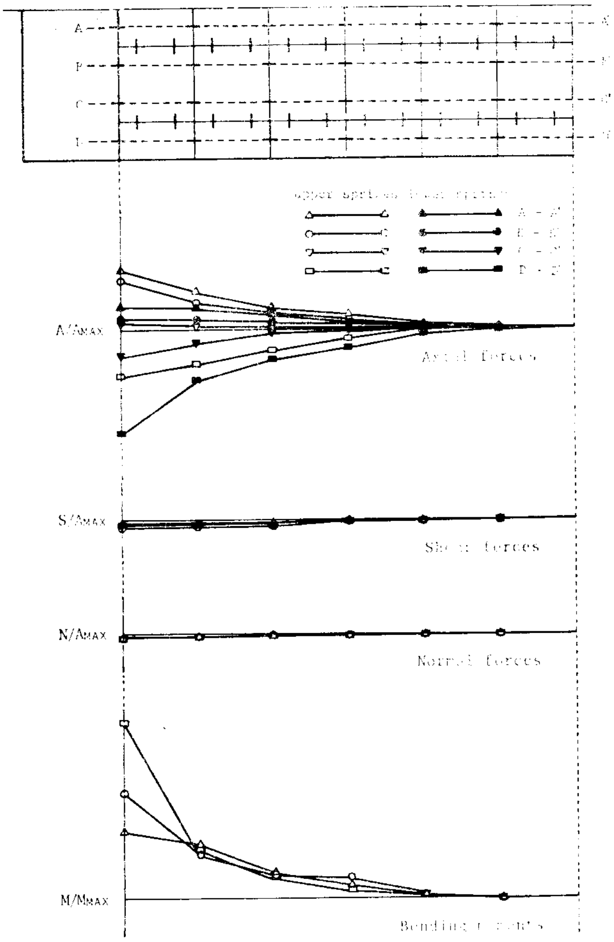


Fig. 8. Internal force diagrams

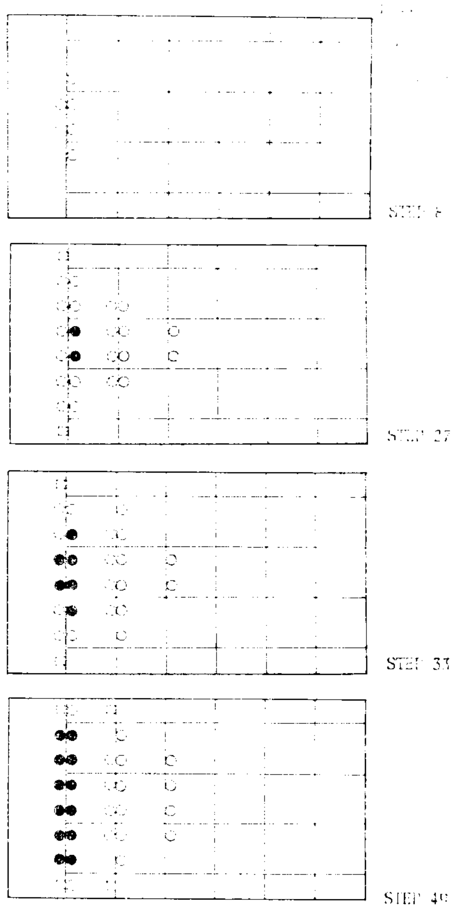


Fig. 9. Crack development diagrams

by the increment load in the elasto-plastic analysis is shown in Fig. 10.

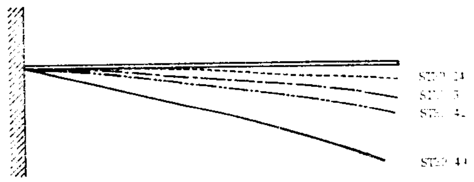


Fig. 10. Deformation state

5. CONCLUSION

In this paper, it has proposed the new elements-the spherical bending element, the cylindrical bending element and the flat bending element, in consideration of the bending, torsion and shear deformation effects, and the procedure of elasto-pastic analysis of shell structures.

And then the rectangle-shaped cylindrical bending elements have applied to the elastic and the elasto-plastic analysis of the reinforced concrete cantilevered cylindrical shell.

Considering the results of numerical analysis, the distribution of stresses is largely expressed the axial force of virtual springs at the fixed end, that is, the tensional stress at the centroid parts of the fixed end and the compressive stress at the edge parts of the fixed end.

And the pattern of cracks is developed from the centroid parts of the fixed end to the others. To obtain more accurate and satisfactory results, with maintenance of the merits; reducing of the degree of freedom and saving the computing time, we will carry on the studies on development of this new approach, as to the spring coefficients for materials and the various problems in according to the element shape, etc.

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