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Markov Chain Properties of Sea Surface Temperature Anomalies at the Southeastern Coast of Korea

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한국 남동연안 이상수온의 마르코프 연쇄 성질

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Abstract

The Markov chain properties of the sea surface temperature (SST) anomalies, namely, the dependency of the monthly SST anomaly on that of the previous month, are studied based on the SST data for 28 years (1957-1984) at 5 stations in the southeastern coast of Korea. We classified the monthly SST anomalies at each station into the low, the normal and the high state, and computed transition probabilities between SST anomalies of two successive months. The standard deviation of SST anomalies at each station is used as a reference for the classification of SST anomalies into 3 states.

The transition probability of the normal state to remain in the same state is about 0.8. The transition probability of the high or the low states to remain in the same state is about one half. The SST anomalies have almost no probability to transit from the high (the low) state to the low (the high) state. Statistical tests show that the Markov chain properties of SST anomalies are stationary in time and homogeneous in space. The multi-step Markov chain analysis shows that the 'memory' of the SST anomalies at the coastal stations remains about 3 months.

요약 : 한국 남동해안 5개 연안 관측점(주문진, 죽변, 장기갑, 울기, 부산)의 28년간(1957 ~ 1984) 월별 수온 자료를 근거로 하여, 매달의 이상수온(water temperature anomalies)이 1개월 전의 이상수온에 의존하는 성질, 즉 마르코프(Markov) 연쇄 성질에 대하여 구명하였다. 각 관측점 이상수온 변동의 표준편차를 기준으로 하여, 월별 이상수온을 저온, 정상 및 고온의 상태로 구분한 후, 연속된 두 달간의 이상수온 상태의 천이 횟수 및 확률을 계산하였다.

정상상태의 이상수온이 그 다음 달에도 정상상태로 유지될 확률은 0.8 정도이다. 저온 또는 고온 상태의 이상수온이 다음 달에도 같은 상태에 남아 있을 확률과 정상상태로 될 확률은 거의 같다. 이상수온이 고온(저온) 상태에 있다가 다음 달에 저온(고온) 상태로 바뀔 확률은 거의 0에 가깝다. 이와 같은 이상수온 변동의 마르코프 연쇄에 대한 통계적 테스트 결과에 의하면, 이상수온 변동은 시간적으로 정상적(stationary)이며, 공간적으로 균질적(homogeneous)이다. 이상수온 변동에 대한 다단계(multi-steps) 마르코프 연쇄분석에 의하면, 연안역 이상수온이 마르코프 연쇄성을 보유하는 '기억'은 약 3개월 정도 유지된다.

INTRODUCTION

Seasonal variations of the sea surface temperature (SST) are deterministic, but those of the SST anomalies, or the deviations of the SST from the multi-year monthly normals, are stochastic. The characteristics of SST anomalies can be studied by means of auto-correlation function or spectral analyses (Kang and Lee, 1984; Kang and Choi, 1985;

Gong and Kang, 1986). These analyses, however, do not show explicitly how the SST anomalies of two successive months are related.

The heat capacity of the ocean is large, and the SST anomaly of the present month is expected to depend on that of the previous month. In other words, the transitions of SST anomalies between two successive months are expected to possess the Markov chain properties. A sequence of transitions of states is called a Markov process if the present value at time t depends on the immediate past value at time t-1. In a Markov process, the 'memory' of the immediate past state is left in the present state. In a random process, on the other hand, the present state has no correlation with the past state. A random process can be understood as a process without any memory of the past, and a deterministic process can be understood as a process with an infinite memory. The SST anomalies are expected to depend on the state of the recent values, but not on the state of long ago.

Since Gabriel and Neumann(1962) applied a two-state Markov chain model to fit daily wet and dry state at Tel Aviv, Israel, the Markov chain models have been successfully applied to the meteorological and geological problems (Harbaugh and Bonham-Carter, 1981; Katz, 1985). In this paper, a three-state Markov chain model is used to fit the monthly SST anomalies at the southeastern coast of Korea.

DATA AND METHOD OF ANALYSIS

a. The Data

The data set used in this study is the monthly SST anomalies for 28 years (1957-1984) at 5 coastal stations (Fig. 1), which had been collected by the Fisheries Research and Development Agency of Korea. The monthly SST anomalies are classified into three states: the low, the normal and the high states. The classification is based on the standard deviation (SD) of the SST anomalies at each station. The monthly SST anomalies are classified into the low (state 1), the normal (state 2) and the high (state 3) states if the anomalies are less than −SD, in the range of [−SD, +SD], and larger than +SD, respectively.

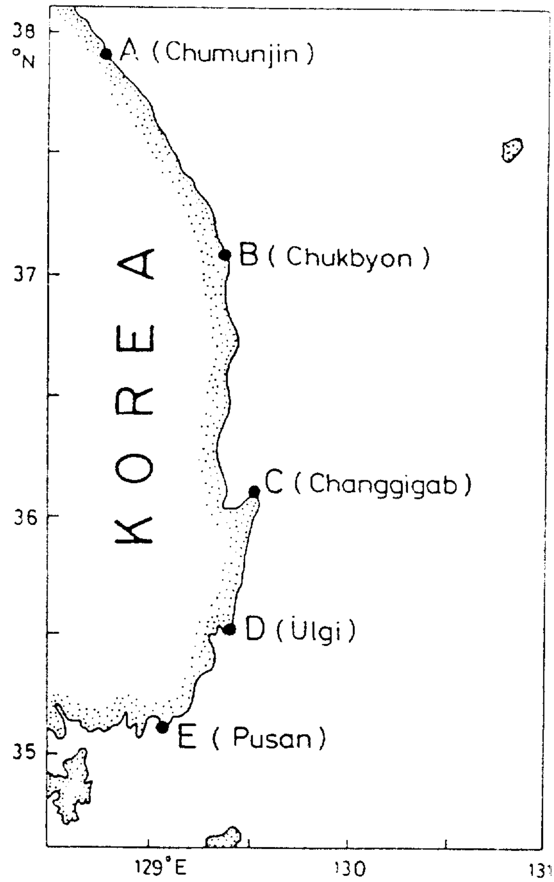

Fig. 1. Locations of coastal stations.

b. Transition Probability

The tally matrix of transitions between two successive states of SST anomalies is computed by counting the numbers of transitions for every possible pair of transitions. For example, (2,3)-th element of the tally matrix is obtained by summing up the numbers of transitions from the normal state (2) to the high state (3).

The transition probability matrix p_{ij} represents the probability from state i to the subsequent state j. For example, the transition probabilities p_{21}, p_{22} and p_{23} represent the probabilities of transitions from the normal state (i = 2) to the low (j = 1), to the normal (j = 2) and to the high (j = 3) states, respectively. The sum of row vector of the transition probability matrix is unity, for example, $p_{21} + p_{22} + p_{23} = 1$.

c. *Markov chain property*

The Markov chain property of the SST anomaly can be statistically tested by checking whether the successive events are independent of each other (the null hypothesis) or they are dependent (the alternative hypothesis). If dependent, they can form a first-order Markov chain. According to Anderson and Goodman (1957), if successive events are independent, then the statistic α defined by

$$\alpha = 2 \sum_{i,j}^{m} n_{ij} \ln (p_{ij}/p_j) \tag{1}$$

is distributed asymptotically as a Chi-squared distribution with $(m-1)^2$ degrees of freedom (Harbaugh and Bonham-Carter, 1981). In (1), m is total number of states (m = 3 in our case), n_{ij} and p_{ij} are the number of occurances and the transition probability from the state i to the state j, respectively, and p_j is the marginal probability for the j-th column of the transition probability matrix, i.e.,

$$p_j = \sum_{i}^{m} n_{ij} / \sum_{i,j}^{m} n_{ij} . \tag{2}$$

d. *Stationarity and Homogeneity*

A Markov process is stationary if its transition probabilities are independent of time. A convenient way to check the stationarity is to divide the whole sequence of events into a few subintervals, and then compute and compare the transition probabilities of each subinterval. A statistic β for a test of stationarity is

$$\beta = 2 \sum_{k}^{K} \sum_{i,j}^{m} n_{ij}(k) \ \ln [p_{ij}(k)/p_{ij}], \tag{3}$$

where K is the number of subintervals, and n_{ij} (k) and p_{ij} (k) are the (i,j)-th elements of the tally matrix and the transition probability matrix of the k-th subinterval, respectively. If a Markov chain is stationary, the statistic β of (3) has a Chi-squared distribution with (K-1)m (m-1) degrees of freedom (Harbaugh and Bonham-Carter, 1981).

Similarly, the spatial homogeneity of the Morkov chain property can be checked as follows. If the Markov chain property of successive events at many locations were homogeneous, then a statistic γ defined by

$$\gamma = 2 \sum_{s}^{S} \sum_{i,j}^{m} n_{ij}(s) \ \ln [p_{ij}(s)/p_{ij}] \tag{4}$$

would be distributed as a Chi-squared distribution with (S-1)m(m-1) degrees of freedom, where S is the number of stations. In (4), $n_{ij}(s)$ and $p_{ij}(s)$ are (i,j)-th elements of the tally matrix and the transition probability matrix of the events at the s-th station, respectively.

RESULTS

The SD's of monthly SST anomalies for 28 years (1957-1984) at the station A (Chumunjin), B (Changgigab), C (Jukbyon), D (Ulgi) and E (Pusan) are 1.57, 1.47, 1.41, 1.66 and 0.90 °C, respectively. These SD's are used in classifying the monthly SST anomalies into the low, the normal and the high states at each station.

a. *Transition Probability*

The tally matrix of SST anomalies for 28 years at all of 5 stations and the associated transition probabilities are shown in Table 1. There were 335 transitions of monthly SST anomaly at each station, and the total number

Table 1. Tally matrix (left) and transition probabilities (right) of SST anomalies for 28 years at 5 stations. Total number of transitions is 1675.

	Low (to)	Norm (to)	High (to)		Low (to)	Norm (to)	High (to)
Low	113	121	5	Low	.47	.51	.02
(from) Norm	120	954	123	(from) Norm	.10	.80	.10
High	5	122	112	High	.02	.51	.47

of transitions at 5 stations is 1675. The probabilities of transitions from the low to the low, to the normal and to the high states are 0.47, 0.51 and 0.02, respectively. The probability of the normal SST anomaly to remain in the same state is 0.80, and that to be raised up to the high state or to be lowered down to the low state is 0.10 each.

The Markov chain properties of SST anomalies at each of 5 stations are shown in Table 2. The distributions of transition probabilities at every station are similar. Roughly speaking, the probabilities of the low, the normal and the high SST anomalies to be found after a normal SST anomaly is about 0.1, 0.8 and 0.1, respectively. The probability of the low or the high SST anomaly to remain at the same state is approximately equal to that to become a normal state. There is almost no probability to have the low state immediately after the high state, and *vice versa*.

Table 2. Tally matrix and transition probabilities of SST anomalies of 336 months (1957-1984) each at 5 stations.

Station	Tally Matrix			Transition Probabilities		
Chumunjin	31	21	1	.58	.40	.02
	20	192	22	.09	.82	.09
	1	22	25	.02	.46	.52
Chukbyon	23	23	1	.49	.49	.02
	23	184	27	.10	.79	.12
	1	27	26	.02	.50	.48
Changgigab	22	25	1	.46	.52	.02
	25	181	30	.11	.77	.13
	1	29	21	.02	.57	.41
Ulgi	10	24	0	.29	.71	.00
	24	234	13	.09	.86	.05
	0	13	17	.00	.43	.57
Pusan	27	28	2	.47	.49	.04
	28	163	31	.13	.73	.14
	2	31	23	.04	.55	.41

b. Dependency of transitions

We checked whether the successive states of SST anomalies are independent (the null hypothesis) or dependent (the alternative hypothesis) by computing a statistic of independence, α, given by (1). The values of α for SST anomalies at stations A, B, C, D and E are 113.8, 81.0, 62.3, 69.0 and 61.7, respectively. Since these values are larger than the Chi-squared value of 9.5 at 5% level or 13.3 at 1% level with 4 degrees of freedom (Selby, 1971), we reject the null hypothesis that the successive transitions are independent process, and accept the alternative hypothesis that the transitions are dependent. In other words, the SST anomaly of the present month is dependent on that of the preceding month, and the transitions of SST anomalies can have the Markov chain property.

In order to study whether the SST anomaly of the present month is related with that of 2 months ago, 3 months ago and so on, we computed the transition probabilities of k-steps ($k = 1,2,\ldots.$) Markov chains, and computed the statistic α of (1) for each step. Fig. 2 shows that the statistic α at each station has a tendency to decrease as the number of steps increases. If a Chi-squared value of 9.5 at 5% level is used as a reference, then the SST anomalies at Stations B (Chukbyon), D (Ulgi) and E (Pusan) have Markov chain properties up to 3 months. The Markov chain property of SST anomaly is limited to 2 months at Station C (Changgigab), and it is extended more than 5 months at Station A (Chumunjin).

c. Stationarity and Homogeneity

In order to check whether the Markov chain properties of SST anomalies are stationary or not, we divided the SST anomalies of 28 years (336 months) at each station into 3 subintervals of 112 months each, and computed

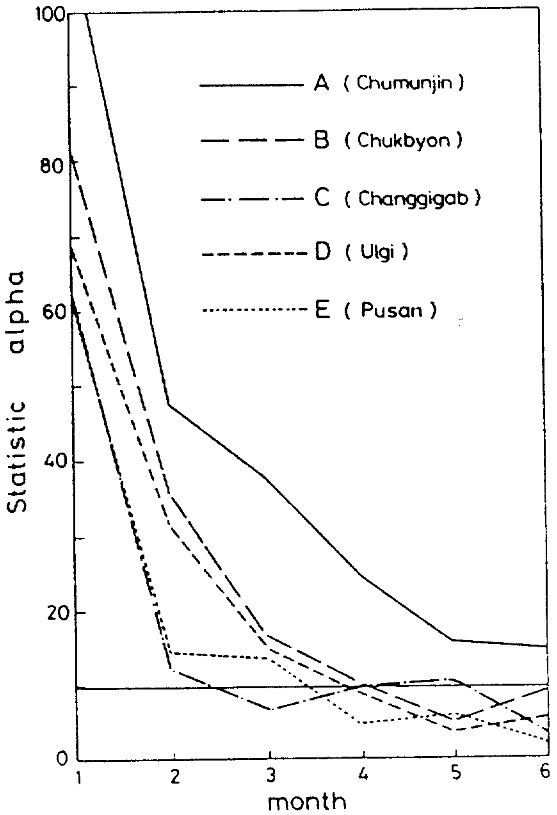

Fig. 2. The statistic of independency test, alpha in (1), as a function of step sizes for the SST anomalies at 5 coastal stations. The horizontal line represents the Chi-squared value of 9.5 at 5% level.

the tally matrix and the transition probabilities of SST anomalies for each subinterval. The result for SST anomalies at station E (Pusan), as an example, is shown in Table 3.

The statistic of stationarity β in (3) for SST

Table 3. Tally matrix and transition probabilities of SST anomalies at Pusan for 3 subintervals of 112 months each.

Subinterval	Tally Matrix			Transition Probilities		
First	5	7	0	.42	.58	.00
	7	54	14	.09	.72	.19
	0	14	10	.00	.58	.42
Second	9	11	2	.41	.50	.09
	12	53	8	.16	.73	.11
	1	8	7	.06	.50	.44
Third	13	10	0	.57	.43	.00
	9	55	9	.12	.75	.12
	1	9	5	.07	.60	.33

anomalies at Stations A, B, C, D and E are 16.6, 18.8, 10.0, 15.8 and 10.9, respectively. Since all of these values are smaller than a Chi-squared value of 21.0 at 5% level with 12 degrees of freedom, we can conclude that the Markov chain properties of SST anomalies are stationary.

The statistic γ in (4) for the spatial homogeneity of the Markov chain properties at 5 stations is 34.1. Since this value is smaller than the Chi-squared value of 36.4 at 5% level with 24 degrees of freedom, we can regard the Markov chain properties of SST anomalies in the southeastern coast of Korea to be spatially homogeneous.

DISCUSSION AND CONCLUSIONS

In this paper we showed that the monthly SST anomalies at the southeastern coast of Korea have Markov chain properties. In other words, transitions of SST anomalies are not random but depend on that of the previous month. We also showed that the Markov chain properties are stationary in time and homogeneous in space.

The Markov chain property of SST anomalies reflects that the thermal 'memory' of the previous month is important in determining the SST anomaly of the present month. Since the heat capacity of the ocean is very large, the SST anomalies depend not only on those of one month ago but also on those of a few months ago. In fact, as shown in this paper, the Markov chain property for the transitions of monthly SST anomalies extend more than 3 months at all stations, except at Station C (Changgigab) where the Markov chain property extends only two months.

The transition probabilities of SST anomalies are almost independent of stations (Table 2) and the time spans considered (Table 3). In other words, the Markov chain properties of SST anomalies are stationary and homogeneous. These features suggest that the informations on the transition probabilities can be useful in a qualitative forecasting of SST anomalies.

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