A Study on Mixed Methods for Reduction of Large Scale System

(고차 시스템의 간소화를 위한 혼합 방법들에 대한 연구)

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要 約

선형 시불변 연속 시스템의 모델 간소화 방법들이 제안되었다. 에너지 분산 방법을 이용하여 모델의 분모 다항식을 결정하고 수식화 유수 또는 타임 모멘트 매칭 방법을 이용하여 분자다항식을 결정하였다. 제안한 방법들은 LUCAS가 제안한 방법과 비교하여 좋은 결과를 얻었다.

Abstract

The model reduction methods of the linear time invariant continuous systems are proposed. The energy dispersion method is used to obtain the model denominator. And the model numerator is found by the modified residue method or the time moment matching method. The methods suggested are compared with the method suggested by Lucas and give good results.

I. Introduction

Simplification of the large scale system to the low order model has been an important subject in control engineering area because of the role in system analysis and controller design. The reduction methods may generally be classified the following categories: 1) the methods which simplify the original systems directly from its overall transfer function [1–7]. 2) the methods which separately determine the denominator (numerator) of the reduced model from the denominator (numerator) of the original system [8–11].

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Mahmoud [3] showed that when the original system satisfied a matrix norm condition, it could be separated by the slow and fast subsystem. He also pointed out the relationship between the modal aggreation and the time scale approach. And he established properties of the closed loop system using approximate feedback control derived from low order subsystems. Eitelberg[1] introduced a time-varying weighting matrix and argued that even unstable linear time invariant models could be reduced by minimizing the weighted equation error. But the characteristic of the reduced model then depends highly on the weighting matrix and reduced models obtained by the equation error method can not be guaranteed to be asymptotically stable and controllable even if the original system satisfies these properties [12]. Terapos[6] proposed the discrete stable equation not to require the bilinear transformation. This must calculate the roots of two polynomials and is espicially time-consuming when the original system is of a high order. Also Lucas[4] showed that the Schwarz approximation had a continued fraction form and was also related to the Routh approximation. He also proposed a mixed method using the moment matching technique to improve the response to the step input.

Alternately Chen[8] proposed the mixed method using the stability equation method and the continued fraction method for reducing SISO system and was expanded in multivariable system. Warwick[10] defined the error polynomial, which was found as the difference between the original system and reduced model output to an identical step input. Here he showed how Markov parameters and H parameters of the reduced model could be matched with those of the system by setting the error polynomial coefficients to zero. This method can not ensure stability of the reduced model even if the original system is stable.

In the recent year Liaw[11] proposed a mixed method of discrete system model reduction retaining the advantages of the power decomposition method and the system identification method. The reduction procedure is fully computer-oriented. The reduced model is always stable if the original one is stable but the system identification requires matrix calculations and is consequently time-consuming.

In this paper the new methods of model reduction based on the energy dispersion method, employed in the linear time-invariant continuous system, are introduced. They show that while the model denominator polynomial is found by means of the energy dispersion method, the model numerator is found by the modified residue method or the time moment matching method. The methods are compared with the method suggested by Lucas. The proposed methods are computationally simple and lead to good results as shown by the example.

II. Energy Dispersion Method

The transfer function of the linear time-variant continuous system can be expressed as

$$G(S) = \frac{Y(S)}{U(S)} = \frac{b_{n-1}S^{n-1} + b_{n-2}S^{n-2} + \dots + b_1 S + b_0}{S^n + a_{n-1}S^{n-1} + \dots + a_1 S + a_0}$$

$$= \sum_{i=1}^{n} \frac{g_i}{S - \lambda_i} \qquad (1)$$

The impulse response of the system g(t) is given as

$$g(t) = \mathcal{L}^{-1}[\,G(S)\,]$$

$$= \sum_{i=1}^{n} g_i \,\mathrm{EXP}(\lambda_i t) \qquad (2)$$

Now let us express the autocovariance function of the output y(t) when the input is the white noise w(t).

Using eqn (2), we can obtain

$$R(\tau) = E[y(t)y(t+\tau)]$$

$$= E[\int_{-\infty}^{t} g(t-\alpha)\; w(\alpha)\, d\alpha \int_{-\infty}^{t+\tau} g(t+\tau-\beta)$$

$$w(\beta)\, d\beta]$$

$$= \sigma_w^2 \int_{-\infty}^{t} g(t-\alpha)\, g(t+\tau-\alpha)\, d\alpha \qquad (3)$$

where σ_W^2 is the variance of the white noise w(t).

By letting $t - \alpha = \beta$ and substituting eqn. (2) into eqn. (3), we can obtain

$$R(\tau) = \sigma_w^2 \sum_{i=1}^{n}\sum_{j=1}^{n} g_i g_j \int_0^{\infty} \mathrm{EXP}((\lambda_i+\lambda_j)\beta)\, d\beta\, \mathrm{EXP}(\lambda_j \tau)$$

$$= \sigma_w^2 \sum_{j=1}^{n}\sum_{i=1}^{n} \frac{-g_i g_j}{\lambda_i + \lambda_j} \mathrm{EXP}(\lambda_j \tau)$$

$$\sum_{j=1}^{n} x_j \,\mathrm{EXP}(\lambda_j \tau) \qquad (4)$$

where

$$x_j = \sigma_W^2 \sum_{i=1}^{n} \frac{-g_i\, g_j}{\lambda_i + \lambda_j} \qquad (5)$$

x_j is the energy contribution of the dynamic mode λ_j to the variance of the output y(t). In case $\tau = 0$, the energy dispersion of the dynamic mode λ_j can be defined as

$$X_j = \frac{x_j}{R(0)} = \frac{x_j}{\sum_{i=1}^{n} x_i} \tag{6}$$

The energy dispersion X_j represents the energy contribution of the dynamic mode λ_j to the total energy of the output. Therefore using X_j, we can determine the dynamic modes of the system.

III. Design of Reduced Model

The transfer function G(S) of the original system is rewritten

$$G(S) = \frac{b_{n-1} S^{n-1} + b_{n-2} S^{n-2} + \dots + b_1 S + b_0}{S^n + a_{n-1} S^{n-1} + \dots + a_1 S + a_0} \tag{7}$$

And the transfer function of the reduced model can be represented as

$$Gr(s) = \frac{d_{k-1} S^{k-1} + d_{k-2} S^{k-2} + \dots + d_1 S + d_0}{S^k + C_{k-1} S^{k-1} + \dots + c_1 S + c_0} \tag{8}$$

Now consider how the numerator of the reduced model is determined. Here we will suggest two methods: the modified residue method and the time moment matching method. These methods are very simple and yield the good approximation of the original system.

1. Modified residue method

The residue corresponding to each dynamic mode can be found in eqn. (7).

$$g_i = G(S)(S-\lambda_i)\Big|_{S=\lambda_i} \qquad i=1,2,\dots,n \tag{9}$$

And let us define the scale factor as

$$\mu \triangleq \frac{b_0/a_0}{\sum_{i=1}^{k} \frac{-g_i}{\lambda_i}} \tag{10}$$

where λ_i (i=1,2, ..., k) are the eigenvalues to be determined in II. and g_i (i=1, 2, ..., k) are the residues corresponding to each dynamic mode. Therefore we can obtain the reduced model in eqns. (9), and (10).

$$Gmres(S) = \frac{\mu \sum_{i=1}^{k} g_i \left(\prod_{j=1}^{k} (S-\lambda_j) \right)}{S^k + c_{k-1} S^{k-1} + \dots + c_1 S_1 + c_0} \quad i \neq j \tag{11}$$

2. Time moment matching method

Eqn. (7) can be written with regard to its power series expansion about S=0.

$$G(S) = l_0 + l_1 S + \dots + l_i S^i + \dots \tag{12}$$

where l_i (i=0,1, ...) can be obtained from the following relationship:

$$\begin{aligned} b_0 &= a_0 \, l_0 \\ b_1 &= a_1 \, l_0 + a_0 \, l_1 \\ &\dots \\ b_{k-1} &= a_{k-1} l_0 + a_{k-2} l_1 + \dots + a_0 \, l_{k-1} \end{aligned} \tag{13}$$

Similary, Gr(S) in eqn. (8) can be written in terms of its power series expansion about S=0.

$$Gr(S) = m_0 + m_1 S^1 + \dots + m_i s^i + \dots \tag{14}$$

m_i (i=0, 1, ...) also satisfy the following relationship.

$$\begin{aligned} d_0 &= c_0 \, m_0 \\ d_1 &= c_1 \, m_0 + c_0 \, m_1 \\ &\dots \\ d_{k-1} &= c_{k-1} m_0 + c_{k-2} \, m_1 + \dots + c_0 \, m_{k-1} \\ &\dots \end{aligned} \tag{15}$$

From eqns.(12) and (14) equating the coefficients up to the k-th moment we can obtain

$$l_i = m_i \quad \text{for } i=0, 1, \dots, k-1 \tag{16}$$

Therefore we can determine the coefficients of the numerator of the reduced model using eqns. (13), (15), and (16).

IV. Example

Consider the following transfer function given from Lucas [4].

$$G(S) = \frac{14.7S^3 + 1363.38S^2 + 28566.8S + 92024.7}{S^4 + 97S^3 + 2268S^2 + 11680S + 14000}$$

where

$\lambda_1 = -1.75$, $\lambda_2 = -4.8$

$\lambda_3 = -25.78$, $\lambda_4 = -64.67$

The energy dispersions of G(S) are represented in Table 1.

Table 1. Energy Dispersion

eigenvalues	G (S)
λ_1	X1 = .8109 (80.37%)
λ_2	X2 = .1822 (18.06%)
λ_3	X3 = .0059 (.58%)
λ_4	X4 = .0100 (.99%)
	Total (100%)

From Table 1. by discarding the dynamic modes with the less important energy contribution (i.e. λ_3, λ_4), we can obtain the denomenator of the reduced model

$$C(S) = S^k + c_{k-1}S^{k-1} + \dots + c_1 S + c_0$$
$$= S^2 + 6.55S + 8.4$$

Using eqns. (9), and (10) we can calculate the g_i's and μ

$g_1 = 10.0709$ $g_2 = 3.9284$

$\mu = 1.0038$

Therefore the transfer function of the modified residue model is

$$\text{Gmres}(S) = \frac{14.0519S + 55.2148}{S^2 + 6.55S + 8.4}$$

Also we can obtain the transfer function of the moment time matching model using eqns. (13), (15), and (16).

$$\text{Gmat}(S) = \frac{14.1296S + 55.2148}{S^2 + 6.55S + 8.4}$$

The unit step responses of the original system and the reduced models are shown in Fig. 1. The models suggested are compared with Lucas [4]. Fig. 2 is showing the output errors between the original system and the reduced models.

The methods suggested in this paper give the better results than that of Lucas as shown from Fig. 2.

V. Conclusion

The mixed methods of model reduction are

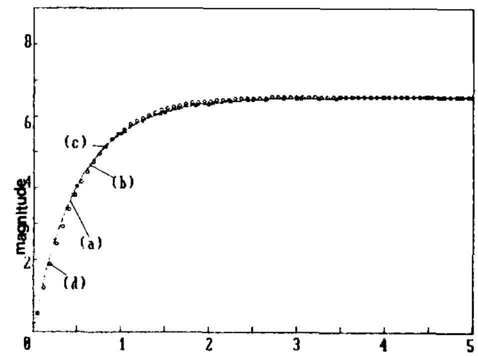

Fig. 1. Step Responses (a)Original (b) Modified Residue (c) Time Moment (d)Lucas

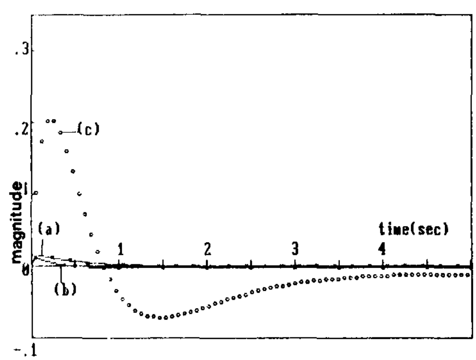

Fig. 2. Response Errors (a)Modified Residue (b)Time Moment (c)Lucas

proposed. The energy dispersion method is used to select the dynamic modes. By discarding the dynamic modes with less energy dispersion, the denominators of the reduced models are obtained. The numerators of them are obtained from the modified residue method and the time moment matching method. As show by an example, the methods suggested yield the good results. Also the models are computationally simple and stable if the original system is stable.

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