

Negative Induced Polarization Responses over a Layered Earth

Hee Joon Kim*

Abstract: Negative induced polarization (IP) responses are examined for a three-layered earth using a digital linear filter method. The negative IP response can occur when the geoelectric section is of type *K* or *Q*. The section of type *K* creates a more pronounced negative effect than that of type *Q*. For such sections, IP coefficients are determined as a function of the resistivity distribution and the electrode configuration, and only the IP coefficient of the first layer can be negative. As a result, the negative IP response can occur when the first layer is polarizable in the section of type *K* or *Q*, and the polarizabilities of the other layers can act to depress the negative response.

INTRODUCTION

Weak negative induced-polarization (IP) response which is accompanied with positive one has been well known. However, since IP phenomena in rocks are defined as positive phenomena, an independent negative IP response is frequently regarded as a failure of measurement, and a sufficient explanation of the negative IP is not usually given.

Seigel (1959) developed a mathematical expression for the IP response of a buried sphere model, and showed that the negative IP response is created by a negative form factor. Bartin (1968) reported that in time-domain measurements negative IP exists in a layered earth model. He obtained negative IP when the first layer is more polarizable than the second one. This idea was extended by Nabighian and Elliot (1976). Based on Seigel's theory (Seigel, 1959), they developed the expression for time-domain IP curves for a layered earth, and explained the existence of negative IP. Roy and Elliot (1980) applied the IP method to ground water survey, and showed a field curve with negative IP

measured in a zone with saline water layer.

Nabighian and Elliot (1976) revealed that negative IP effects can occur whenever the geologic section is of type *K* or *Q*. For such sections, the IP function (as defined by Seigel) of the first layer can be negative. As a result, if the first layer is polarizable, the response from deeper layers can be severely masked solely as a function of the resistivity distribution.

In this paper, negative IP responses are examined for a three-layered earth model. The IP responses are calculated by Anderson's digital linear filter (Anderson, 1979). Model parameters are electrode configuration, thickness ratio between the first and second layers, and resistivity ratio between the first and bottom layers.

IP EFFECTS

From the potential solution, IP responses can be readily derived, following the theoretical development of Seigel (1959). According to his theory, the apparent IP response (M_a) is obtained by a weighted summation of the intrinsic IP responses (M_i) of each layer:

$$M_a = \sum_i B_i M_i, \quad (1)$$

and the weighting function B_i are obtained from

* Department of Applied Geology, National Fisheries University of Pusan

$$B_i = \frac{\partial \log \rho_a}{\partial \log \rho_i} = \frac{\rho_i \partial \rho_a}{\rho_a \partial \rho_i}, \quad (2)$$

where ρ_a and ρ_i are the apparent and layer resistivities, respectively. The dimensionless coefficients B_i are a function of resistivity distribution and geometric relationship of all layers with reference to the measuring point. The B_i must satisfy the relationship (Seigel, 1959)

$$\sum_i B_i = 1. \quad (3)$$

Once the dilution factors (B_i) are known, the apparent IP response can be completely described. In order to know the B_i , apparent resistivity and its partial derivative should be estimated.

Fig. 1 shows the three sounding arrays considered here: Wenner, Schlumberger and dipole-dipole. For these arrays, apparent resistivities for a n -layered earth are given by

$$\rho_a^W(a) = 2a \int_0^\infty T(\lambda) [J_0(a\lambda) - J_1(2a\lambda)] d\lambda, \quad (4)$$

$$\rho_a^S(s) = s^2 \int_0^\infty T(\lambda) \lambda J_1(s\lambda) d\lambda, \quad (5)$$

and

$$\rho_a^D(r) = \frac{r^2}{2} \int_0^\infty T(\lambda) \lambda J_1(r\lambda) d\lambda - \frac{r^3}{2} \int_0^\infty T(\lambda) \lambda^2 J_0(r\lambda) d\lambda, \quad (6)$$

where the superscripts W , S and D denote Wenner, Schlumberger and dipole-dipole arrays, respectively, and J_0 and J_1 represent the Bessel functions of order 0 and 1, respectively. In (4), (5) and (6), $T(\lambda) \equiv T_1(\lambda)$ is the resistivity transform (Koefoed, 1970).

$$T_i(\lambda) = \rho_i \frac{(\rho_i + T_{i+1}) - (\rho_i - T_{i+1}) \exp(-2\lambda d_i)}{(\rho_i + T_{i+1}) + (\rho_i - T_{i+1}) \exp(-2\lambda d_i)}, \quad (7)$$

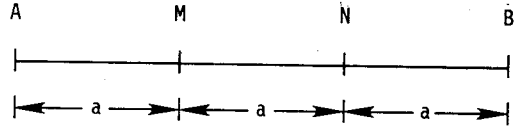
$$i = n-2, n-3, \dots, 1,$$

and

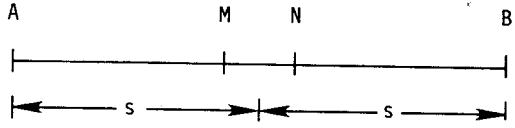
$$T_{n-1}(\lambda) = \rho_n \frac{(\rho_{n-1} + \rho_n) - (\rho_{n-1} - \rho_n) \exp(-2\lambda d_{n-1})}{(\rho_{n-1} + \rho_n) + (\rho_{n-1} - \rho_n) \exp(-2\lambda d_{n-1})}, \quad (8)$$

where ρ_i and d_i are the resistivity and thickness of the i -th layer, respectively.

WENNER



SCHLUMBERGER



DIPOLE-DIPOLE

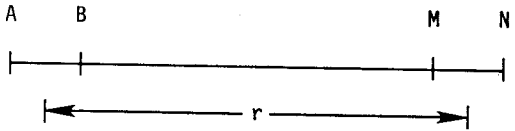


Fig. 1 Resistivity sounding arrays. A and B indicate the current electrodes, and M and N the potential electrodes.

Partial derivatives of the apparent resistivity are obtained by differentiating the resistivity transform $T(\lambda)$ with respect to the layer resistivities. Since the ρ_i enter T_i only through T_{i+1} , we have

$$\frac{\partial T_i}{\partial \rho_j} = \frac{\partial T_i}{\partial T_{i+1}} \frac{\partial T_{i+1}}{\partial \rho_j}, \quad (9)$$

$$j = n, n-1, \dots, i+1.$$

The recursive formula is (Kim, 1981)

$$\frac{\partial T_i}{\partial T_{i+1}} = \frac{4\rho_i^2 \exp(-2\lambda d_i)}{[(\rho_i + T_{i+1}) + (\rho_i - T_{i+1}) \exp(-2\lambda d_i)]^2}, \quad (10)$$

$$\frac{\partial T_i}{\partial \rho_i} = \frac{T_i}{\rho_i} - \frac{T_{i+1}}{\rho_i} \frac{\partial T_i}{\partial T_{i+1}}, \quad (11)$$

$$\frac{\partial T_{n-1}}{\partial \rho_n} = \frac{4\rho_{n-1}^2 \exp(-2\lambda d_{n-1})}{[(\rho_{n-1} + \rho_n) + (\rho_{n-1} - \rho_n) \exp(-2\lambda d_{n-1})]^2}, \quad (12)$$

and

$$\frac{\partial T_{n-1}}{\partial \rho_{n-1}} = \frac{T_{n-1}}{\rho_{n-1}} - \frac{\rho_n}{\rho_{n-1}} \frac{\partial T_{n-1}}{\partial \rho_n}. \quad (13)$$

NUMERICAL RESULTS

For a three-layered earth, negative IP responses appear only for sequences of type *K* ($\rho_1 < \rho_2 > \rho_3$) and *Q* ($\rho_1 > \rho_2 > \rho_3$). For resistivity sequences of type *H* ($\rho_1 > \rho_2 < \rho_3$) and *A* ($\rho_1 < \rho_2 < \rho_3$), the negative IP responses are not encountered.

Fig. 2 shows the dilution factors for Schlumberger array over a three-layered earth of type *K*. Only the B_1 curve shows negative responses in region over about 10m of spacing s , but the B_2 and B_3 curves have no negative values. This means that negative IP effects can occur when the first layer is polarizable, and the polarizabilities of the second and bottom layers may depress the negative IP response.

The B_1 value varies with layer thickness. An increase of thickness of the first layer decreases the negative IP effect (Fig. 3), but an increase of thickness of second layer increases it (Fig. 4). Therefore, it can be concluded that an increase of thickness ratio between the first and the second layers (d_2/d_1) increases the negative IP effect. Spacings s with negative minima increase with an decrease of the thickness ratio.

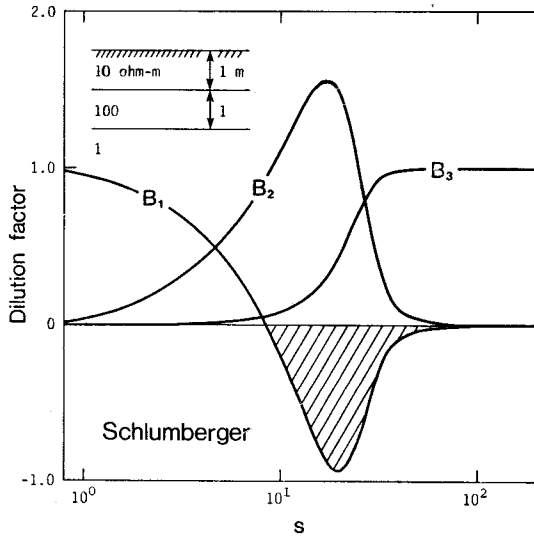


Fig. 2 Dilution factors B_1 , B_2 and B_3 plotted as sounding curves for Schlumberger array on a three-layered earth of type *K*.

The B_1 value also varies with a resistivity ratio between the first and the bottom layers (ρ_3/ρ_1). An increase of the resistivity ratio decreases the negative IP effect (Fig. 5). In the case of $\rho_1 < \rho_3$, the negative IP effect is not so apparent even in *K* type. In spite of changes in the resistivity ratio, spacings s with negative peaks are nearly the same.

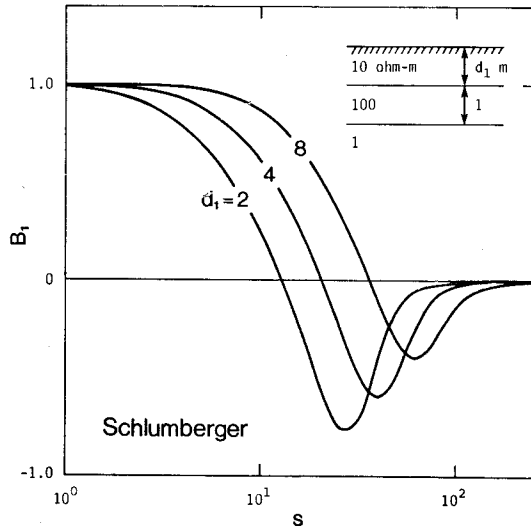


Fig. 3 B_1 curves for Schlumberger array on a three-layered earth of type *K*, with variable thickness of the first layer.

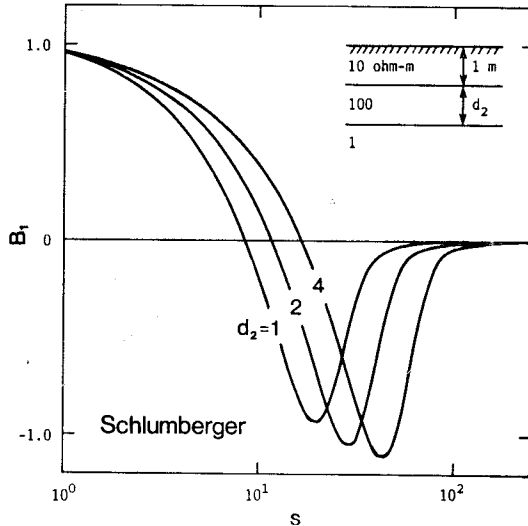


Fig. 4 B_1 curves for Schlumberger array on a three-layered earth of type *K*, with variable thickness of the second layer.

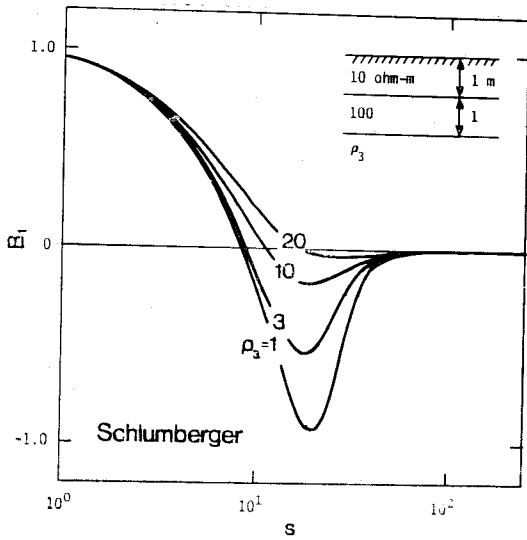


Fig. 5 B_1 curves for Schlumberger array on a three-layered earth of type K , with variable resistivity of the bottom layer.

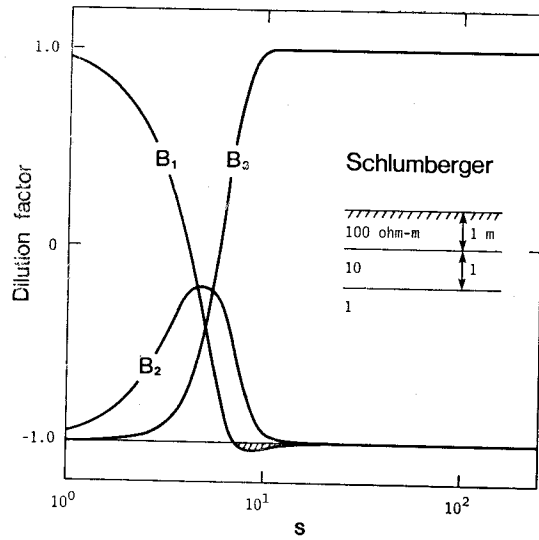


Fig. 7 Dilution factors B_1 , B_2 and B_3 plotted as sounding curves for Schlumberger array on a three-layered earth of type Q .

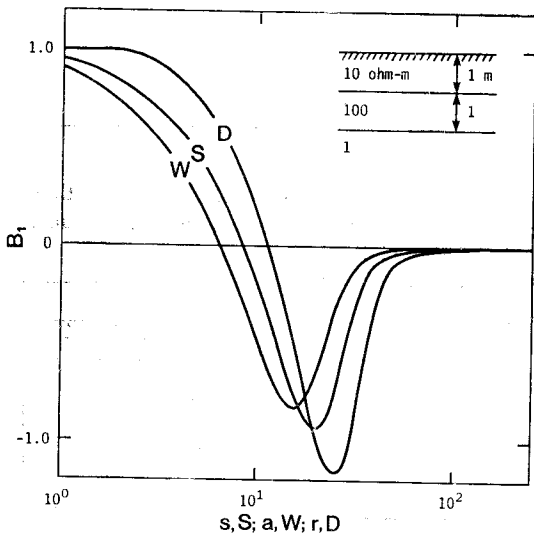


Fig. 6 Comparison of B_1 curves obtained from Wenner, Schlumberger and dipole-dipole arrays on a three-layered earth of type K .

Fig. 6 compares the B_1 curves for Wenner, Schlumberger and dipole-dipole arrays. From this illustration, one can see that the negative IP effect is the most apparent in the dipole-dipole array.

A three-layered earth of type Q can also give negative IP responses (Fig. 7). In this case,

like the type K , only the B_1 curve has negative values. However, the negative IP coefficient in type Q is less apparent than that in type K .

DISCUSSION AND CONCLUSIONS

The IP response of a horizontal layered earth is obtained by the summation of the products of weighting coefficients B_i and intrinsic IP responses M_i for each of the layers. From this, suitable theoretical apparent IP curves M_a can be readily calculated for interpretation of field data. Negative IP coefficients can be obtained whenever there is a type K or type Q resistivity distribution. Only B_1 is a negative coefficient, and B_2 and B_3 are always positive for both types (Figs. 2 and 7). This means that the negative IP response can occur when the first layer is polarizable, while the polarizability of the second and the bottom layers can act to depress the negative IP response. The negative IP effect is more apparent in type K than in type Q (Figs. 2 and 7), and the most apparent in the dipole-dipole array (Fig. 6).

If we assume that only the first layer is polar-

izable, then the pattern of IP curve is coincided with that of B_1 curve. In this case, a careful interpretation of IP data is required because the polarizability of the first layer produces negative IP response in deeper plotting point of data. A serious problem may occur in the native interpretation using a pseudo-sectional representation of field data alone. The resistivity sequence of the layering, therefore, is of utmost importance and should be known before planning an IP survey or attempting quantitative interpretation of IP data. There exists a paradoxical situation in the apparent resistivity curves when there is a layered section with a conductive layer underlying a more resistive layer. This situation influences drastically the corresponding IP curves.

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층상대지의 음수 유도분극 응답

김 회 준

요약 : 3층 구조 대지의 음수 유도분극 반응을 디지털 선형 필터법으로 구하여 검토하였다. 음수의 유도분극 반응은 전기비저항 구조가 K 나 Q 형일 때 나타나며, K 형 구조의 경우 Q 형보다 더 뚜렷한 음수의 반응을 보여 준다. 이러한 구조일 때, 유도분극 계수는 전기비저항 분포와 전극 배치의 함수로 주어지며, 표층의 계수만이 음수의 값을 가질 수 있다. 따라서 음수의 유도분극 반응은 표층이 분극성일 때 생길 가능성이 있으며, 그 외의 층의 분극성은 오히려 음수의 유도분극 반응을 줄이는 역할을 한다.