論 文

다단상호결합 네트윅을
이용한 Star의 성능분석

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Performance Analysis of Star using
Multistage Interconnection Network

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要 約   본 논문에서는 다중프로세서 시스템을 구성하는 중요한 요소인 다단상호결합 네트윅의 성능을 고찰한다. Ba-
seline 네트윅을 이용한 Star 네트윅 시스템의 Hardware 적인 구성을 고찰하며 Analytical Model 로 Request 가 받아들여
질 확률과 Clustering 확률을 구한다. 그리고 Baseline 네트윅 대신에 Delta 네트윅을 이용하여 위에서 언급한 확률을 구
한 다음 네트윅의 성능을 비교한다.

ABSTRACT   In this paper we consider the performance Analysis of multistage interconnection network, which is
major parts of multi-processor system. We review the Hardware configuration of STAR network system using base-
line interconnection network and obtain the probability of clustering basing on analytical model. In addition, Instead
of Baseline interconnection system, mentioned above, STAR network system using delta network is considered and
TWO probability mentioned above is obtained, finally the comparative result is shown in the figure.

I. INTRODUCTION

During the last decade interest has increased
in large scale Multiprocessors composed of thou-
sand of processors sharing a common memory,
and several subnetwork system, multistage inter-
connection network have been proposed [1~8]

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Multistage interconnection Network (mins) are
made of more than one stage of switching ele-
ments. Many different mins have been proposed
including the baseline [3], SW banyan [5],
Omega [6], Flip [7], indirect binary n-cube,
delta and gamma network.

They share the common characteristics in
that the order of the number of stage is LOG N,
where N is the number of processors in the net-
work.

A network which can handle all possible
connections without blocking is called a non-

blocking network. All the mins mentioned above have blocking property.

On the performance analysis, patel's [2] work is one of the earliest on the analysis of mins.

An analytical model [8] based on the Markov process is also presented for the circuit switching network.

A recursive rule is used to divide an N × N network into N/2 × N/2 networks in each stage.

This paper will consider the performance of starnetwork using baseline interconnection network and compare it with. The star-net using delta interconnection network.

We begin by reviewing the star network and the performance of the network are investigated with analytical models.

Performance is compared to each network metioned above.

II. STAR NETWORK

Star is a multiprocessor system which can be configured to emulate various Topologies which are often used in parallel algorithms.

When compared to other multiprocessor system that incorporate to either a bus or cross-bar network, Star is cost efficient when the number of processors is large.

The hardware model of star shown in figure 1 is composed of a collection of N processor nodes and a communication subnet. Each processor node is connected to B baseline networks [3] through an interface unit [IU]. By properly setting the control of the individual switching elements of the baseline networks, circuit path can be established to allow interprocessor communication.

With N processors, each baseline network is made of log N stage, and each stage consists of N/2 switching elements. Each switching element connects two input ports with two output ports in various combinations.
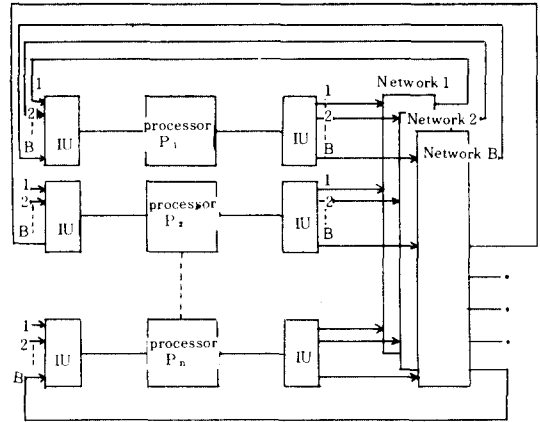

Fig 1   Harware model of star network.

III. ANALYTICAL MODEL OF STAR

Performance evaluation measures the capability of a computer system in terms of parameters that represent major characteristics of the system. The performance of star depends on both the performance of the interconnection network and the host processors. Since star is not designed specifically for a particular processor, only the performance of interconnection network will be investigated and compared.

The network to be analyzed is made of B baseline networks. Each network is numbered from 1 to B. Each baseline network connects N input nodes to N output nodes. To compare the performance of subnet made of B baseline networks whose size is N × N is also considered. The effectiveness of a network can be measured by examining how the network can realize the connection that makes up a given cluster. The analysis is based on the following three assumpttions for the multistage interconnection network. Assumption (1): At the beginning of a task, each processor generates random and independent request to network 1 with probability r.

The requests are uniformly distributed among the N receivers. Assumption (2): If more networks are available, the requests blocked in the network

(j) will be resubmitted to the network (j+1). Assumption (3): The resubmitted request is independent of the previously blocked request. In other word, the destination of the new request is newly generated.

The uniform randomness assumption (1) is used because there is no better alternative distribution. It also simplifies the analysis. There might be a better approach submitting a request to the network than the approach used in the assumption (2). However, this approach is quite reasonable and can be a good candidate in a actual system.

The baseline network is blocking network However, When more than one request is submmitted to a same destination node, only one connection can be made. To enhance the network performance, more than one baseline network can be used; although it will add more cost to the already costly baseline network. However it is important to understand the performance of much a system.

Since there is no intermediate blocking in the baseline network, exact analysis is possible using assumption (1) and (2). The following events are defined to clarify the baseline network model development.

A(ij) : Node i generates a request to node j and the request is accepted.

Sij : Node i generates a request to node j.

Bijk : Node i and k-1 other nodes generate requests to node j. In other word a total of k requests are submitted node j.

With the above definitions, if exact k nodes generate requests to node j; the probability that one of them is accepted is b/k. If k<B, them all request are accepted.

This is written as follows;

$$P|Aij| = \sum_{K=1}^{B-1} P|Bijk| + \sum_{K=B}^{N} B/K \ P|Bijk| \qquad (1-1)$$

IF B=1, equation (1-1) is simplified to

$$P|Aijk| = \sum_{K=1}^{N} 1/KP|Bijk| \qquad (1-2)$$

This case (B=1) has been solved by patel (9). The result from his works is used here with a short explanation. Let q(i) be the probability that i requests are generated for an N×N baseline network, then

$$q(i) = \binom{N}{i} r^i (1-r)^{N-1} \qquad (1-3)$$

Let E(i) be the expected number of requests accepted by an N × N baseline network, given that i requests are submitted. To evaluate E(i), consider the number of ways that i random requests can map to N distinct destination, which is N^i. Suppose now that a particular destination is not requested. Then the number of ways to map i requests to the remaining (N-1) nodes is $(N-1)^i$ Thus, $N^i - (N-1)^i$ the number of maps in which a particular destination node is always requested. Thus, the probability that a particular destination is requested is $[N^i - (N-1)^i]/N^i$. For every destination if it is requested, it means one request is accepted by the baseline network, Therefore the expected number of acceptance, given i request, is

$$E(i) = \frac{N^i - (N-1)^i}{N^i} N$$
$$= [1 - (\frac{N-1}{N})^i] N \qquad (1-4)$$

Thus the expected number of accepted requests, bandwidth (BW) is;

$$BW = \sum_{0 \le i \le N} E(i) \cdot q(i)$$

Which simplifies to

$$BW = N - N(1-r/N)^N \qquad (1-5)$$

The probability of acceptance of a single request is

$$PA\ (B=1\ )=\frac{BW}{N_r}=\frac{1}{r}[\ 1-(\ 1-r/N)^N] \tag{1-6}$$

From the definition of Aij and Sij, the probability of acceptance can be written as equation (1-7)

$$PA=P\{\ Aij\ |\ Sij\ \}=P\{\ Aij\ \}\ /\ P\{\ Sij\ \}$$

and $\quad P\{\ Sij\ \}=r/N \tag{1-7}$
$$\tag{1-8}$$

Therefore,

$$P\{\ Aij\ \}(B=1\ )=P\{\ Sij\ \}\cdot PA(B=1\ )$$

$$=\frac{1}{N}[\ 1-(\ 1-\frac{r}{N})^N] \tag{1-9}$$

The acceptance probability, When B = 1, can be obtained from (1-1) and (1-9). P [Bij1] is the probability that only one request is submitted to j and can be expanded as shown in equation (1-11).

$$P\{\ Aij\ \}(B=2\ )=P\{\ Bij\ 1\ \}+\sum_{K=2}^{N}\ 2\ /K\ P\{\ BijK\ \}$$

$$=2\sum_{K=1}^{N}\ 1\ /K\ P\{\ BijK\ \}-P\{\ Bij\ 1\ \}$$

$$=2\ P\{\ Aij\ \}(B=1\ )-P\{\ Bij\ 1\ \}$$

$$=\frac{2}{N}[\ 1-(\ 1-\frac{r}{N})^N]-r/N$$

$$-r/N\ (\ 1-r/N)^{N-1} \tag{1-10}$$

$$PA(B=2\ )=N/r\ P\{\ Aij\ \}\ B=2$$

$$=2\ /r\{\ 1-(\ 1-r/N)^N\}$$

$$-(\ 1-r/N)^{N-1} \tag{1-11}$$

Likewise, if B= 3, then from (1 − 1)

$$P\{\ Aij\ \}(B=3\ )=P\{\ Bij\ 1\ \}+P\{\ bij\ 2\ \}$$

$$+\sum_{K=3}^{N}\ 3\ /k\ P\{\ BijK\ \}$$

$$=3\sum_{K=1}^{N}\ 1\ /K\ P\{\ BijK\ \}-2\ P\{\ Bij\ 1\ \}-$$

$$-1\ /\ 2\ P\{\ Bij\ 2\ \}$$

$$=3\ /N[\ 1-(\ 1-r/N)^N]-2\ (r/N)(\ 1-r/N)^{N-1}$$

$$=1\ /\ 2\ (r/N)\ \binom{N-1}{1}\ (\ 1-r/N)^{N-2} \tag{1-12}$$

$$PA(B=3\ )=N/r\ P\{\ Aij)\ B=3$$

$$=3\ /r[\ 1-(\ 1-r\ /N)^N]$$

$$-2\ (\ 1-r\ /N)^{N-1}$$

$$=1\ /\ 2(\ r\ /N)\ (\ 1-r\ /N)^{N-2}$$

$$\cdot\ (N-1\ ) \tag{1-13}$$

This recursive relation cab be repeated with little trouble if B is small, which is reasonable in the actual system.

$$PA=N/r\ P\{\ Aij\ \} \tag{1-14}$$

The probability of acceptance is easily computed.

$$PC=PA^{Nr} \tag{1-15}$$

The probability of acceptance is obtained.

IV. PERFORMANCE ANALYSIS OF STARNET USING DELTA

A delta network is defined as on $a \times b$, switching network with n stages consisting of $a \times b$ crossbar modules. Thus, there are b^n processors connected to a^n memory dodules, we, we apply the result of eq (1-5) for a N×N baseline network to an $a \times b$ crossbar and then extend the analysis for the complete delta network.

Given the request rate r at each of the a inputs of on a × b crossbar module. The expected number of request that it passes per time unit is obtained as follows.

$$b - b \left(1 - \frac{r}{b} \right)^a \qquad (2-1)$$

and divided by b, requests on any of b output lines: that is,

$$1 - \left(1 - \frac{r}{b} \right)^a \qquad (2-2)$$

Thus for any stage of a delta network, output rate of request, r out is a function of input rate, rin, and is given by

$$r \text{ out} = 1 - \left(1 - \frac{rin}{b} \right)^a \qquad (2-3)$$

The bandwith $B(a^n, b^n)$ of an $a^n \times b^n$ delta network, given r,
The rate of requests generated by each processors.

$$B(a^n, b^n) = b^n r_n \qquad (2-4)$$

$$ri = 1 - \left(1 - \frac{r_{i-1}}{b} \right)^a, \quad ro = r \qquad (2-5)$$

Their probability of request acceptance is

$$PA = \frac{b^n r_n}{a^n r}$$
$$= \frac{b^n \left(1 - \left(1 - \frac{rin}{b} \right)^a \right)}{a^n \cdot r} \text{ and } ro = r$$
$$(2-6)$$

Now in this paper let's assume $a^n = b^n = 2^n$
then, $2^n = N$, $n = \log_2 N$
from the definition of Aij and Sij, mentioned in previous section. The probability of acceptance can be written as equation (2-7)

$$PA = P\{Aij|Sij\} = P\{Aij\} / P\{Sij\} \qquad (2-7)$$

$$\text{and} \quad P\{Sij\} = r/N \qquad (2-8)$$

Therefore,

$$P\{Aij\}(b=1) = P\{Sij\} \cdot PA(b=1)$$
$$= \frac{r_n}{N} \qquad (2-9)$$

The acceptance probability, when $b \neq 1$, can be obtained form (1-1) and (1-9) P |Bij 1| is the probability that only one request is submitted to j and can expanded as shown in equation (1-10)

$$P[Aij|b=2 = P\{Bij 1\} + \sum_{K=2}^{N} 2/K P\{BijK\}$$
$$= 2 \sum_{K=1}^{N} 1/K P\{BijK\} - P\{Bij 1\}$$
$$= 2 \frac{r_n}{N} - \frac{r}{N} \left(1 - \frac{r}{N} \right)^{N-1} \qquad (2-10)$$

Thus,

$$PA(b=2) = \frac{N}{r} P\{Aij\}(b=2)$$
$$= \frac{N}{r} \left\{ 2 \frac{r_n}{N} - \frac{r}{N} \left(1 - \frac{r}{N} \right)^{N-1} \right\}$$
$$= \frac{1}{r} \left\{ 2 r_n - r \left(1 - \frac{r}{N} \right)^{N-1} \right\}$$

Likewise, if b=3, then from (1-1)

$$P\{Aij\}(b=3) = P\{Bij 1\} + P\{Bij 2\}$$
$$+ \sum_{K=3}^{N} 3/K P\{Bij k\}$$
$$= 3 \sum_{K=1}^{N} 1/K P[Bijk] - 2 P\{Bij 1\}$$
$$- 1/2 P\{Bij 2\}$$
$$= 3 \frac{r_n}{N} - 2 \left(\frac{r}{N} \right) \left(1 - \frac{r}{N} \right)^{N-1}$$
$$- \frac{1}{2} \left(\frac{r}{N} \right)^2 \binom{N-1}{1} \left(1 - \frac{r}{N} \right)^{N-2}$$

$$PA(b=3) = \frac{N}{r} P(Aij)(b=3)$$

$$= \frac{N}{r} \{ 3 \frac{r_n}{N} - 2 (\frac{r}{N}) (1 - \frac{r}{N})^{N-1}$$

$$- \frac{1}{2} (\frac{r}{N})^2 \binom{N-1}{1} (1 - \frac{r}{N})^{N-2}$$

$$= \frac{1}{r} \{ 3 r_n - 2 r (1 - \frac{r}{N})^{N-1}$$

$$- r/2 (\frac{r}{N}) \binom{N-1}{1} (1 - \frac{r}{N})^{N-2}$$

$$(2-11)$$

This recursive relation can be expected with little trouble if b is small, which is reasonable in the actual system.

$$PA = N / r \ P|Aij| \qquad (2-12)$$

The probability of acceptance is obtained and the probability of clustering is easily computed.

$$PC = PA^{Nr} \qquad (2-13)$$

Because the MIN model uses one unrealistic assumption, the result obtained from model should be cautiously interpreted. In Fig 2, the probability of acceptance is plotted against the
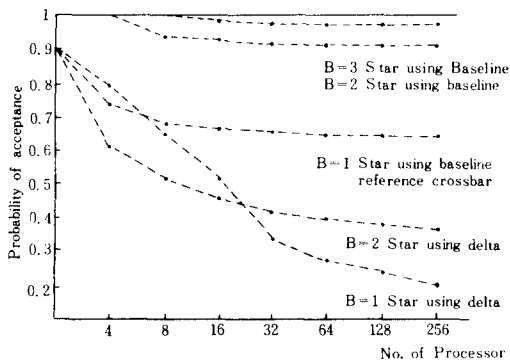

Fig 2   Probability of acceptance for two Network.

number of processors for r=1.0. It can be seen that the probability of acceptance for STAR using baseline improves rapidly when B increases. Another important point from this figure is that when N increase the probability of acceptance for STAR using baseline varies constantly with
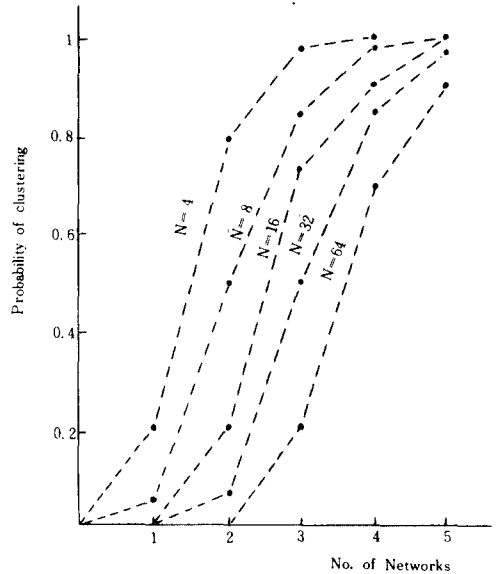

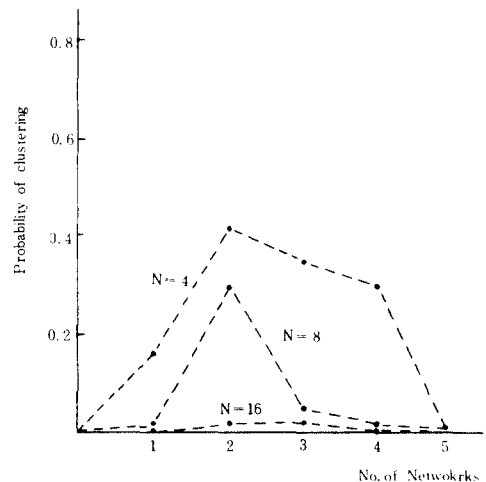Fig 3   Probability of clustering for Star using baseline.

Fig 4   Probability of clustering for Star using delta.

N > 8. From this result, it can be concluded that one network is not enough if most processors generate request at random pattern. For STAR using delta, when B increases the probability of acceptance of improves a little but when N increases the probability of acceptance falls down it indicates that this analysis is too pessimistic and not available for multiple circuit switching network. In Fig 3, the probability of clustering obtained from the analytical model is plotted against the number of network for STAR using baseline with r=1.00. Fig 4 shows the same results obtained from STAR using delta. These two figures show that for STAR using baseline more than one network should be used to enhance the performance of network. But for STAR using delta, it is not good for clustering. Analytical model of star using baseline and delta are shown, those models are useful tools to compare the relative effectiveness of each network if actual system generates similar request pattern.

V. CONCLUSION

The performance evaluation measures the capability of computer system in terms of parameters that represent major characteristics of the system. The performance of network depends on both the performance of min, and Host processors.

The performance observation indicate that probability of clustering for starnet using Baseline interconnection network is available than that of starnet using delta inter-connection network.

The principle shown in fig 2 and fig 3 give us the result that, to enhance the performance of network, more than one network should be used for starnet using baseline network, and for starnet using delta network it is not available for multiple circuit switching network. As shown in fig. 2, crossbar network is used as a reference network and has a constant value when N>16. The proba-

bility of acceptance for star-net using baseline network approaches to approximately 1 when B and N increase. But the probability of acceptance for star-net using delta-network falls down when B and N increase. This result will be good guide for network designer.

參 考 文 獻

(1) T. Feng, "A survey of Interconnection Network", computer, vol. 14, no, 12 DEC, 1981

(2) J. H. Patel, "Performance of Processor memory interconnection for multiprocessor," IEEE Trans, comput., vol, C - 30 OCT, 1981.

(3) C. L WU and T. Y. Feng, "On a class of multistage interconnection network." IEEE Trans, comput., vol C-29 Aug. 1980.

(4) S. Thanawastien and V. P. Nelson, "Interference analysis of shuttle / exchange network." IEEE Trans, comput., vol. C-30 Aug. 1981.

(5) L. K. Goke and G. J. Lipovski, "Banyan Networks for partitianing multiprocessing systems," First Annual computer Architecture conference, Dec. 1973.

(6) D. H Lawrie, "Access and Aligment of data in an Array processor," IEEE Trans, computers, vol. C-24, no. 12, Dec, 1975

(7) K. E. Batcher, "Ther Flip Network in STARAN", 1976 International Conference an parallel Processing, Aug, 1976.

(8) C. Wu and T. Feng, and M, Lin, "STAR: A local network system for real-time management of Imagery Data", IEEE Trans, computers, vol, C-31, no, 10, OCT. Oct. 1982.

(9) C. Wu al., "Prototype of star Architecture·A status Report", Proceeding of the national computer conference, 1985.

(10) C. Wu and T. feng, Tutorial : Interconnection Networks for parallel and Distributed processing, IEEE computer society, 1984.

(11) M. C. Pease, "The Indirect Binary n-cube Micorprocessor Array, " IEEE Trans Computer, Vol. C-26, 15, May, 1977.

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