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ROTATIONAL DIRECTED TRIPLE SYSTEMS

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1. Introduction

A *directed triple* is a set of three ordered pairs of the form $\{(a,b), (b,c), (a,c)\}$ that we will always denote it by $[a,b,c]$. A *directed triple system* DTS(v) of order v is a pair (V,B) where V is a v-set and B is a collection of directed triples of elements of V (called blocks) such that every ordered pair of distinct elements of V belongs to exactly one block. It is well-known [3] that a DTS(v) exists if and only if $v\equiv 0$ or 1 (mod 3). An *automorphism* of a DTS(v) (V,B) is a permutation α on V which preserves B. A DTS(v) is said to be *k-rotational* if it admits an automorphism α consisting of a single fixed element and exactly $k\,\frac{v-1}{k}$-cycles ; and α is called a *k-rotational automorphism.* If a permutation α of degree v consists of a single v-cycle, then a DTS(v) admitting α as its automorphism is called *cyclic.* It is shown by Colbourn and Colbourn [2] that a cyclic DTS(v) exists if and only if $v\equiv 1, 4$ or 7 (mod 12).

In this paper, we obtain a necessary and sufficient condition for the existence of k-rotational DTS(v).

A *Steiner triple system* STS(v) of order v is a pair (V,B) where V is a v-set and B is a collection of 3-subsets of V (called triples) such that every 2-subset of V belongs to exactly one triple. It is well-known that a STS(v) exists if and only if $v\equiv 1$ or 3(mod 6), and Peltesohn [5] first shows that a cyclic STS(v) exists if and only if $v\equiv 1$ or 3(mod 6) and $v\neq 9$.

An (A,k)-*system* (a (B,k)-and a (C,k)-system, respectively) is a set

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of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \cdots, k\}$ such that $b_r - a_r = r$ for $r=1, 2, \cdots, k$, and $\bigcup_{r=1}^{k} \{a_r, b_r\} = \{1, 2, \cdots, 2k\}$ ($= \{1, 2, \cdots, 2k-1, 2k+1\}$ and $= \{1, 2, \cdots, k, k+2, \cdots, 2k+1\}$, respectively), An (A, k)-system and a (B, k)-system are essentially the same as what have been called by Roselle [7] a *Skolem* $(2, k)$*-sequence* and a *hooked Skolem* $(2, k)$*-sequence*, respectively. It is well-known [see 4, 6, 8] that an (A, k)-system (a (B, k)-and a (C, k)-system, respectively) exists if and only if $k \equiv 0$ or $1 \pmod 4$ ($k \equiv 2$ or $3 \pmod 4$ and $k \equiv 0$ or $3 \pmod 4$, respectively). An *(E, k)-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \cdots, k\}$ such that $b_r - a_r = r$ for $r=1, 2, \cdots, k$, and $\bigcup_{r=1}^{k} \{a_r, b_r\} = \{1, 2, \cdots, (k+1)/2-1, (k+1)/2+1, \cdots, 2k+1\}$. An (E, k)-system exists if and only if $k \equiv 1 \pmod 2$ [see 1].

2. The neccessary Condition for the Existence of k-Rotational DTS(v)

Let Z denote the set of all integers and let Z_v be the group of residue classes of Z modulo v. For a fixed block $b = [x, y, z]$ in a k-rotational DTS(v) with α as its k-rotational automorphism, define the set

$$C(b) = \{[\alpha^n(x), \alpha^n(y), \alpha^n(z)] \mid n \in Z\}.$$

A collection of starter blocks of a k-rotational DTS(v) with blocks B is a subset S of B for which $\{b \mid b \in C(s),\ s \in S\} = B$.

LEMMA 2.1. *For each block b in a k-rotational DTS(v), $|C(b)| = \frac{v-1}{k}$.*

Proof. This follows from the fact that if $[x, y, z]$ is a block, then its cyclically shifted blocks $[x, y, z]$, $[y, z, x]$ and $[z, x, y]$ are distinct.

A basic necessary condition for the existence of k-rotational DTS(v) is $v \equiv 0$ or $1 \pmod 3$, since this is the spectrum for DTS(v). It is a trivial exercise to see that if (V, B) is a DTS(v), then $|B| = \frac{v(v-1)}{3}$. Thus, if there exists a k-rotational DTS(v) then both $\frac{1}{3}v(v-1)/\frac{1}{k}(v-1) = \frac{kv}{3}$ and $\frac{v-1}{k}$ are integers. Hence, we have the following necessary condition.

LEMMA 2.2. *If there exists a k-rotational DTS(v), then (i) $k \equiv 1, 2 \pmod 3$, $v \equiv 0 \pmod 3$ and $v \equiv 1 \pmod k$ or (ii) $k \equiv 0 \pmod 3$ and*

$v \equiv 1 (mod\ k)$.

3. The Existence of k-Rotational DTS(v)

Let α be a permutation of degree v with type $[j_1, j_2, \cdots, j_v]$, i.e. consists of precisely j_i i-cycles, and $\sum_{i=1}^{v} ij_i = v$. If α is of type $[1, 0, \cdots, 0, 1, 0]$ and $v \equiv 1$ (mod k), then α^k is of type $[1, 0, \cdots, 0, k, 0, \cdots, 0]$, i.e. $j_{(v-1)/k} = k$. If α is of type $[1, 0, \cdots, 0, 3, 0, \cdots, 0]$, i.e. $j_{(v-1)/3} = 3$, and if $v \equiv 1$(mod $3k$) then α^k is of type $[1, 0, \cdots, 0, 3k, 0, \cdots, 0]$, i.e. $j_{(v-1)/3k} = 3k$. Thus, to show that the necessary condition for the existence of k-rotational DTS(v) which is given in Lemma 2.2 is also sufficient, it is enough to construct k-rotational DTS(v) for

(i) $k=1$ and $v \equiv 0$(mod 3),

(ii) $k=3$ and $v \equiv 1$(mod 3).

Let us assume that the set of elements of our k-rotational DTS(v) is $V = (Z_{(v-1)/k} \times Z_k) \cup \{\infty\}$ and the corresponding k-rotational automorphism is $\alpha = (\infty) \prod_{i=0}^{k-1} (0_i\ 1_i \cdots ((v-1)/k-1)_i)$; here, instead of (x, i) we write x_i. In the case $k=1$, we also write for brevity $V = Z_{v-1} \cup \{\infty\}$ instead of $V = (Z_{v-1} \times Z_1) \cup \{\infty\}$.

Lemma 3.1. *If $v \equiv 3$ or 6(mod 12), then there exists a 1-rotational DTS(v).*

Proof. Let $v = 3t$, $t \equiv 1$ or 2(mod 4) and let $\{(a_r, b_r) \mid r = 1, 2, \cdots, t-1\}$ be an $(A, t-1)$-system. Then

$\{[1, \infty, 0]\}$,

$\{[0, r, b_r + t - 1] \mid r = 1, 2, \cdots, t-1\}$

are a collection of starter blocks of a 1-rotational DTS($3t$) where $t \equiv 1$ or 2(mod 4).

Lemma 3.2. *If $v \equiv 0$ or 9(mod 12), then there exists a 1-rotational DTS(v).*

Proof. Let $v = 3t$, $t \equiv 0$ or 3(mod 4) and let $\{(a_r, b_r) \mid r = 1, 2, \cdots, t-1\}$ be a $(B, t-1)$-system. Then

$\{[2, \infty, 0]\}$,

$\{[0, r, b_r + t - 1] \mid r = 1, 2, \cdots, t-1\}$

are a collection of starter blocks of a 1-rotational DTS ($3t$) where $t \equiv 0$

or 3(mod 4).

THEOREM 3.3. *A 1-rotational DTS (v) exists if and only if* $v\equiv 0$ $(mod\ 3)$.

LEMMA 3.4. *If* $v\equiv 16(mod\ 18)$, *then there exists a 3-rotational* $DTS(v)$.

Proof. Let $v=18t+16$, $t\geq 0$ and let $B=B_1\cup B_2\cup B_3$ where

$$B_1:\ \{[0_i, \infty, (3t+2)_i]\,|\,i=0,1,2\}$$

$$B_2:\ \{[0_i, r_i, (b_r+2t+1)_i]\,|\,i=0,1,2\ ;\ r=1,2,\cdots,2t+1\}$$

where $\{(a_r, b_r)\,|\,r=1,2,\cdots,2t+1\}$ is an $(E, 2t+1)$-system,

$$B_3:\ \{[0_0, r_1, (2r)_2], [(2r)_2, r_1, 0_0]\,|\,r=0,1,\cdots,6t+4\}.$$

Then B is a collection of starter blocks of a 3-rotational DTS$(18t+16)$, $t\geq 0$.

LEMMA 3.5. *If* $v\equiv 4(mod\ 18)$, *then there exists a 3-rotational* $DTS(v)$.

Proof. Let $v=18t+4$, $t\geq 0$ and let $B=B_1\cup B_2\cup B_3$ where

$$B_1:\ \{[\infty, 0_0, 0_1], [0_0, \infty, 0_2], [0_2, 0_1, \infty], [0_1, 0_2, 0_0]\},$$

$$B_2:\ \{[a_i, b_i, c_i], [c_i, b_i, a_i]\,|\,\{a, b, c\}\in C, i=0,1,2\}$$

where C is a collection of starter triples of a cyclic STS $(6t+1)$,

$$B_3:\ \{[0_0, r_1, (2r)_2], [(2r)_2, r_1, 0_0]\,|\,r=1,2,\cdots,6t\}.$$

Then B is a collection of starter blocks of a 3-rotational DTS$(18t+4)$, $t\geq 0$.

LEMMA 3.6. *There exists a 3-rotational* $DTS(28)$.

Proof. A collection of starter blocks of a 3-rotational DTS(28) is

$$\{[1_i, \infty, 0_i]\,|\,i=0,1,2\},$$

$$\{[3_0, 0_0, 0_1], [3_1, 0_1, 0_2], [0_0, 3_2, 0_2], [6_2, 0_1, 0_0], [0_2, 6_1, 0_0], [3_2, 0_0, 3_1], [3_1, 0_0, 6_2]\},$$

$$\{[0_0, r_1, (9-r)_2], [(9-r)_2, r_1, 0_0]\,|\,r=1,2,4,5,7,8\},$$

$$\{[0_i, 1_i, 4_i], [0_i, 2_i, 7_i]\,|\,i=0,1,2\}.$$

LEMMA 3.7. *If* $v\equiv 10(mod\ 18)$, *then there exists a 3-rotational*

$DTS(v)$.

Proof. The case $v=28$ has been treated in Lemma 3. 6.

Let $v=18t+10$, $t\neq 1$, and let $B=B_1\cup B_2\cup B_3\cup B_4$ where

B_1 : $\{[0_i, \infty, (2t+1)_i] \mid i=0, 1, 2\}$,

B_2 : $\{[(2t+1)_0, 0_0, 0_1], [(2t+1)_1, 0_1, 0_2], [0_0, (2t+1)_2, 0_2], [(4t+2)_2, 0_1, 0_0][0_2, (4t+2)_1, 0_0], [(2t+1)_2, 0_0, (2t+1)_1], [(2t+1)_1, 0_0, (4t+2)_2\}$,

B_3 : $\{[a_i, b_i, c_i], [c_i, b_i, a_i] \mid \{a, b, c\} \in C, i=0, 1, 2\}$

where $C\cup\{\{0, 2t+1, 4t+2\}\}$ is a collection of starter triples of a cyclic STS $(6t+3)$,

B_4 : $\{[0_0, r_1, (6t+3-r)_2], [(6t+3-\mathrm{r})_2, r_1, 0_0] \mid r=1, 2, \cdots, 2t, 2t+2, \cdots, 4t+1, 4t+3, \cdots, 6t+2\}$.

Then B is a collection of starter blocks of a 3-rotational DTS$(18t+10)$, $t\neq 1$.

LEMMA 3. 8. *If* $v\equiv 1$ *or* $19(mod\ 24)$, *then there exists a 3-rotational* $DTS(v)$.

Proof. In this case $v\equiv 1$ or $19(\text{mod } 24)$, we obtain a 3-rotational DTS(v) from a 3-rotational STS(v) which is constructed by Cho [1] (such a system exists if and only if $v\equiv 1$ or $19(\text{mod } 24)$), by replacing each triple $\{a, b, c\}$ not containing ∞ of the 3-rotational STS(v) with two cyclic triples $[a, b, c]$, and $[c, b, a]$, and each triple $\{\infty, a, b\}$ containing ∞ with $[a, \infty, b]$.

LEMMA 3. 9. $v\equiv 7(mod\ 24)$, *then there exists a 3-rotational* $DTS(v)$.

Proof. Let $v=24t+7$, $t\geq 0$ and let $B=B_1\cup B_2\cup B_3$ where

B_1 : $\{[0_1, \infty, 0_0], [0_2, \infty, (4t+1)_1], [0_0, \infty, 0_2]\}$,

B_2 : $\{[0_i, r_i, (b_r)_{i+1}], [(b_r)_{i+1}, r_i, 0_i] \mid i=0, 1, 2\ ;\ r=1, 2, \cdots 4t\}$

where $\{(a_r, b_r) \mid r=1, 2, \cdots, 4t\}$ is a $(C, 4t)$-system,

B_3 : $\{[0_2, 0_1, (4t+1)_0], [0_2, 0_0, (4t+1)_2], [0_i, (4t+1)_i, 0_{i+1}] \mid i=0, 1\}$.

Then B is a collection of starter blocks of a 3-rotational DTS$(24t+7)$, $t\geq 0$.

LEMMA 3. 10. *There exists a 3-rotational* $DTS(37)$.

Proof. A collection of starter blocks of a 3-rotational DTS(37) is

$$\{[0_i, \infty, 6_i], [0_i, 1_i, 8_i], [8_i, 1_i, 0_i] \mid i=0,1\},$$
$$\{[0_2, \infty, 4_2], [0_2, 1_2, 11_2], [0_2, 2_2, 8_2]\},$$
$$\{[0_0, 2_0, 9_1], [9_1, 2_0, 0_0], [0_0, 3_0, 11_1], [11_1, 3_0, 0_0],$$
$$[0_1, 2_1, 2_2], [2_2, 2_1, 0_1], [0_1, 3_1, 11_2], [11_2, 3_1, 0_1],$$
$$[0_2, 5_2, 5_0], [5_0, 5_2, 0_2], [0_3, 3_3, 10_0], [10_0, 3_2, 0_2]\},$$
$$\{[0_0, 0_1, 4_2], [0_0, 1_1, 11_2], [0_0, 2_1, 8_2], [0_0, 3_1, 10\ ],$$
$$[4_2, 0_1, 0_0], [11_2, 1_1, 0_0], [8_2, 2_1, 0_0], [10_2, 3_1, 0_0],$$
$$[0_0, 4_1, 1_2], [0_0, 5_1, 6_2], [0_0, 6_1, 9_2], [0_0, 10_1, 3_2],$$
$$[1_2, 4_1, 0_0], [6_2, 5_1, 0_0], [9_2, 6_1, 0_0], [3_2, 10_1, 0_0]\}.$$

DEFINITION 3.11. A *(P,k)-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1,2,\cdots,k\}$ such that $b_r - a_r = r$ for $r=1,2,\cdots,k$, $b_{(k+1)/2} = k+1$, and $\bigcup_{r=1}^{k} \{a_r, b_r\} = \{1, 2, \cdots, \frac{3k+1}{2}, \frac{3k+1}{2}+2, \cdots, 2k+1\}$.

LEMMA 3.12. *A (P,k)-system exists if and only if $k \equiv 1 \pmod 4$ and $k \neq 5$.*

Proof. If $\{(a_r, b_r) \mid r=1,2,\cdots,k\}$ is a (P,k)-system, then we have

$$\sum_{r=1}^{k} (b_r - a_r) = \frac{k(k+1)}{2}$$

and

$$\sum_{r=1}^{k} (b_r + a_r) = \frac{(2k+1)(2k+2)}{2} - \frac{3k+3}{2}$$

and hence

$$\sum_{r=1}^{k} b_r = \frac{5k^2+4k-1}{4}.$$

Since $\sum b_r$ is an integer, $5k^2+4k-1 \equiv 0 \pmod 4$ and so $k \equiv 1 \pmod 4$.

For sufficiency, let $k=4t+1$. Then the following ordered pairs form a (P,k)-system.

(I). $t \equiv 0$ or $2 \pmod 4$.

$$(2t+2-r, 4t+1+r), \quad r=1,2,\cdots,t+1,$$
$$(t+1-r, 2t+1+r), \quad r=1,2,\cdots,\frac{t}{2},$$
$$(r, 4t+2-r), \quad r=1,2,\cdots,\frac{t}{2},$$
$$\left(\frac{5t}{2}+1+r, \frac{7t}{2}+2-r\right), \quad r=1,2,\cdots,\frac{t}{2},$$

$(5t+2+r,\ 8t+4-r),\qquad r=1,2,\cdots,\ \frac{t}{2},$

$\left(\frac{11t}{2}+2+r,\ \frac{15t}{2}+3-r\right),\ r=1,2,\cdots,\ \frac{t}{2},$

$(6t+3+r, 7t+3-r),\qquad r=1,2,\cdots,\ \frac{t}{2}-1,$

$\left(\frac{13t}{2}+3,\ \frac{15t}{2}+3\right).$

(II). $t\equiv 1$ or $3 \pmod 4$.

It is easy to check that there is no $(P,5)$-system.

$t=3$: (24, 25), (9, 11), (19, 22), (23, 27), (3, 8), (20, 26), (7, 14), (10, 18), (6, 15), (2, 12), (5, 16), (1, 13), (4, 17).

$t>3$, we distinguish four cases and each case contains the following ordered pairs in common.

$(r,\ 4t+2-r),\qquad r=1,2,\cdots,\ \frac{t+1}{2},$

$\left(\frac{t+1}{2}+r,\ \frac{5t-1}{2}+2-r\right),\ r=1,2,\cdots,\ \frac{t-1}{2},$

$(2t+2-r,\ 4t+1+r),\qquad r=1,2,\cdots,t+1,$

$\left(\frac{5t-1}{2}+1+r,\ \frac{7t-1}{2}+2-r\right),\ r=1,2,\cdots,\ \frac{t-1}{2},$

$(3t+1,\ 5t+3),$

$(5t+3+r,\ 8t+4-r),\qquad r=1,2,\cdots,\ \frac{t-3}{2},$

$\left(\frac{11t-1}{2}+2+r,\ \frac{15t+1}{2}+3-r\right),\ r=1,2,\cdots,\ \frac{t+1}{2}.$

Case 1. $t\equiv 3 \pmod 4$.

$t=7$: (47, 48), (46, 49), (51, 56), (50, 57).

$t=11$: (71, 72), (73, 76), (74, 79), (70, 77), (78, 87), (75, 86).

$t\geq 15$:

$\left(\frac{13t+1}{2}+3+r,\ \frac{15t+1}{2}+5-r\right),\qquad r=1,2,$

$(6t+3+r,\ 7t+1-r),\qquad r=1,2,\cdots,\ \frac{t-7}{4},$

$\left(\frac{25t-7}{4}+3+r,\ \frac{25t-7}{4}+8-r\right),\qquad r=1,2,$

$\left(\frac{25t-7}{4}+7+r,\ \frac{27t+7}{4}+1-r\right),\qquad r=1,2,\cdots,\frac{t-7}{4}-2,$

$$\left(\frac{13t-7}{2}+5+r,\quad 7t+3-r\right),\qquad r=1,2.$$

Case 2. $t\equiv 5 \pmod{12}$.

$$\left(\frac{13t+1}{2}+3+r,\quad \frac{15t+1}{2}+5-r\right),\qquad r=1,2,$$

$$(6t+5+r,\quad 7t+3-r),\qquad r=1,2,\cdots,\ \frac{t-8}{3},$$

$$\left(\frac{19t+7}{3}+1,\quad \frac{19t+7}{3}+4\right),$$

$$\left(6t+3+r,\ \frac{19t+7}{3}+4-r\right),\qquad r=1,2,$$

$$\left(\frac{13t+1}{2}+6,\quad \frac{13t+1}{2}+7\right),$$

$$\left(\frac{19t+7}{3}+4+r,\quad \frac{20t+17}{3}-r\right),\qquad r=1,2,\cdots,\ \frac{t-5}{6}-2.$$

Case 3. $t\equiv 1 \pmod{12}$.

$$\left(\frac{13t+1}{2}+4,\quad \frac{15t+1}{2}+4\right),$$

$$(6t+3+r,\quad 7t+3-r),\qquad r=1,2,\cdots,\ \frac{t-1}{6}-1,$$

$$\left(\frac{41t+1}{6}+3,\quad \frac{15t+1}{2}+3\right),$$

$$\left(\frac{37t-1}{6}+3,\quad \frac{13t+1}{2}+3\right),$$

$$\left(\frac{37t-1}{6}+3+r,\quad \frac{41t+1}{6}+3-r\right),\qquad r=1,2,\cdots,\ \frac{t-1}{6}-1,$$

$$\left(\frac{19t-1}{3}+3,\quad \frac{19t-1}{3}+4\right),$$

$$\left(\frac{13t+1}{2}+3-r,\quad \frac{13t+1}{2}+4+r\right),\qquad r=1,2,\cdots,\ \frac{t-1}{6}-1.$$

Case 4. $t\equiv 9 \pmod{12}$.

$$\left(\frac{13t+1}{2}+3,\quad \frac{15t+1}{2}+3\right),$$

$$\left(\frac{41t+3}{6}+3,\quad \frac{15t+1}{2}+4\right),$$

$$\left(\frac{13t+1}{2}+2,\quad \frac{41t+3}{6}+2\right),$$

$$(6t+3+r,\quad 7t+3-r),\qquad r=1,2,\cdots,\ \frac{t-9}{6},$$

$$\left(\frac{37t-3}{6}+2+r,\ \frac{41t+3}{6}+2-r\right),\qquad r=1,2,\cdots,\ \frac{t-3}{6},$$
$$\left(\frac{19t}{3}+2,\ \frac{19t}{3}+3\right),$$
$$\left(\frac{19t}{3}+3+r,\ \frac{20t}{3}+3-r\right),\qquad r=1,2,\cdots,\ \frac{t-9}{6}.$$

LEMMA 3.13. *If $v\equiv 13$ (mod 24), then there exists a 3-rotational $DTS(v)$.*

Proof. The case $v=37$ has been treated in Lemma 3.10.

Let $v=24t+13$, $t\neq 1$, and let $B=B_1\cup B_2\cup B_3\cup B_4$ where

B_1 : $\{[0_0, \infty, 0_1], [0_2, \infty, 0_0], [0_1, \infty, 0_2]\}$,

B_2 : $\{[0_i, (2t+1)_i, (6t+3)_i] \mid i=0,1,2\}$,

B_3 : $\{[0_i, r_i, (b_r)_{i+1}], [(b_r)_{i+1}, r_i, 0_i] \mid i=0,1,2\ ;\ r=1,2,\cdots,2t, 2t+2,\cdots,4t+1\}$

where $\{(a_r, b_r) \mid r=1,2,\cdots,4t+1\}$ is a $(P, 4t+1)$-system,

B_4 : $\{[0_1, 0_0, (2t+1)_2], [0_0, (4t+2)_2, (4t+2)_1], [(6t+3)_2, (4t+2)_1, 0_0]$ $[0_0, (2t+1)_1, 0_2], [(4t+2)_2, 0_0, (6t+3)_1], [(2t+1)_1, 0_0, (6t+3)_2],$ $[(2t+1)_2, (6t+3)_1, 0_0]\}$.

Then B is a collection of starter blocks of a 3-rotational DTS$(24t+13)$, $t\neq 1$.

Now, we have the following theorem.

THEOREM 3.14. *A 3-rotational $DTS(v)$ exists if and only if $v\equiv 1$ (mod 3).*

Finally, we conclude the following theorem.

THEOREM 3.15. *A k-rotational $DTS(v)$ exists if and only if*
(i) $k\equiv 1, 2$ (mod 3), $v\equiv 0$ (mod 3) and $v\equiv 1$ (mod k) or
(ii) $k\equiv 0$ (mod 3) and $v\equiv 1$ (mod k).

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