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Approximate Interpolator for Direct Fourier Reconstruction in Diffraction Tomography

(회절 단층법에서 직접 푸리에 재구성을 위한 근사적 보간 함수에 관한 연구)

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要 約

本 論文에서는 廻折斷層法에서 直接 푸리에 再構成 알고리즘을 위한 여러가지 補間 函數들에關하여 言及하고, 表本化 定理 및 圓上 表本化 定理를 利用하여 近似的인 補間 函數를 提案하였다.

同軸上의 두 圓筒型을 被寫體로 하여 行한 컴퓨터 시뮬레이션의 結課로써, 本 論文에서 提案된 補間 函數에 의하여 影像뿐 아니라 數值에서도 原 補寫體에 가장 가까운 影像 再構成을 얻을 수 있다는 것을 보였다.

Abstract

In this paper, the interpolation schemes for Direct Fourier Reconstruction in Diffraction Tomography are discussed. The interpolator using circular sampling theorem is modified so that the reconstructed image may be closer to original object than those produced with other interpolators. Reconstructed images obtained by computer simulations with several interpolators including that derived in this paper are compared to original object: two concentric cylinders.

I. Introduction

The image reconstruction algorithms for diffraction tomography have been studied for ten years. In diffraction tomography, informations of Fourier transform of measured data are located on the circular arcs in spatial frequency domain according to Fourier Diffraction projection Theorem [1]. Image reconstruction technique with such informations needs interpolation schemes that interpolate the values from the circular arcs to points on a uniform grid to do inverse fast Fourier Transform. There are two fundamental algorithms for image reconstruction from diffracted projections: "Direct Fourier Reconstruction Algorithm" and "Filtered-Backpropagation Algorithm" [2-4]. The Filtered-Backpropagation Algorithm derived by Devaney takes spatial domain interpolation scheme, which is fundamentally based on the concept of the Filtered-Backprojection algorithm for X-ray Tomography. On the other hand, the Direct Fourier Reconstruction Algorithm takes

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spatial frequency domain interpolation scheme.

With the Direct Fourier Reconstruction Algorithm, one can obtain the reconstructed image directly by only doing inverse Fourier transform of the interpolated informations. Pan and Kak [2] showed that by the direct Fourier Reconstruction Algorithm with bilinear interpolation technique one can obtain reconstructions of quality which is comparable to that produced by the Filtered-Backpropagation Algorithm, and computation time is very much saved also. Many interpolation schemes have been studied. According to sampling theorem and circular sampling theorem and circular sampling theorem, the band limited signal can be exactly interpolated from moderately sampled data. But Pan and KaK obtain the conclusion after their computer simulation that the bilinear interpolation sheme is superior to the interpolation scheme using circular sampling theorem [2]. In this paper, the interpolator using circular sampling theorem is modified slightly, and the reconstructed images with such interpolator are compared to those with several other interpolation schemes (e.g. Nearest-Neighbor Interpolation, Interpolations using circular sampling theorem).

II. Coordinate Transformation for Interpolation

According to Fourier Diffraction Projection Theorem, informations of Fourier transform of diffracted projection data are located on the semi-circle of radius ko, where ko is the wave number of incident plane wave. In fig. (1), space domain and spatial frequency domain coordinate systems are shown, where α is projection angle, k is ζ-directed spatial frequency.

As shown in fig. (1), data obtained by Fourier transform of diffracted projection are represented by (α, K) coordinate system. For inverse fast Fourier Transform, informations on the points of uniform grid are needed, and the grid points are represented by (u,v) cartesian coordinate as in fig. (2). Thus, one must transform the grid points represented by (u,v) coordinate to (α, K) coordinate, or vice versa. Transformation equations between (α, K) and

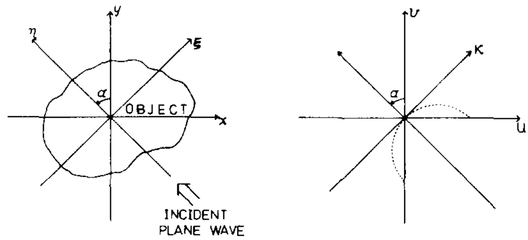

Fig. 1. Coordinate system. a) Space domain b) Spatial frequence domain.

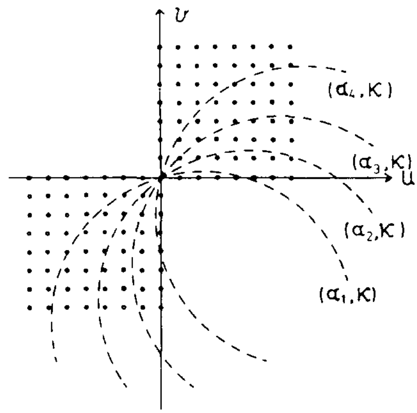

Fig. 2. Circular arcs for various α (dashed line) and points on the uniform grid for IFFT (small circles).

(u,v) coordinates are as follows [2], [3].

1) $(u,v) \rightarrow (\alpha, k)$
for $k \geqslant 0$,

$$k = k_c \cdot \sin\left(2\arcsin\sqrt{\frac{u^2+v^2}{2k_o}}\right)$$

$$\alpha = \arctan\frac{v}{u} + \arcsin\sqrt{\frac{u^2+v^2}{2k_o}} \qquad \text{(1-a)}$$

and for $k < 0$,

$$k = -k \cdot \sin\left(2\arcsin\sqrt{\frac{u^2+v^2}{2k_o}}\right)$$

$$\alpha = \arctan\frac{v}{u} - \arcsin\sqrt{\frac{u^2+v^2}{2k_o}} \qquad \text{(1-b)}$$

2) $(\alpha, k) \rightarrow (u,v)$

$$u = (k_o - \sqrt{k_o^2 - k^2})\cdot\sin\alpha + k\cdot\cos\alpha$$

$$v = -(k_o - \sqrt{k_o^2 - k^2})\cdot\cos\alpha + k\cdot\sin\alpha \quad (2)$$

III. Interpolation Schemes for Direct Fourier Reconstruction in Diffraction Tomography

Interpolation schemes for X-ray Tomography (e.g. Largrange interpolation, Spline algorithm, Sinc interpolator, etc.) enable us to obtain nearly exact reconstructed image [5]. In Diffraction Tomography, however, the locations of the data-known points in the spatial frequency domain are different to them in straightpath-like tomography. So the interpolation schemes in Diffraction Tomography are somewhat complicated and the reconstructed images with them are not satisfiable, too. Several interpolation schemes have been studied since the concept of the Direct Fourier Reconstruction Algorithm for the Diffraction Tomography was brought about. The brief description about them is given below 2-4. At first, the grid point (u,v) at which the value must be defined is transformed to (α, k) coordinate according to the transformation equations (1-a) and (1-b), then, the value at the grid point, f(u,v), is interpolated from the values at the points (α_i, k_j), $f(\alpha_i, k_j)$, according to the following equation (3).

$$f(\alpha(u,v), k(u,v)) = \sum_{i=1}^{N}\sum_{j=1}^{M} H(\alpha(u,v), k(u,v); \alpha_i, k_j)\, f(\alpha_i, k_j) \quad (3)$$

where,

$(\alpha(u,v), k(u,v))$ is transformed (u,v) point, $H(\alpha(u,v), k(u,v); \alpha_i, k_j)$ is the interpolating function.

The interpolating function H in(3) is defined as follows [2],[4].

1) Nearest Neighbor Interpolation

$$H(\alpha(u,v), k(u,v); \alpha_i, k_j) = h_1(\alpha-\alpha_i)\cdot h_2(k-k_j) \quad (4)$$

where

$$h_1(\alpha-\alpha_i) = \begin{cases} 1 : \text{abs}(\alpha-\alpha_i) = \min[\text{abs}(\alpha-\alpha_m); m=1,\cdots, N] \\ 0 : \text{other wise} \end{cases}$$

$$h_2(k-k_j) = \begin{cases} 1 : \text{abs}(k-k_j) = \min[\text{abs}(k-k_n); n=1,\cdots, M] \\ 0 : \text{other wise} \end{cases}$$

2) Bilinear Interpolation

$$H(\alpha(u,v), k(u,v); \alpha_i, k_j) = h_1(\alpha-\alpha_i)\cdot h_2(k-k_j) \quad (5)$$

where,

$$h_1(\alpha-\alpha_i) = \begin{cases} 1 - \dfrac{|\alpha-\alpha_i|}{\Delta\alpha} & |\alpha-\alpha_i| \leqslant \Delta\alpha \\ 0 & \text{other wise} \end{cases}$$

$$h_2(k-k_j) = \begin{cases} 1 - \dfrac{|k-k_j|}{\Delta k} & |k-k_j| \leqslant \Delta k \\ 0 & \text{other wise} \end{cases}$$

$\Delta\alpha$ and Δk are the sampling intervals for α and k, respectively.

3) Interpolation using the Circular Sampling Theorem

$$H(\alpha(u,v), k(u,v); \alpha_i, k_j) = h_1(\alpha-\alpha_i)\cdot h_2(k-k_j) \quad (6)$$

where,

$$h_1(\alpha-\alpha_i) = \frac{\sin(\frac{N}{2}(\alpha-\alpha_i))}{N\sin(\frac{\alpha-\alpha_i}{2})}$$

$$h_2(k-k_j) = \text{sinc}\left(\frac{\pi M}{2k_o}(k-k_j)\right)$$

If N is even number, $h_1(\alpha-\alpha_i)$ in (6) is corrected as (7)[6].

$$h_1(\alpha-\alpha_i) = \frac{\sin(\frac{N}{2}(\alpha-\alpha_i))}{N\sin(\frac{\alpha-\alpha_i}{2})}\cos\left(\frac{\alpha-\alpha_i}{2}\right) \quad (7)$$

In this section, N is a total of projections and M is a number of sample points for each projection.

IV. Approximate Interpolator using sampling Theorem

With the transformation equation (2), the data-known points (α_i, k_j) on the circular arcs are transformed to (u,v) coordinate. Then the value at the grid point, $f(u_k, v_\ell)$, is interpolated from the values at the points on the circular arcs, $f(\alpha_i, k_j)$. With full angle projections the circular arcs are overlapped. If only the quadrants (i.e. $k \leqslant 0$, or $K \geqslant 0$) are selected for interpolation, as shown in fig. (3), the data-known points are spaced uniformly along the circle of which the center is (0,0) and unequally along the radial direction. That is, let (u_{ij}, v_{ij}) represent the transformed point of $\alpha_i, k_j)$ and (R_{ij}, ϕ_{ij}) be its polar coordinate description, then $(\phi_{ij}-\phi_{(i-1)}\ j)$ is constant for all i, but $(R_{ij}-R_{i(j-1)})$ is dependent upon j.

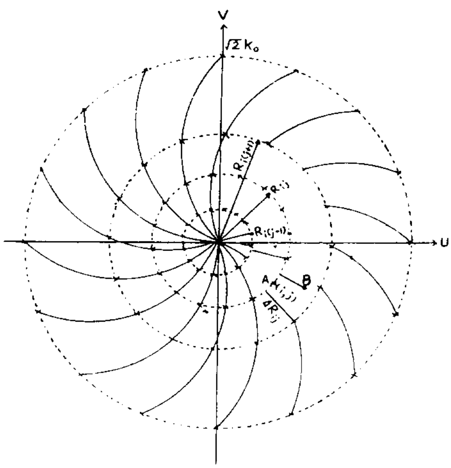

Fig. 3. Allocation of data-known points(x) for derivation of approximate interpolator.

For applying the sampling theorem, the Radial direction sampling interval ΔR_{ij} was approximated as (8)

$$\Delta R_{ij} = \begin{cases} \text{abs}\,(R_{ij}-R_{i(j-1)}) & R < R_{ij} \\ \text{abs}\,(R_{i(j+1)}-R_{ij}) & R \geqslant R_{ij} \end{cases} \quad (8)$$

where, $R = \sqrt{u_k^2 + v_\ell^2}$

(u_k, v_ℓ) is the grid point B in fig. (3).

The circular sampling interval, on the other hand, is uniformly $\Delta\phi = N \ 2\pi$, where N is the number of projections. The angular distance between the two points (u_k, v_ℓ) and (u_{ij}, v_{ij}) is given as $\Delta\phi_{ij} = |\phi - \phi_{ij}|$, where $\phi = \arctan(V_\ell/U_k)$.

Then the approximate interpolator using sampling theorem can be represented as (9).

$$H(u_k, v_\ell\,; u_{ij}, v_{ij}) = H(R, \phi; R_{ij}, \phi_{ij}) = h_1(R, R_{ij})\, h_2(\phi, \phi_{ij}) \quad (9)$$

where,

$$h_1(R, R_{ij}) = \text{sinc}\left(\frac{\Pi(R-R_{ij})}{\Delta r R_{ij}}\right)$$

$$h_2(\phi, \phi_{ij}) = \text{sinc}\left(\frac{\Pi(\phi-\phi_{ij})}{\Delta\phi}\right)$$

In (9), the usual sampling function was approximately given as the circular interpolator $h_2(\phi, \phi_{ij})$.

The computational procedure for reconstructing an image from diffracted projections with approximate interpolator (9) may be presented in the form of the following steps.

step 1. obtain the projection data. Computer simulations in V were performed with the projection data obtained from first order Bessel function, Fourier transform of cylinder function.

step 2. coordinate transformation, (α_i, k_j) on circular arcs and (U_k, V_ℓ) on uniform grid are transformed to polar coordinates (R_{ij}, ϕ_{ij}) and (R, ϕ), respectively.

step 3. determine the sampling interval ΔR_{ij}, $\Delta\phi$ according to (8).

step 4. calculate the interpolated value with approximate interpolator (9).

step 5. reconstruct the image by doing inverse fast Fourier transform with the interpolated value obtained in step 4.

V. Results of Computer Simulation

The interpolators in this paper are tested

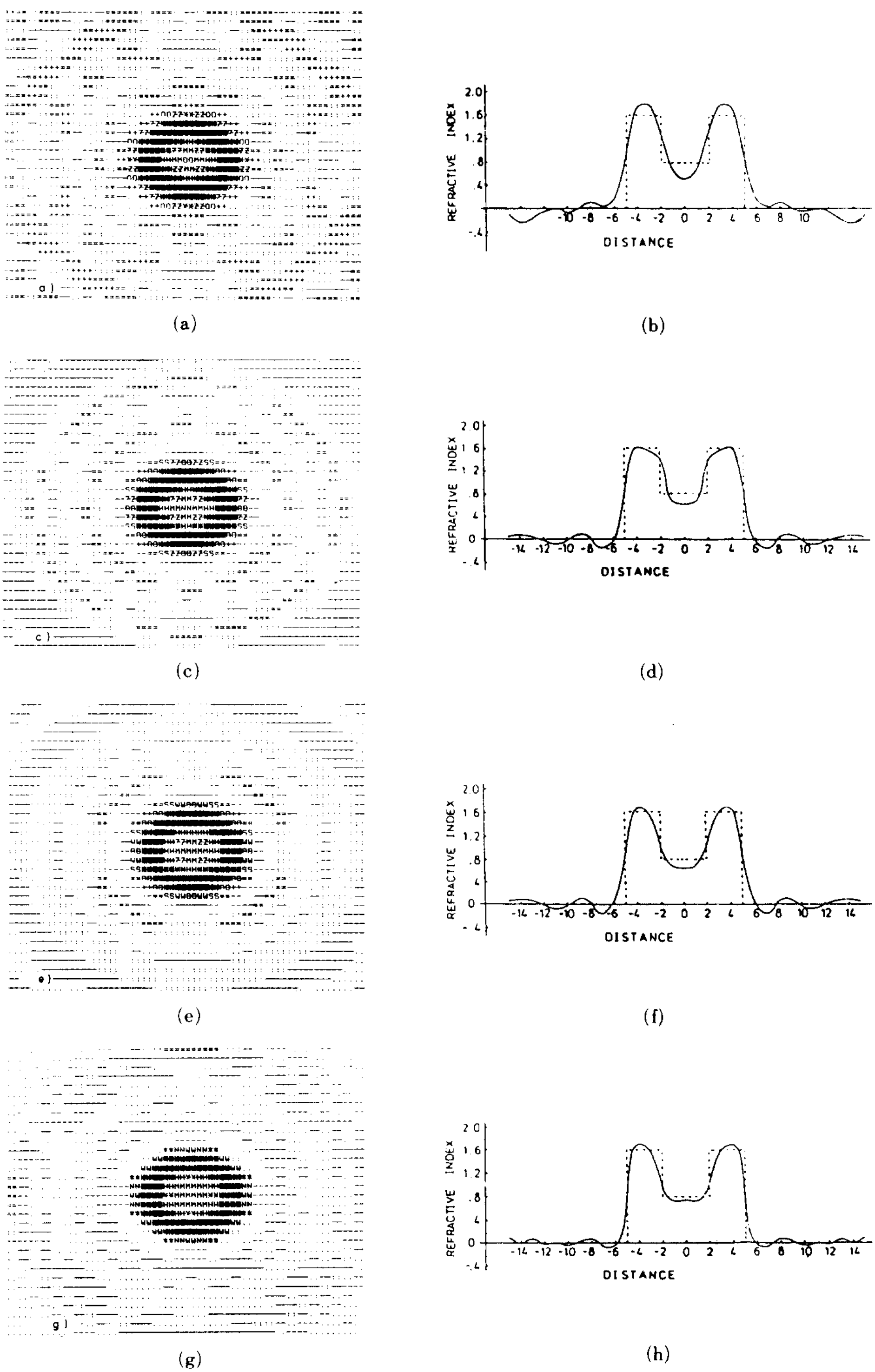

Fig. 4. Computer simulation results. a), c), e) and g) are 32×32 reconstruction images; b), d), f) and h) are numerical comparisons of the true and reconstructed values on the center line through the object (solid line represents the reconstructed values and dotted line the true values). a) and b) with nearest neighbor interpolation, c) and d) with interpolator using circular sampling theorem. e) and f) with corrected interpolator using sampling theorem, g) and h) with approximate interpolator using sampling theorem.

by computer simulation. A coaxial cable-shaped object was chosen as that to be reconstructed.

The ratio of the object size to the wave length of the incident wavelength was assumed 4. Each of 32 projections were used. Only the half of the sampled points (i.e. $K \geqslant 0$) were need for interpolation to prevent the circular arcs from overlap.

Computer simulation results are shown in fig. (4). Shown in figs. (4a), (4c), (4e), and (4g) are 32x32 reconstruction images and in figs. (4b), (4d), (4h) and (4f) numerical comparisons of true and reconstructed values on the center-line through the original object.

In fig. (4), all of the reconstructions and the numerical values are only the real parts of inverse Fast Fourier Transform of interpolated values. The imaginary parts are all below 10^{-2}. Results in figs. (4c), and (4d) are obtained with interpolator using circular sampling theorem as (6). Results in figs. (4e) and (4f) are same as in figs. (4c) and (4d) except that circular interpolator $h_1(\alpha)$, in (6) is corrected as (7).

Comparing results in fig. (4), one can see that results obtained with the approximate interpolator (8) using sampling theorem (figs. (4g) and (4h)) are the closet to original object among them. Mean Square Error (MSE) in each case is as follows :0.1511 in figs. (4a) and (4b), 0.1145 in figs. (4c) and (4d), 0.1133 in figs. (4e) and (4f), and 0.0996 in figs. (4g) and (4h) the smallest value.

In the case of noisy data, the fact causes SNR to be down that interpolation with the approximate interpolator in this paper is performed using only the half of the sample points. But SNR can be compensated by e.g. arithmatic mean of interpolated data using two data sets respectively, since the half of the sample points unused are allocated in exactly the same pattern with the other half used in interpolation.

VI. Conclusions

Several interpolation schemes for Direct Fourier Reconstruction Algorithm in Diffraction Tomography were briefly discussed.

Using the sampling theorem, Approximate Interpolator was derived in this paper.

Computer simulations were performed with those interpolators, and results of reconstructions, numerical values, and mean square errors were obtained. Comparing computer simulation results, we can conclude that in the respects of displayed image, numerical values and MSE are the results obtained with Approximate Interpolator using sampling theorem more satisfiable than those with other interpolators.

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