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On Fuzzy Convergence Spaces

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Introduction

In section 1, we shall list the concepts of the termilogies and some properties. In section 2, we shall define new concepts i.e., fuzzy convergence structure, fuzzy limit structure and fuzzy pretopological structure and show that the category *FzConv* of fuzzy convergence spaces and fuzzy continuous maps is properly fibred topological. In section 3, we investigate relationships amongst categories *FzConv*, *FzLim*, *FzPrTop* and *FzHConv*, i.e., in certain we prove that *FzLim* is bireflective in *FzConv* in Theorem 3.2, *FzPrTop* is bireflective in *FzConv* in Theorem 3.3, and *FzHConv* is epireflective in Theorem 3.6.

We define new concepts of fuzzy convergence structure, fuzzy limit structure, fuzzy pretopological structure and fuzzy Hausdorff convergence structure and investigate some properties about them.

I. Preliminaries

Throughout this paper, we adopt *R*. Lawen's definition of a fuzzy topological (6) Let N_x be the family of all fuzzy neighborhoods of x in a fuzzy topological space (X, δ). Then $(N_x)_{x\in X}$ determines the fuzzy topology on X (See (12)).

We adopt K.C. Min's definition of a fuzzy filter, where $F_x(X)$ denotes the collection of all fuzzy filters at x on a set X and $F(X)=\bigcup_{x\in X} F_x(X)$. The followings are the results investigated by K.C. Min (10):

(1) For any F, $G\in F_x(X)$, we denote $F\subseteq G$ in $F_x(X)$ iff for any $\mu\in F$, there exists $\nu\in G$ such that $\nu\leq_x\mu$, where $\nu\leq_x\mu$ means $\nu\leq\mu$ and $\nu(x)=\mu(x)$.

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Then $(F_x(X), \subseteq)$ is a partially ordered set. For any F, $G \in F_x(X)$, $F \cap G = \{\tau \in I^X \mid \mu^\vee \nu \leq_x \tau,\ \mu \in F,\ \nu \in G,\ \mu(x) = \nu(x)\}$, where I and denote the unit interval and the supremum, respectively.

(2) Let $f: X \longrightarrow Y$ be a map and $F \in F_x(X)$. Then $f(F) = \{\mu \in I^Y \mid f(\nu) \leq \mu$ with $0 < \nu(x) = \mu(f(x))$ for some $\nu \in F\} \in F_{f(x)}(Y)$.

(3) Let $f: X \longrightarrow Y$ be a map. Then

(i) For any $x \in X$, $f(\dot{x}) = f(\dot{x})$, where $\dot{x} = \{\mu \in I^X \mid \mu(x) > 0\} \in F(X)$.

(ii) If $F \subseteq G$ in $F_x(X)$, then $f(F) \subseteq f(G)$ in $F_{f(x)}(Y)$.

(iii) If F, $G \in F_x(X)$, then $f(F \cap G) = f(F) \cap f(G)$

(4) Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be maps. Then for any $F \in F_x(X)$, $(g \circ f)(F) = g(f(F))$.

Definition 1.1. Let(X, δ) and (Y, γ) be fuzzy topological spaces. A map $f: (X, \delta) \longrightarrow (Y, \gamma)$ is fuzzy continuous iff for any $\nu \in \gamma$, $f^{-1}(\nu) \in \delta$, where $f^{-1}(\nu) = \nu \circ f$.

Remark 1.2. $f: (X, \delta) \longrightarrow (Y, \gamma)$ is fuzzy continuous iff for each $x \in X$ and each $\mu \in N_{f(x)}$, there exists $\nu \in N_x$ such that $f(\nu) \leq \mu$ and $\nu(x) = \mu(f(x))$, where $f(\nu)$ $(y) = \bigvee_{z \in f^{-1}(y)} \nu(z)$ if $f^{-1}(y) \neq \phi$, and 0, otherwise [11].

Definition 1.3. Let $\underline{A}$ be a category.

(1) A *source in* $\underline{A}$ is a pair $(X, f_i)_I)$, where X is an $\underline{A}$-object and $(f_i: X \longrightarrow X_i)_I$ is a family of $\underline{A}$-morphisms each with domain X. [To simply notation a source $(X, (f_i)_I)$ is often denoted by (X, f_i)].

(2) A source (X, f_i) is called a *mono-source* provided each f_i is an $\underline{A}$-monomorphism.

Dual Notion. *sink in* A; (f_i, X); *epi-sink*

Definition 1.4. Let $\underline{A}$ be a concrete category and $((Y_i, \xi_i))_{i \in I}$ a family of objects in $\underline{A}$ indexed by a class I, and let X be a set and $(f_i: X \longrightarrow Y_i)_{i \in I}$ a source of maps indexed by I. An $\underline{A}$-structure on X is called on *initial structure with respect to* $(X, (f_i), ((Y_i, \xi_i)))$ if the following conditions are satisfied:

(1) For each $i \in I$, $f_i: (X, \xi) \longrightarrow (Y_i, \xi_i)$ is an $\underline{A}$-morphism,

(2) If (Z, ξ) is an $\underline{A}$-object and $g: Z \longrightarrow X$ is a map such that for each $i \in I$, the map $f_i \cdot g: (Z, \xi) \longrightarrow (Y_i, \xi_i)$ is an $\underline{A}$-morphism, then, $g: (Z, \xi) \longrightarrow (X, \xi)$ is an $\underline{A}$

-morphism. In this case, the source $(f_i: (X, \xi) \longrightarrow (Y_i, \xi_i))_{i\in I}$ is also called *an initial source*.

Dual Notion. *final structure*; *final sink.*

Definition 1.5. Let $\underline{A}$ be a concrete category.

(1) The $\underline{A}$-*fibre* of a set X is the class of all $\underline{A}$-structures on X.

(2) $\underline{A}$ is called a *properly fibred category* if it eatisfied the following conditions:

(i) For each set X, the A-fibre of X is a set,

(ii) For each singleton set X, the $\underline{A}$-fibre of X has precisely one element,

(iii) If ξ and η are $\underline{A}$-structures on X such that I_x: $(x, \xi) \longrightarrow (X, \eta)$ and I_x: $(X, \eta) \longrightarrow (X, \xi)$ are $\underline{A}$-morphisms, then $\xi = \eta$

Definition 1.6. A concrete category $\underline{A}$ is called a *topological category* if for each set X, for any family $((Y_i, \xi_i))_{j\in I}$ of A-objects, and for any source $(f_i: X \longrightarrow Y_i)_{\in iI}$ of maps, there exists an initial $\underline{A}$-structure on X with respect to $(X, (f_i))$, $((Y_i, \xi_i))$. Dually we define *cotopological* categories.

Definition 1.7. Let A be a subcategory of $\underline{B}$

$\underline{A}$ is called an *isomorohism-closed subcategory* of $\underline{B}$ if every B-object that is isomorphic with some $\underline{A}$-object is itself a A-object.

Definition 1.8. Let $\underline{A}$ be a subcategory of $\underline{B}$ with embedding functor F: $A \hookrightarrow B$.

(1) An E-universal map (r_B, A_B) for a $\underline{B}$-object B is called on A-*reflection* of B.

(2) $\underline{A}$ is called *reflective in* $\underline{B}$ or a *reflective subcategory* of $\underline{B}$ if there exists on $\underline{A}$-reflection for each $\underline{B}$-object.

(3) $\underline{A}$ is said to be *epireflective* (resp. *monoreflective*) in $\underline{B}$ if for each $\underline{B}$-object B, there exists an $\underline{A}$-reflection (r_B, A_B) such that each r_B is a $\underline{B}$-epimorphism (resp. a B-monomorphism).

Theorem 1.9. ([5]). Let $\underline{A}$ be a full, isomorphism closed subcategory of a properly fibred topological category $\underline{B}$.

(1) $\underline{A}$ is epireflective in $\underline{B}$ iff $\underline{A}$ is closed under the formation of initial monosources, i,e., for any initial monosource $(f_i: A \longrightarrow A_i)_{i\in I}$ in $\underline{B}$ with $A_i \in \underline{A}$ for all $i \in I$, the A also belongs to $\underline{A}$.

(2) $\underset{\sim}{A}$ is bireflective in $\underset{\sim}{B}$ iff $\underset{\sim}{A}$ is closed under the formation of initial sources.

Theorem 1.10 ([5]). If B is a (properly fibred, resp.) topological category and $\underset{\sim}{A}$ is a full isomorphism closed bireflective subcategory of $\underset{\sim}{B}$ then $\underset{\sim}{A}$ is also a (properly fibred, resp.) topological category.

2. Fuzzy convergence spaces.

By using fuzzy filters, we will introduce a concept of fuzzy convergence. Let $P(F(X))$ denote the power set of $F(X)$ and $I_0=(0, 1]$.

Definition 2.1. Let X be a set. A map $\triangle: X \longrightarrow P(F(X))$ is a fuzzy *convergence s tructure* on X iff the following properties hold for all $x \in X$;

($\triangle_0$) $\triangle(x) \in F_x(X)$,

($\triangle_1$) $\dot{x} \in \triangle(x)$,

($\triangle_2$) If $F \in \triangle(x)$ and $F \subseteq G$ in $F_x(X)$, then $G \in \triangle(x)$,

($\triangle_3$) If $F \in \triangle(x)$, then $F \cap \dot{x} \in \triangle(x)$.

The pair $(X, \triangle)$ is called a *fuzzy convergence space.*

Notation. Let $(X, \triangle)$ be a fuzzy convergence space. If $F \in \triangle(x)$, then x is called a *fuzzy limit* of F, or F is said to *fuzzily converge to* x, and we write $F \xrightarrow{\triangle} x$ or $F \longrightarrow x$.

Example. (1) In a set X, we define $\triangle: X \longrightarrow P(F(X))$ by $\triangle(x)=\{\dot{x}\}$ $(x \in X)$. Then $\triangle$ is clearly a fuzzy convergence structure on X. In this case, $\triangle$ is called the *discrete fuzzy convergence structure* on X and $(X, \triangle)$ is called the *discrete fuzzy convergence space.*

(2) Let (X, δ) be a fuzzy topological space. We define $\triangle_\delta: X \longrightarrow P(F(X))$ by: for each $x \in X$, $\triangle_\delta(x)=\{F \in F_x(X) \mid N_x \subseteq F\}$.

Then $\triangle_\delta$ is a fuzzy convergence structure on X.

Definition 2.2. Let $(X, \triangle)$, $(Y, \triangle')$ be fuzzy convergence spaces. Then

(1) a map $f: (X, \triangle) \longrightarrow (Y, \triangle')$ is *fuzzy continuous at* $x \in X$ iff for any $F \in \triangle(x)$, $f(F) \in \triangle'(f(x))$, i.e., $F \xrightarrow{\triangle} x \Longrightarrow f(F) \xrightarrow{\triangle'} f(x)$.

(2) a map $f: (X, \triangle) \longrightarrow (Y, \triangle')$ is *fuzzy continuous* iff f is fuzzy continuous at each

$x \in X$.

(3) a map f: $(X, \triangle) \longrightarrow (Y, \triangle')$ is an isomorphism iff f is bijective and f and f^{-1} are fuzzy continuous.

We can immediately obtain the following results from the definition of fuzzy continuity.

Proposition 2.3. (1) For any fuzzy convergence space $(X, \triangle)$, the identity map 1_x: $(X, \triangle) \longrightarrow (X, \triangle)$ is fuzzy continuous.

(2) If f: $(X, \triangle) \longrightarrow (Y, \triangle')$ and g: $(Y, \triangle') \longrightarrow (Z, \triangle'')$ are fuzzy continuous, respectively, then $g \circ f$: $(X, \triangle) \longrightarrow (Z, \triangle'')$ is fuzzy continuous.

Remark 2.4. It is clear by proposition 2.3 that the collection of all fuzzy convergence spaces and fuzzy continuous maps between them forms a concrete category, which will be denoted by *FzConv*.

Proposition 2.5. The category *FzConv* is properly fibred.

Proof. Let X be any set. Then clearly, the class of all fuzzy convergence structures on X is a set. Hence the *FzConv*-fibre of X is a set.

Let $X = \{p\}$ be any singleton set. Then X has the only one fuzzy convergence structure $\triangle(p) = \{p\} = \{\{\alpha \mid \alpha \in I.\}\}$ Hence the *FzConv*-fibre of $X = \{p\}$ is a singleton set.

Now let $\triangle$ and $\triangle'$ be any fuzzy convergence structures on a set X. Suppose 1_x: $(X, \triangle) \longrightarrow (X, \triangle')$ and 1_x: $(X, \triangle') \longrightarrow (X, \triangle)$ are fuzzy continuous, respectively. Enough to show that $\triangle(x) = \triangle'(x)$, for all $x \in X$. Let $F \in \triangle(x)$. Then $l_x(F) \in \triangle'(l_x(x))$, since l_x: $(X, \triangle) \longrightarrow (X, \triangle')$ is fuzzy continuous. On one hand, $1_x(F) = F$, $\triangle'(1_x(x)) = \triangle'(x)$. Thus $F \in \triangle'(x)$ and hence $\triangle(x) \subseteq \triangle'(x)$. Similarly $\triangle'(x) \subseteq \triangle(x)$. Thus $\triangle(x) = \triangle'(x)$ for all $x \in X$, and hence $\triangle = \triangle'$. Therefore *FzConv* is properly fibred.

Proposition 2.6. Let X be a set, $((X_j, \triangle_j))_J$ a family of fuzzy convergence spaces and $(f_j: X \longrightarrow X_j)_J$ any source of maps. Then there exists a fuzzy convergence structure $\triangle$ on X, which is initial with respect to $(X, (f_j), (X_j, \triangle_j))_J$.

Proof. Let $\triangle$: $X \longrightarrow P(F(X))$ be the map defined by: for each $x \in X$, $F \in \triangle(x)$ iff $F \in F_x(X)$ and $f_j(F) \in \triangle_j(f_j(x))$, for each $j \in J$.

Then by the definition of fuzzy convergence structure, we can show that $\triangle$ is a fuzzy convergence structure on X. Moreover, from the definition of $\triangle$, $f_j: (X, \triangle) \longrightarrow (X_j, \triangle_j)$ is fuzzy continuous for each $j \in J$.

Let $(Y, \triangle')$ be any fuzzy convergence space and $g: Y \longrightarrow X$ any map. Suppose$(f_j \circ g: (Y, \triangle') \longrightarrow (X_j, \triangle_j))_J$ is a source in *FzConv*. For any $y \in Y$, let $F \in \triangle'(y)$. Then $f_j(g(F)) = f_j \circ g(F) \in \triangle_j(f_j \circ g(y))$, for all $j \in J$. Thus $g(F) \in \triangle(g(y))$. Hence $g: (Y, \triangle') \longrightarrow (X, \triangle)$ is fuzzy continuous. Therefore $\triangle$ is the initial fuzzy convergence structure on X with respect to $(f_j)_J$. ///

Immediately, from Proposition 2.6, we can obtain the following result.

Theorem 2.7. *FzConv* is a topological category.

Definition 2.8. Let $(X, \triangle)$ be a fuzzy convergence space and A a subset of X. Then there exists the initial fuzzy convergence structure $\triangle_A$ on A with respect to the inclusion map $j: A \hookrightarrow X$. In this case, $\triangle_A$ is called the *relative fuzzy convergence structure on* A of $\triangle$ and $(A, \triangle_A)$ is called the subspace of $(X, \triangle)$.

Remark 2.9. For any fuzzy convergence space $(Y, \triangle')$ and any map $f: Y \longrightarrow A$, $f: (Y, \triangle') \longrightarrow (A, \triangle_A)$ is fuzzy continuous iff $j \circ f: (Y, \triangle') \longrightarrow (X, \triangle)$ is fuzzy continuous.

Definition 2.10. Let $((X_j, \triangle_j))_J$ be a family of fuzzy convergenc spaces indexed by a set J. Then there exists the initial fuzzy convergence structure $\triangle$ on ΠX_j with respect to $(\Pi X_j, (Pr_j), (X_j))_J$, where for each $j \in J$, $pr_j: \Pi X_j \longrightarrow X_j$ is the j-th projection. In this case, $\triangle$ is called the *product fuzzy convergence structure* of $(\triangle_j)_J$ and written $\Pi \triangle_j$, and $(\Pi X_j, \Pi \triangle_j)$ is called the product *fuzzy convergence space* of $(X_j, \triangle_j))_J$.

Remark 2.11. (1) For any fuzzy convergence space $(X, \triangle)$ and any source $(f_j: (X, \triangle) \longrightarrow (X_j, \triangle_j))_J$ in *FzConv*, there exists a unique fuzzy continuous map $f: (X, \triangle) \longrightarrow (\Pi X_j, \Pi \triangle_j)$ with for each $j \in J$, $pr_j \circ f = f_j$. In the following, f will be denoted by Πf_j.

(2) Let $((f_j: (X_j, \triangle_j) \longrightarrow (Y_j, \triangle'_j))_J$ be any family of fuzzy continuous maps between convergence spaces. Then there exists a unique fuzzy continuous map $f: (\Pi X_j, \Pi \triangle_j) \longrightarrow (\Pi Y_j, \Pi \triangle'_j)$ with for each $j \in J$, $pr'_j \circ f = f_j \circ pr_j$, where $pr'_j: \Pi Y_j$

$\longrightarrow Y_j$ is the j-th projection. In the following, f will be denoted by Πf_j.

Definition 2.12. Let X be a set and $\triangle$: $X \longrightarrow P(F(X))$ a map. Consider the following properties;

(L) If F, $G \in \triangle(x)$, then $F \cap G \in \triangle(x)$.

(pr) For any $x \in X$, $\bigcap\{F \mid F \in \triangle(x)\} \in \triangle(x)$.

(1) the map $\triangle$ is a *fuzzy limit structure* on X iff $\triangle$ satisfies ($\triangle_0$), ($\triangle_1$), ($\triangle_2$) of Definition 2.1 and (L). In this case, $(X, \triangle)$ is called a *fuzzy limit space.*

(2) the map $\triangle$ is a *fuzzy pretopological structure* on X iff $\triangle$ satisfies ($\triangle_0$), ($\triangle_1$), ($\triangle_2$) of Definition 2.1 and (Pr). In this case, $(X, \triangle)$ is called a *fuzzy pretopological space.*

From the Definition 2.1 and 2.12, immediately, we obtain the following implications for a map $\triangle$: $X \longrightarrow P(F(X))$.

Proposition 2.13. $\triangle$ is a fuzzy pretopological structure $\Rightarrow$ a fuzzy limit structure $\Rightarrow$ a fuzzy convergence structure.

Notation: (1) *FzLim* denotes the category of fuzzy limit spaces and fuzzy continuous maps between them.

(2) *FzPrTop* denote the category of fuzzy pretopological spaces and fuzzy continuous maps between them.

It is clear that *FzLim* and *FzPrTop* are full subcategories of *FzConv*, respectively.

3. Some properties of *FzLim*, *FzPrTop* and *FzHConv*.

Immediately, we can see the category *FzLim* defined above is identical with the category *FzConv* defined by K.C. Min [10].

Proposition 3.1. *FzLim* (*FzPrTop*) is *an* isomorphism closed subcategory of *FzConv*.

Proof. Let $(X, \triangle)$ be any fuzzy convergence space such that is isomorphic with some fuzzy limit space $(Y, \triangle')$. Then there exists an isomorphism f: $(X, \triangle) \longrightarrow (Y, \triangle')$. Suppose F, $G \in \triangle(x)$, for each $x \in X$. Then $f(F)$, $f(G) \in \triangle'(f(x))$. Thus $f(F) \cap f(G) \in \triangle'(f(x))$, since is a fuzzy limit structure on X. On one hand, $f(F) \cap f(G) = f(F \cap G)$.

Thus $f(F\cap G)\in\triangle'(f(x))$. Since f^{-1}: $(Y,\triangle')\longrightarrow(X,\triangle)$ is fuzzy continuous and f: $X\longrightarrow Y$ is bijective, $F\cap G=f^{-1}(f(F\cap G))\in\triangle(f^{-1}(f(x)))=\triangle x$. Hence $\triangle$ is a fuzzy limit structure on X, i.e., $(X,\triangle)$ is a fuzzy limit space.

Therefore *FzLim* is an isomorphism closed subcategory of *FzConv*. ///

Theorem 3.2. *FzLim* is bireflective in *FzConv* and hence is a properly fibred topological category.

Proof. From Theorem 1.9, [2], it is sufficient to show that *FzLim* is closed under the formation of initial sources.

Let X be any set and $(f_j$: $X\longrightarrow X_j)_J$ any initial source in *FzConv* such that for all $j\in J$, $(X_j, \triangle_j)\in$ *FzLim*. Let $\triangle$ be the initial fuzzy convergence structure on X with respect to $(f_j$: $X\longrightarrow X_j)_J$. For each $x\in X$, suppose F, $G\in\triangle(x)$. Then for all $j\in J$, $f_j(F)$, $f_j(G)\in\triangle_j(f_j(x))$. Since f_j: $(X, \triangle)\longrightarrow(X_j, \triangle_j)$ is fuzzy continuous, for all $j\in J$. Thus for all $j\in J$, $f_j(F\cap G)=f_j(F)\cap f_j(G)\in\triangle_j(f_j(x))$, since $\triangle_j$ is a fuzzy limit structure on X_j, for all $j\in j$. Hence by the definition of the initial fuzzy convergence structure $\triangle$ on X, $F\cap G\in\triangle(x)$.

Thus $\triangle$ is a fuzzy limit structure on X, i.e., $(f_j$: $X\longrightarrow X_j)_J$ is an initial source in *FzLim*. Therefore *FzLim* is bireflective in *FzConv*. ///

Theorem 3.3. *FzPrTop* is bireflective in *FzConv* and hence is a properly fibred topological category.

Proof. From Theorem 1.9, [2] and Proposition 3.1, it is enough to show that *FzPrTop* is closed under the formation of initial sources.

Let X be a set and $(f_j$: $X\longrightarrow X_j)_J$ an initial source in *FzConv* such that for each $j\in J$, $(X_j, \triangle_j)\in$ *FzPrTop*. Let $\triangle$ be the initial fuzzy convergence structure on X with respect to $(f_j)_J$. For each $x\in X$, consider the collection $\{F\,|\,F\in\triangle(x)\}$. Then clearly, $\{f_j(F)\,|\,f_j(F)\in\triangle_j(f_j(x))\}$ since f_j: $(X, \triangle)\longrightarrow(X_j, \triangle_j)$ is fuzzy continuous, for each $j\in J$. Thus for each $j\in J$, $\bigcap\{f_j(F)\,|\,f_j(F)\in\triangle_j(f_j(x))\}\in\triangle_j(f_j(x))$ and $\bigcap f_j(F)=f_j(\bigcap F)$. Hence $\bigcap F\in\triangle(x)$. Thus $\triangle$ is a fuzzy pretopological structure on X, i.e., $(f_j$: $X\longrightarrow X_j)_J$ is an initial source in *FzPrTop*. Therefore *FzPrTop* is bireflective in *FzConv*. ///

Theorem 3.4. *FzPrTop* is bireflectve in *FzLim*.

Definition 3.5. A fuzzy convergence space $(X, \triangle)$ is a *fuzzy Hausdorff convergence* space iff $F\in\triangle(x)$ and $F\in\triangle(y) \Rightarrow x=y$.

Notation. *FzHConv* denotes the category of fuzzy Hausdorff convergence spaces and fuzzy continuous maps between them: *FzHLim*$=$*FzHConv*$\cap$*FzLim*; *FzHPrTop* $=$*FzHConv*$\cap$*FzPrTop*.

Theorem 3.6. *FzHConv* is epireflective in *FzConv*.

Proof. Since *FzConv* is properly fibred topological, it is sufficient to show that *FzHConv* is closed under the formation of initial monosources in *FzConv*.

Suppose $(f_j: X \longrightarrow X_j)_J$ is an initial monosource in *FzConv* such that for each $j\in J$, $(X_j, \triangle_j)$ is a fuzzy Hausdorff convergence space. Let $\triangle$ be the initial fuzzy couvergence structure on X with respect to $(f_j: X \longrightarrow X_j)_J$. Assume that a fuzzy filter F on X has two fuzzy limits x, y. Then $F\in\triangle(x)$ and $F\in\triangle(y)$. Thus for each $j\in J$, $f(F)\in\triangle_j(f_j(x))$ and $f_j(F)\in\triangle_j(f_j(y))$, since f_j: $(X, \triangle)\longrightarrow(X_j, \triangle_j)$ is fuzzy continuous. Hence $f_j(x)=f_j(y)$ for each $j\in J$, since $\triangle_j$ is a fuzzy Hausdorff convergence space for all $j\in J$. Thus $x=y$, since (f_j) is a monosouce. Hence $(X, \triangle)$ is a fuzzy Hausdorff convergence space.

Therefore *FzHConv* is epireflective in *FzConv*. ///

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