〈研究論文〉

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Closest Open Location Rule in AS/RS

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Abstract

This article deals with the expected travel distances under Closest Open Location rule. In this paper it is shown that uniform distribution assumption over a rack, which is usually used when modeling randomized storage, is not valid under COL rule, where arrival loads follow Poisson process and duration times are independent of each other and have finite mean. We give both exact and approximate methods for the expected travel distances. Warehouse sizing problem is also included.

1. Introduction

In modeling randomized storage for Automated Storage and Retrieval System (AS/ RS), it is commonly assumed that each location over a rack is equally likely selected for storage [2]. If we randomly select an open location among available open locations, the utilization of each location will be equal, therefore the uniform distribution assumption over the rack will be valid. But if we choose the closest open location to Input/Output (I/O) point for storage among the available open locations and if the duration time per each storage is independent of each other, this uniform distribution assumption over the rack does not hold anymore.

The primary purpose of this article is to show that, under Closest Open Location (COL) rule, the degree of utilization of each location is different from each other. For locations close to I/O point, the degree of utilization is very high. It is very stable for some range but drops sharply as the travel distance increases. Our model does not depend on duration time distribution, but the assumption of Poisson input process is crucial.

Secondly, we drive expected travel distances under COL rule. At first, we deal with linear storage rack, but soon our model extends to rectangular storage rack. For ractangular storage rack, expected travel distances are obtained in terms of Tchevychev metric. The warehouse sizing problem is also included.

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2. Ordered Hunt Approach

Assume that arriving loads follow Poisson process with rate λ and is cleared immeadiately when there is no available open location in the rack. Further assume that the duration time per each storage is independent of each other and has finite mean τ. This is a well-known M/G/N/N Queueing system, where N is the total number of locations in the rack.

Suppose that the locations are numbered 1,2,...,N, such that location j is the jth closest location to I/O point. Under COL rule, each arriving load takes the lowest-numbered location which is available.

Let

$$a = \lambda \tau \quad \cdots\cdots (1)$$

and define

$$B(k, a) = \frac{a^k/k!}{\sum_{j=0}^{k} (a^j/j!)} \quad \cdots\cdots (2)$$

a is called the offered load, which provides a measure of the demand to the system. $B(k, a)$ is well-known Erlang loss formula. Note that $B(N, a)$ is the probability that all the locations in the rack are full.

Let $L(j)$ be the load carried by the jth ordered location in equilibrium. From [3] and [7], we have the followings.

$$L(j) = a[B(j-1, a) - B(j, a)],\ j = 1, 2, \cdots, N. \quad \cdots\cdots (3)$$

$$L(1) > L(2) > \cdots > L(N) \quad \cdots\cdots (4)$$

Note that $L(j)$ equals the degree of utilization of jth location, i.e., the probability that jth location is occupied in equilibrium. ⟨Fig. 1⟩ shows $L(j)$ for various a's. We see that $L(j)$ is very stable for some range but drops sharply as j increases.

Let $p(j)$ be the probability that an arriving load is stored in jth location. From [4], we have

$$p(j) = B(j-1, a) - B(j, a),\ j = 1, 2, \cdots, N. \quad \cdots\cdots (5)$$

Let $p'(j)$ be the probability of hunting jth location, on condition that the input load is stored in the rack. Then from (4) and (5), we have the followings.

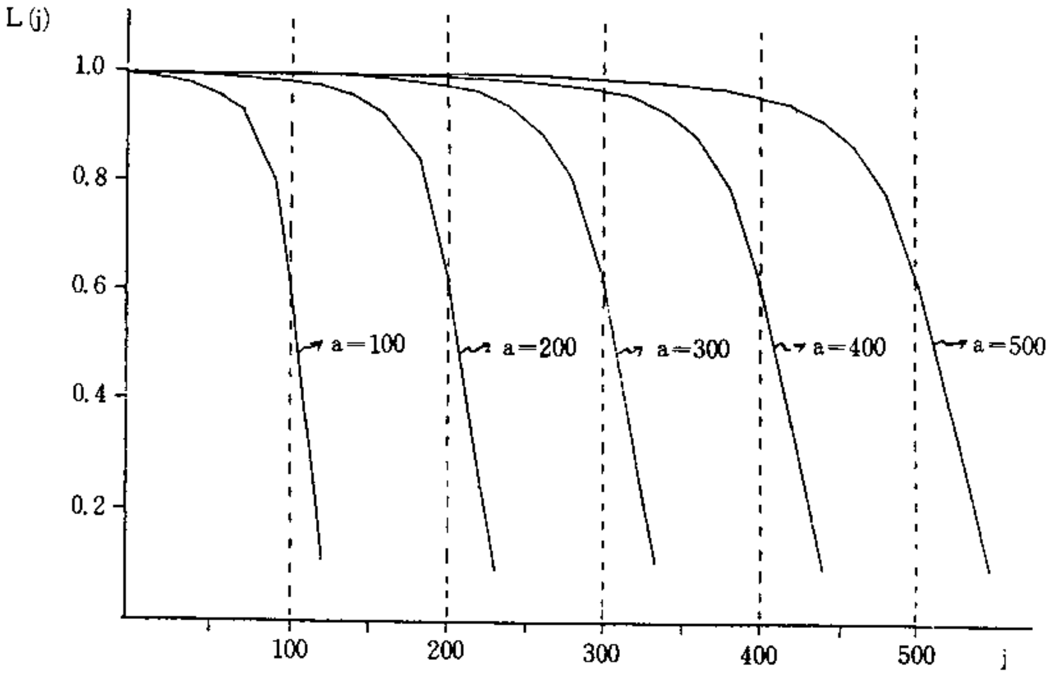
$$p'(j) = [B(j-1, a) - B(j, a)] / [1 - B(N, a)]. \quad \cdots\cdots (6)$$

$$p'(1) > p'(2) > \cdots > p'(N). \quad \cdots\cdots (7)$$

$p'(j)$ represent the relative access frequency of location j in equilibrium. We know from (7) that uniform distribution assumption over the rack does not hold anymore under COL rule. Note that

$$p'(j) = L(j) / [a\{1 - B(N, a)\}]. \qquad (8)$$

So we can indirectly figure out p′ (j) from ⟨Fig. 1⟩.



⟨Fig. 1⟩ Carried Load

3. Expected Travel Distances under COL rule

Consider a linear storage rack. Assume that each opening has the same size and I/O point is located at a-half opening size away from location 1.

The expected travel distance from I/O point to the closest open location is given by

$$D(N, a) = \sum_{j=1}^{N} j p'(j)$$

$$= [\sum_{j=0}^{N-1} B(j, a) - NB(N, a)] / [1 - B(N, a)]. \qquad (9)$$

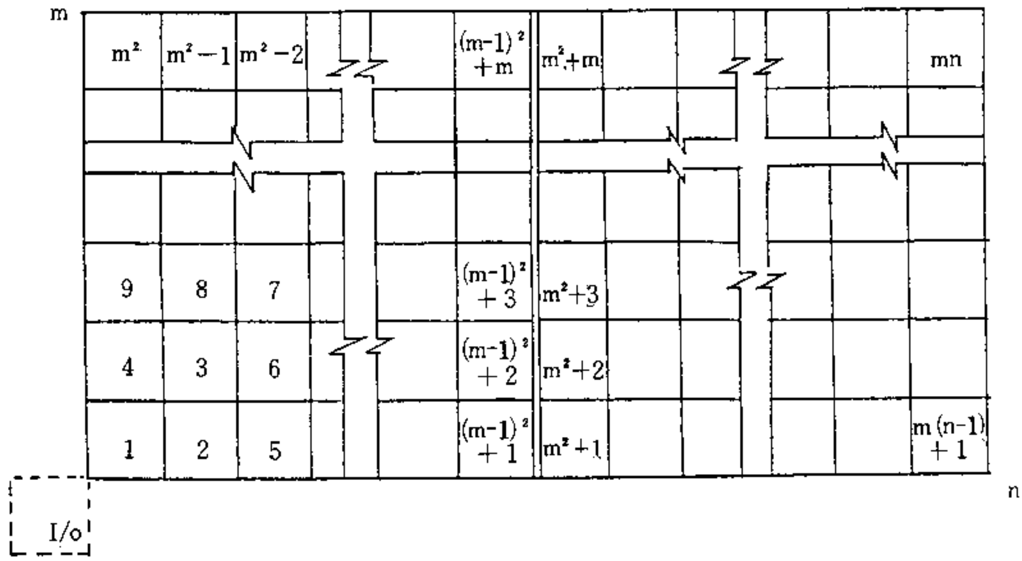
For practical purpose assume that N is infinte. Because both B(N, a) and NB(N, a) converage to zero as N increases, we have

$$D_1(\infty, a) = \sum_{j=0}^{\infty} B(j, a). \qquad (10)$$

If N is relatively larger than a, (10) gives very good approximation to (9).

Interestingly $D_1(\infty, a)$ goes to a/2 as a increases, which tells that when N is relatively larger than a, the expected travel distances under COL rule are near a-half of offered load and approximately equal to those in uniformly distributed rack with N=a.

Next, consider the following rectangular storage rack in ⟨Fig. 2⟩. Without loss of generality, we can assume that $m \leq n$, where $N=mn$.



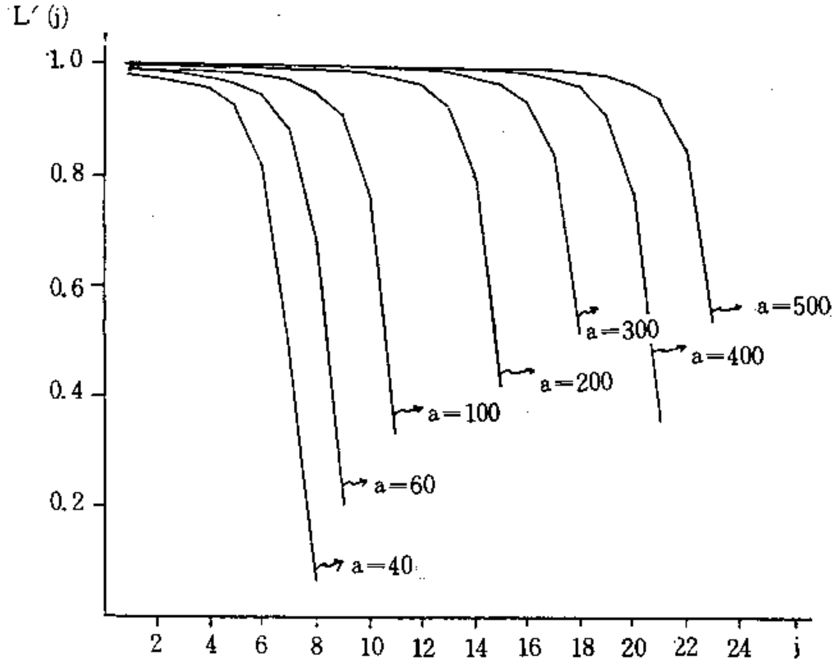
⟨Fig. 2⟩ Rectangular Storage Rack

Assume that the rack is arranged such that the vertical and horizontal travel distances between two adjacent locations are the same. Furthermore, we assume that the Storage and Retrieval (S/R) machine is capable of travelling simultaneously in the aisle both vertically and horizontly. Hence the travel distance is given in terms of Tchebychev metric and is the maximum of the vertical and horizontal travel distances.

Let $L'(j)$ be the degree of utilization of jth ordered location. Then, for $j \leq m$, $L'(j)$ is given by

$$L'(j) = [L((j-1)^2+1) + L((j-1)^2+2) + \cdots + L(j^2)] / (2j-1). \quad \cdots\cdots (11)$$

$L'(j)$ is shown in ⟨Fig. 3⟩



〈Fig. 3〉 Carried Load for Rectangular Rack

The expected travel distance from I/O point to the closest open location in equilibrium is given by

$$D_2(N, a) = \sum_{k=1}^{m} k \sum_{j=(k-1)^2+1}^{k^2} p'(j) + \sum_{k=m+1}^{n} k \sum_{j=m(k-1)+1}^{mk} p'(j)$$

$$= \left[\sum_{k=0}^{m-1} B(k^2, a) + \sum_{k=m}^{n-1} B(mk, a) - nB(N, a) \right] / \left[1 - B(N, a) \right] \quad \cdots\cdots (12)$$

We can show that

$$D_2(\infty, a) = \sum_{k=0}^{m-1} B(k^2, a) + \sum_{k=m}^{n-1} B(mk, a) \quad \cdots\cdots (13)$$

or for sufficiently large m,

$$D_2(\overset{\downarrow}{\infty}, a) = \sum_{k=0}^{\infty} B(k^2, a) \quad \cdots\cdots (14)$$

As a increases, (14) goes to $\frac{2}{3}\sqrt{a}$, which tells that when N is relatively larger than a, the expected travel distances under COL rule in square rack are approximately equal to those in uniformly distributed square rack with N=a.

On the other hand, we can apply continuous approximation to discrete rack to obtain expected travel distance. Let (x, y), $o \le x \le n$, $o \le y \le m$, $m \le n$, represent a random location in a rack. Suppose that the degree of utilization of each location (or relative access frequency of each location) can be described by the following function.

$$f(x, y) = \begin{cases} f(x), & \text{if } x \geq y \\ f(y), & \text{if } x < y. \end{cases}$$

Then the expected travel distance is given by

$$D_3(m, n) = \left[2\int_0^m t^2 f(t)dt + m\int_m^n tf(t)dt\right] / \left[2\int_0^m tf(t)dt + m\int_m^n f(t)dt\right]. \quad \cdots\cdots\cdots(15)$$

We like to point out that (15) is a general fomula to be used in diverse situations such as randomized storage or dedicated storage. As an example, suppose an uniformly distributed rack with f (x, y) = 1/mn. The expected travel distance by continuous approximation is

$$D_3(m, n) = m/6n + n/2 \quad \cdots\cdots\cdots(16)$$

while exact solution for discrete rack is
(m−1)/6n + (n+1)/2.

4. Storage Sizing

Let X denote the number of occupied locations at arbitrary instant in equilibrium. Then the probability that among N locations, k locations are occupied is given by

$$P(X=k) = \begin{cases} \dfrac{a^k/k!}{\sum_{j=0}^{N} (a^j/j!)}, & \text{for } k=0,1,2,\cdots,N \\ 0, & \text{Otherwise,} \end{cases} \quad \cdots\cdots\cdots(17)$$

and mean and variance of X are as follows [1].

$$E(X) = a[1 - B(N, a)] \quad \cdots\cdots\cdots(18)$$

$$VAR(X) = E(X) - aB(N, a) \ [N - E(X)] \quad \cdots\cdots\cdots(19)$$

Because mean E(X) denotes the mean number of occupied locations in equilibrium, E(X)/N represents the degree of average rack utilization. Note that both mean and variance of X converge to a as N increases.

Remember that B(N, a) denotes the probability that an arriving load is blocked. Using the following relationship,

$$B(N, a) = \left[\int_0^\infty (1 + x/a)^N e^{-x} dx\right]^{-1} \quad \cdots\cdots\cdots(20)$$

we can prove the followings.

For $0 < N < \infty$,

$$\lim_{a \to 0} B(N, a) = 0,$$

$$\lim_{a \to \infty} B(N, a) = 1. \quad \cdots\cdots (21)$$

Furthermore, for $0 < a < \infty$,

$$\lim_{N \to \infty} B(N, a) = 0,$$

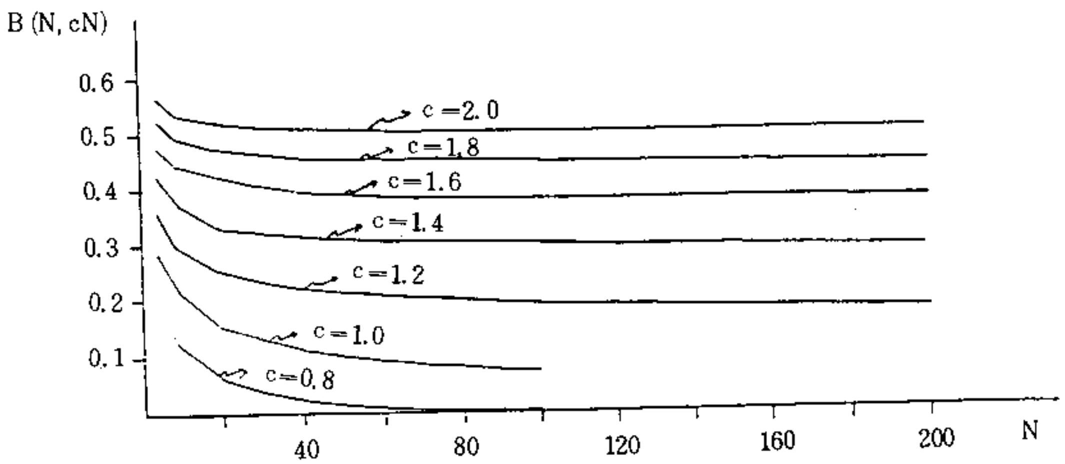
$$\lim_{N \to 0} B(N, a) = 1. \quad \cdots\cdots (22)$$

If both N and a increase to infinity, we have the following.

$$\lim_{N \to \infty} B(N, cN) = \begin{cases} 0, & \text{if } 0 \le c \le 1 \\ 1 - 1/c, & \text{if } c > 1, \end{cases} \quad \cdots\cdots 23$$

where $c = a/N$

⟨Fig. 4⟩ shows B (N, cN)'s for various c's. We see that (23) gives very good approximations for N>100.



⟨Fig. 4⟩ Blocking Probability

(23) gives good interpretation. In order to keep constant blocking probability as N increases, it must be that a should increase proportionally to N with $c>1$. If $c\leq 1$, blocking probability converges to zero as N increases.

$B(N, a)$ gives important information concerning about rack sizing. Suppose that the offered load a and the service level of the rack α are given as extraneous factors. Then the total number of rack openings N^* can be determined from the following relationship.

$$\alpha = B(N^*, a). \qquad (24)$$

Furthermore, when duration times are exponentially distributed, the over-all warehouse sizing problem with r racks and buffer storage can be easily solved by the equivalent random method [3].

5. Concluding Remark

Suppose that there are n classes of different arrival streams, each stream following independent Poisson process with rate λ_i, $i=1,2,\ldots,n$. Further assume that mean duration time of each stream is τ_i, $i=1,2,\ldots,n$, and independent of each other. Because each arrival stream follows independent Poisson process, over-all input process follows Poisson process with rate λ, where $\lambda = \Sigma\lambda_i$. Again, since each stream is Poisson, each stream sees the same state distribution [10]. Then the proportion of stored loads which are of class i turns out to be λ_i/λ. So we have composite mean duration time τ,

$$\tau = \sum \tau_i \lambda_i/\lambda. \qquad (25)$$

Thus our model can be easily adapted even when there are several classes of stream, where each stream is different from each other in input process and duration time.

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