A STRESS-STRAIN THEORY FOR COMPACTED ROCKFILL

다짐된 砂礫材料의 應力—變形 理論

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ABSTRACT

Based on observation emerged from the undrained tests and the anisotropic consolidation tests, an incremental stress-strain theory for rockfill is proposed in a manner similar to that developed by Cambridge Group for normally consolidated soils; the volumetric strain due to stress increment is the same as the increment due to an undrained component followed by an increment along the constant stress ratio path. The strains in drained tests are predicted from those in the undrained tests and in the anisotropic consolidation tests. An expression for the undrained stress path is derived based on the bilinear relationship between the pore pressure developed and the stress ratio observed during undrained tests. Good agreement is found between the calculated and measured strains. This trend in behaviour would be helpful in establishing a stress-strain model for rockfill using the elasto-plastic behaviour with the concept of plastic potentials and flow rules.

1. Introduction

To date, numerous stress-strain theories have been developed, which in general concentrate on relatively loose and/or soft soils. However, only a few stress-strain theories for compacted dense material like rockfill were proposed due to the difficulty in formulating the material properties in a mathematical form. The compaction energy applied to the rockfill material in the field and laboratory may cause a somewhat complicated stress history. Due to the large particle size of the material in a real rockfill structure, it is almost impossible to simulate the prototype behaviour in the laboratory. Assuming that modelling with parallel gradation does not affect the results of the tests, the method of sample preparation is seen to play an important role in the stress-strain behaviour.

A comprehensive set of laboratory investigations was carried out to study the general behaviour of rockfill under various stress conditions.[1] Three different apparatus such as the large-scaled oedometer(ϕ730mm), the large triaxial apparatus. (ϕ300mm) and the conventional triaxial apparatus

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(ϕ100mm) were employed.

The index properties of the crushed rockfill (Graywacke origin) indicates that the material is relatively sound, homogeneous and subangular.

In this paper, a stress-strain theory for rockfill is proposed based on the concept of ROSCOE & POOROOSHASB[2]. The application of the incremental stress-strain theory in rockfills seems more complex since the undrained stress path cannot be normalized with respect to the pre-shear consolidation stress. Parameters estimated from experimental observations under undrained condition can be adopted to formulate the equation of undrained stress paths. The proposed model is derived from the test results(both undrained and anisotropic consolidation tests) for the triaxial specimens having the specimen diameter of 100mm, the maximum particle size of 13mm and the uniformity coefficient of 10.

2. Definitions

The stress parameters p and q are defined by:

$$p=(\sigma_1'+2\sigma_3')/3$$

$$q=(\sigma_1'-\sigma_3')$$

where σ_1', σ_2' and σ_3' are the principal effective compressive stresses, and $\sigma_2'=\sigma_3'$ under the triaxial stress system. Similarly the incremental strain parameters dv and $d\varepsilon$ are given by:

$$dv=d\varepsilon_1+2d\varepsilon_3$$

$$d\varepsilon=2(d\varepsilon_1-d\varepsilon_3)/3$$

where $d\varepsilon_1$, $d\varepsilon_2$ and $d\varepsilon_3$ are the principal incremental compressive strains and $d\varepsilon_2$ is equal to $d\varepsilon_3$ under the triaxial stress system. The stress ratio, η, is equal to q/p.

3. Incremental Stress-Strin Theory

3.1 General

An incremental stress-strain theory for normally consolidated clay was proposed by ROSCOE & POOROOSHASB[2] of the form:

$$d\varepsilon_1=(d\varepsilon_1/d\eta)_v d\eta+(d\varepsilon_1/dv)_\eta dv \quad (1)$$

where $(d\varepsilon_1/d\eta)_v$ corresponds to the variation of $d\varepsilon_1$ with $d\eta$ in an undrained test and $(d\varepsilon_1/dv)_\eta$ represents the variation of $d\varepsilon_1$ with dv in a constant-η stress path. dv and $d\varepsilon_1$ are the incremental volumetric and axial strains. Equation (1) can be presented in a slightly different form as:

$$(d\varepsilon)_{drained}=(d\varepsilon)_{undrained}+(d\varepsilon/dv)_\eta dv \quad (2)$$

This equation was derived based on the assumption that:

(a) the undrained stress path can be normalized with respect to the pre-shear consolidation stress (p_0). Therefore, the shear strain in an undrained test is only a function of the stress ratio.

(b) the slope $(d\varepsilon/dv)_\eta$ in the (v, ε) plane during anisotropic consolidation (constant-η) tests is only a function of the stress ratio.

(c) the volumetric strain(v) is a function of η and p throughout the state boundary surface.

However, the undrained stress path of rockfill cannot be normalized with respect to p_0[1]. The shape of the undrained stress path changed continuously with changes in p_0. Therefore in the derivation of the incremental stress-strain theory for rockfill, the shear strain during an undrained test would be formulated as a function of η and p_0. The strain increment ratio, $(d\varepsilon/dv)_u$, can be obtained from anisotropic consolidation tests.

3.2 Equation of the Undrained Stress Path

It is a normal practice in soil mechanics to use the equation of the undrained stress path as a volumetric yield locus or work-hardening yield cap. But in most of the models the volumetric yield locus is represented as a function of an ellipse[3], a hyperbola[4] or a parabola[5],[6]. This is due to the difficulty in formulating the actual form of a hardening cap. In this regard it is essential to obtain the equation of undrained stress path using some fundamental parameters similar to those used in the theories developed by ROSCOE & BURLAND[7] and ROSCOE et al[8].

As presented in Fig. 1 the shape of the undrained stress path for rockfills (at different consolidation stresses) appears to be somewhat similar, but cannot be normalized with respect to the pre-shear consolidation stress. Fig. 2 shows a typical undrained stress path and the relationship between the stress ratio (η) and the pore pressure(u). It reveals that the relationship between

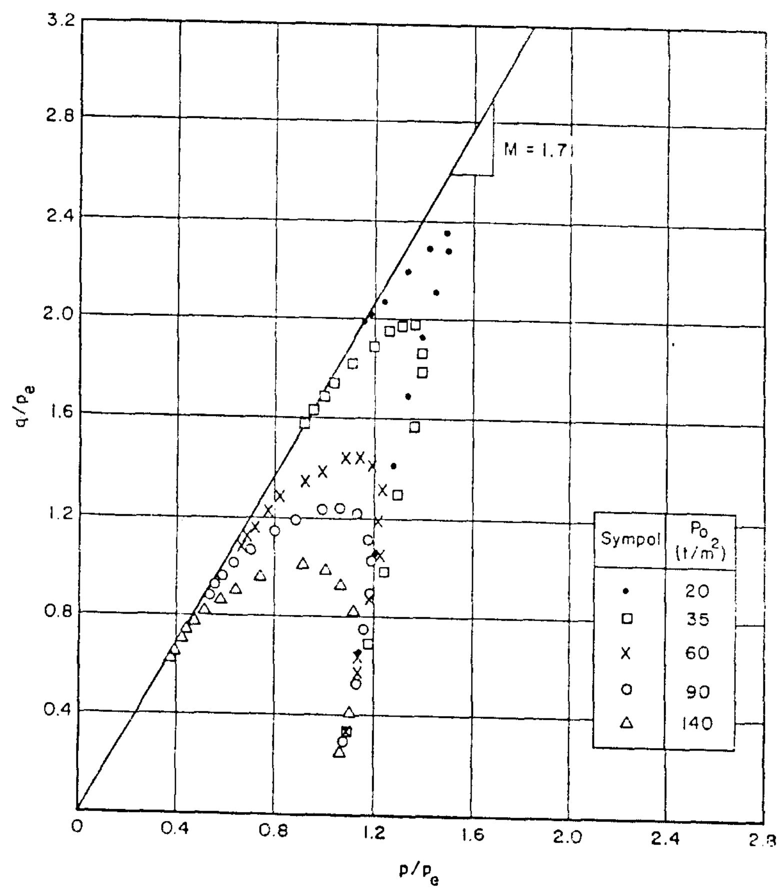

Fig. 1 Normalized Stress Paths with respect to p_0 during Undrained Shear Tests

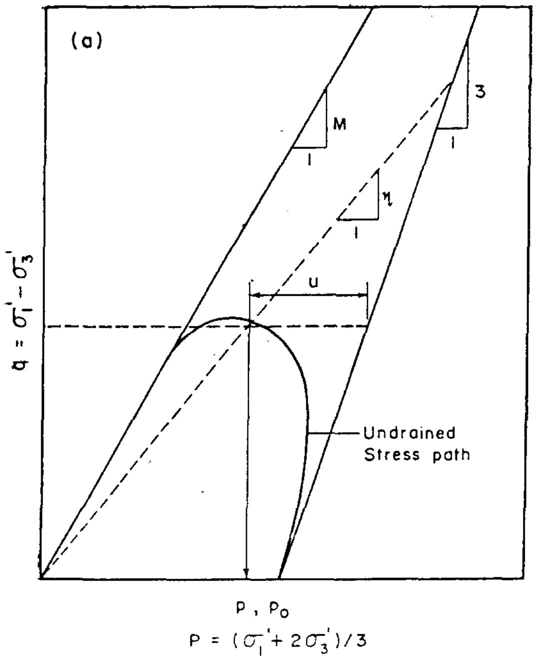

Fig. 2(a) Typical Shape of Undrained Stress Path

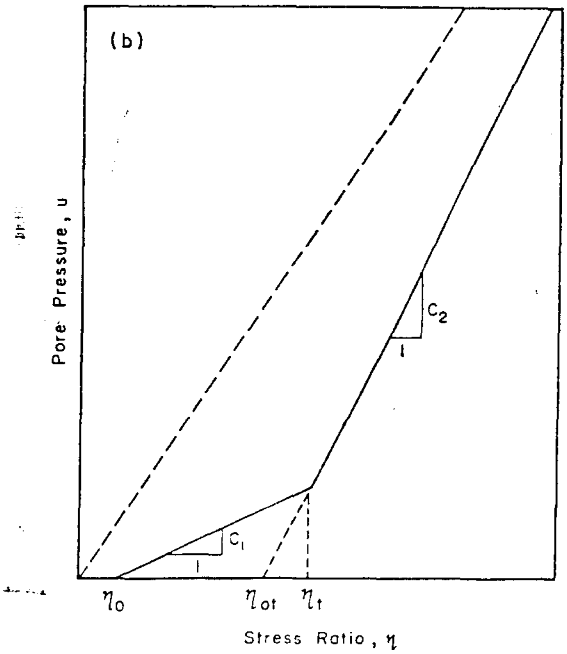

Fig. 2(b) Typical Relationship Between Pore Pressure and Stress Ratio

the pore pressure and the stress ratio is bilinear for rockfills. It is then possible to formulate the undrained stress path of rockfill by combining the relations in Figs. 2(a) and 2(b).

If we assume that the pore pressure has a linear relationship with the stress ratio (passing through the origin as shown in Fig. 2b), the following equations can be derived:

$$u=p_0+q/3-p$$
$$u=C_i\eta \qquad (3)$$

where, u=pore pressure

p_0=pre-shear consolidation stress

C_i=a material constant for a given p_0

From Eq. (3) the mean effective stress(p) of the undrained stress path can be expressed as:

$$p=3(p_0-C_i\eta)/(3-\eta) \qquad (4)$$

However, the $(u-\eta)$ relationship for rockfill is bilinear as shown in Fig. 3, which implies that there is a transition point. The stress ratio at the transition point is named as the 'transition stress ratio (η_t)'. It also appears that the pore pressure remains initially zero up to a certain value of the stress ratio. The explicit expression of Eq. (4) for the zone before and after the transition can be expressed as:

$$\text{For } \eta\leq\eta_t: p=3\{p_0-C_i(\eta-\eta_0))/(3-\eta) \qquad (5)$$
$$\text{For } \eta>\eta_t: p=3\{p_0-C_2(\eta-\eta_{0t})\}/(3-\eta) \qquad (6)$$

where, $C_1=du/d\eta$ before transition

$C_2=du/d\eta$ after transition

η_0=intercept on $\eta-$axis of the $(u-\eta)$ line before transition

η_{0t}=intercept on $\eta-$axis of the $(u-\eta)$ line after transition

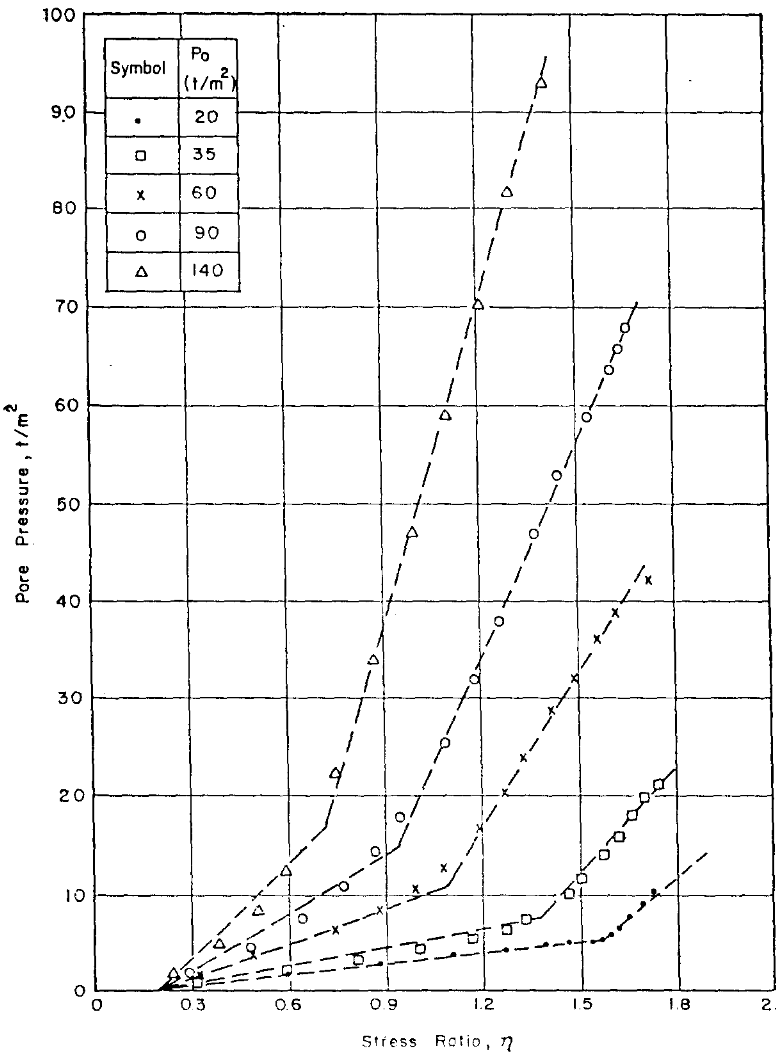

Fig. 3 Relationships Between Pore Pressure and Stress Ratio of Rockfill

Totally five parameters(C_1, C_2, η_0, η_{0t} and η_t)are required for the estimation of the mean effective stress(p) at any stress ratio. Then the deviator stress (q) can be accordingly obtained by using the relationship of $q=p\eta$.

3.3 Shear Strain during Undrained Tests

Figure 4 shows the relationship between the undrained shear strain and the stress ratio at different pre-shear consolidation stress (p_0). The nonlinear stress ratio-strain curve can be approximated by a hyperbola, with a high degree of accuracy (up to a shear strain of 7.0%) as shown in Fig. 5. For a given pre-shear consolidation stress (p_0) the transformed hyperbolic plot of (ε/η) against ε is a straight line, which can be expressed as:

$$\varepsilon/\eta=a_i+b_i\varepsilon \qquad (7)$$

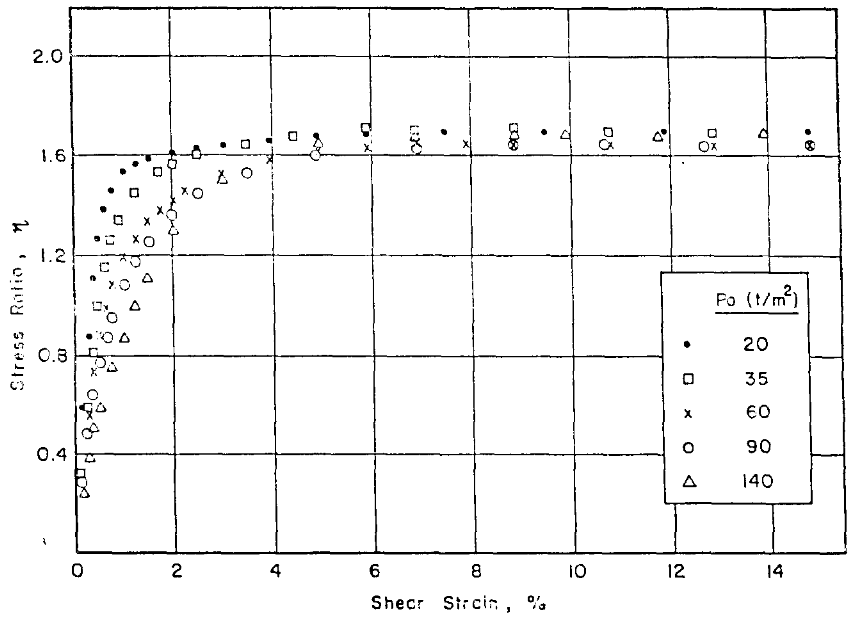

Fig. 4 ($\eta-\varepsilon$) Relationships of Rockfill during Undrained Shear Tests

where 'a_i' and 'b_i' are the intercept and the slope of the resulting straight line respectively. DUNCAN & CHANG[9] also made use of the hyperbolic equation proposed by KONDNER[10] in order to simulate the stress-strain relationships of soils fitting into the finite element program. They used the deviator stress, $(\sigma_1-\sigma_3)$, instead of the stress ratio(η).

Then the differental form of Eq. (7) can be presented as:

$$d\varepsilon=\frac{a_i(1-b_i\eta)d\eta+a_ib_i\eta d\eta}{(1-b_i\eta)^2} \tag{8}$$

It is noted here that the parameters 'a_i' and 'b_i' are dependent on the pre-shear consolidation stresses; as p_0 increases, 'a_i' increases while 'b_i' decreases.

3.4 Strains during Anisotropic Consolidation

(1) Successive Shifting of the Undrained Stress Path

The equation of the undrained stress path was formulated at a given pre-shear consolidation stress in the previous section. However, the undrained stress path keeps on changing in shape and position in the stress space as p_0 changes. Figure 6 illustrates the successive shifting of the undrained stress path. Thus the stress increment $dp_{a,b}$, followed by the undrained stress path (stress point 'a' to 'b' in Fig. 6) is dependent on the initial consolidation stress. It is assumed here that for a given stress state, (η, p), the rockfill remembers the initial stress ratio and the preshear consolidation stress.

The shifted value of p_0 corresponding to the stress state(η, p)should be evaluated to determine the stress increment for any stress ratio.

Rewriting Eq. (5) or (6) for the undrained stress path, and by substituting the expression of 'C_i' as a function of p_0 we get:

$$K_ip_a^{(1-n)}(\eta-\eta_{0i})p_0^n-p_0+p(3-\eta)/3=0 \tag{9}$$

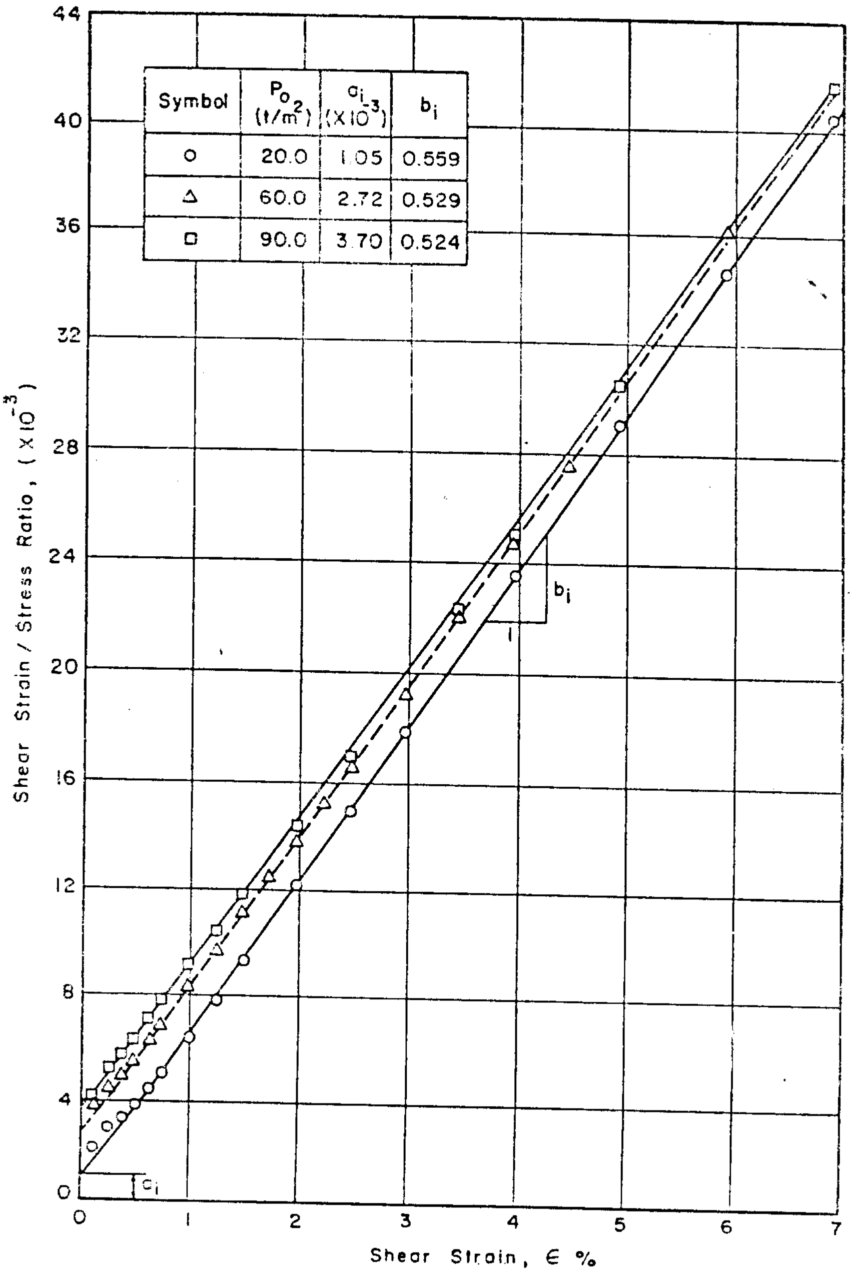

Fig. 5 Transformed Hyperbolic Plots of ε/η against ε Measured from Undrained Tests of Rockfill

where, K_i and n are the intercept and slope of the straight line in the (C_i/p_a vs p_0/p_a) plot respectively.

To solve Eq. (9) for the shifted value of p_0, a numerical technique (Newton-Raphson's iteration method for this model) should be adopted. The calculated value of p_0 from Eq. (9) is more or less similar to the 'mean equivalent stress, p_e'[7]. It is seen in Fig. 6 that the shape of the

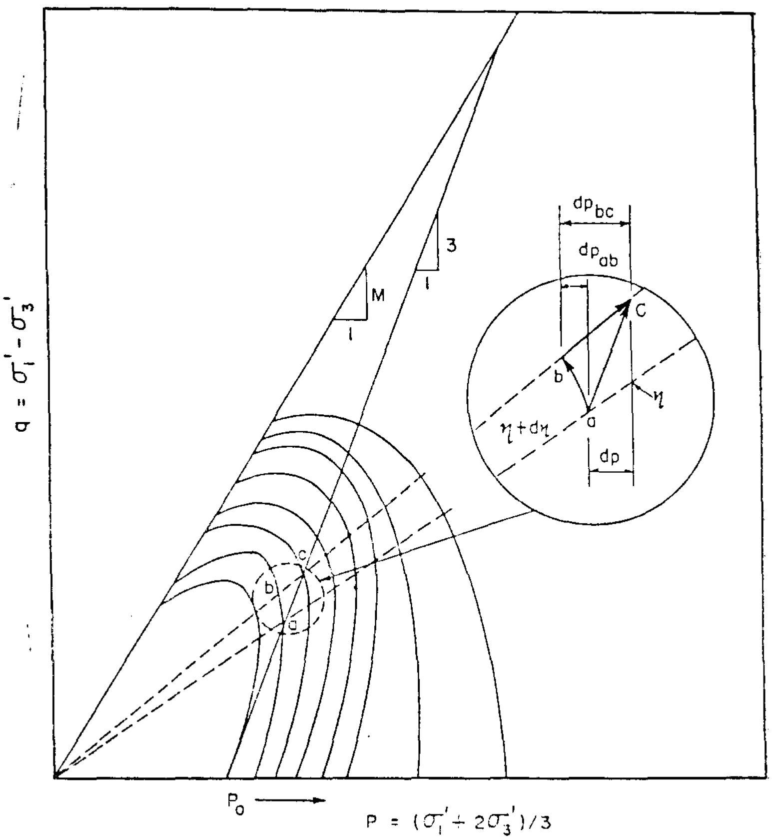

Fig. 6 Successive Shifting of Undrained Stress Paths of Rockfill with Changes in p_0

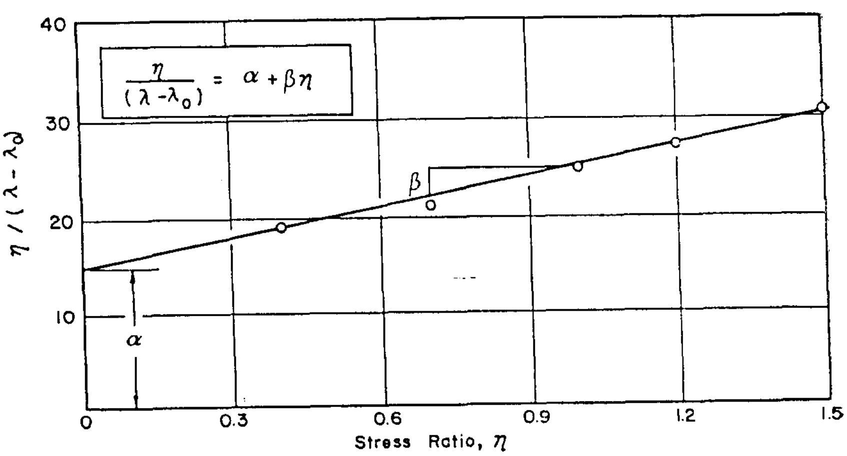

Fig. 7 Transformed Plot of $\eta/(\lambda-\lambda_0)$ against η of Rockfill During Anisotropic Consolidation

undrained stress path is not unique but depends on the stress history of the material.

(2) Volumetric Strain Increments in Constant-η Path

The magnitude of the volumetric strain increment followed by a constant-η stress path can be determined based on the following:

(a) the shape of the undrained stress path is known.

(b) the successive shifting of the undrained stress path (commencing from the same initial stress ratio, but different pre-shear consolidation stresses) can be formulated.

(c) the slope of the (e-ln p) curves (λ) for a series of anistropic consolidation tests at a constant-η varies as a funciton of the stress ratio. Thus the volumetric strain caused by an increment in p at a particular constant-η path is given by:

$$(dv)_{\eta}=\frac{-\lambda dp}{p(1+e)} \tag{10}$$

The variation of the λ-value as a function of stress ratio is presented in Fig. 7. These data provide sufficient information for the determination of the total volume change caused by a

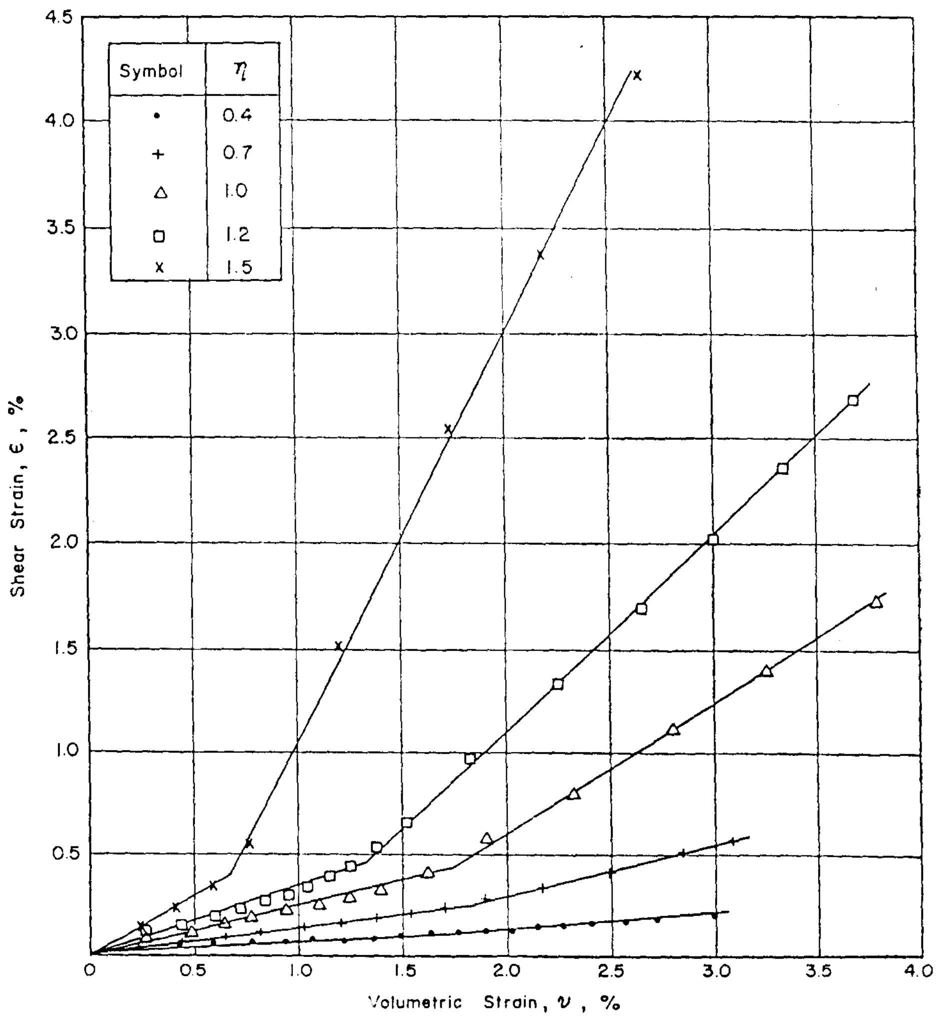

Fig. 8 Strain Paths During Anisotropic Consolidation of Rockfill

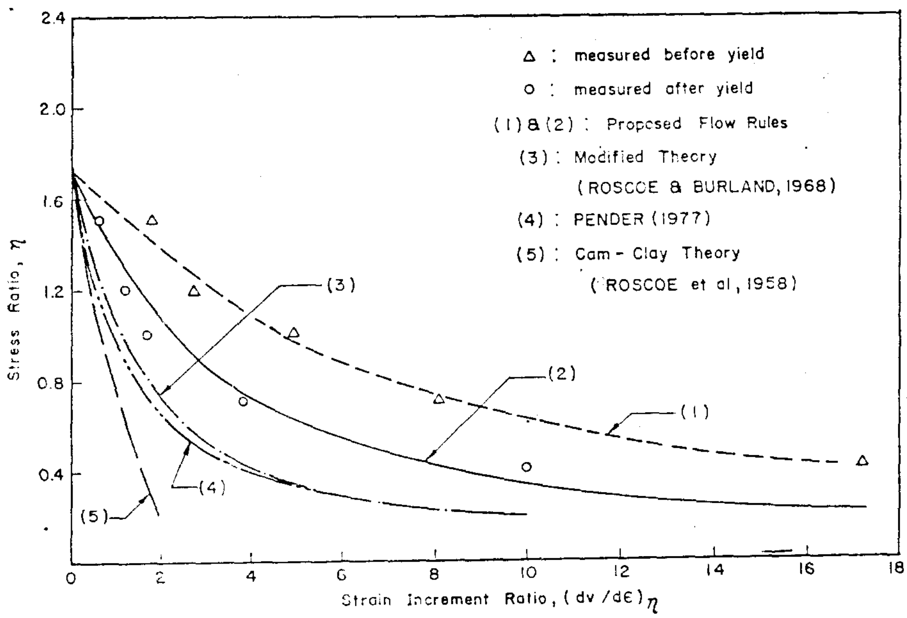

Fig. 9 Comparison Between Measured Strain Increment Ratio and Various Flow Rules

general stress increment along the constant-η path. Thus it is possible to adopt a concept similar to that of ROSCOE & POOROOSHASB[2]; the volumetric strain due to a stress increment $(d\eta, dp)$ is the same as the increment due to an undrained component followed by an increment in the constant-η path. In Fig. 6, the increment in p for the stress path from 'a' to 'c' is dp. The dp can be divided into two components; the stress increment(or decrement)due to undrained stress path (dp_{ab}) and that due to constant-η stress path (dp_{bc}). This can be written as:

$$dp=dp_{ab}+dp_{bc} \tag{11}$$

Then a differential form of Eq. (4) is given by:

$$\frac{d\eta}{dp}=\frac{C_n(3-\eta)^2}{3p_0(C_n+\eta_{0t}-3)} \tag{12}$$

where C_n is the non-dimensional parameter of (p_0/C_i) and is a constant at a given p_0. The stress increment 'dp_{ab}' due to an undrained stress path from 'a' to 'b' can be directly obtained from Eq. (12), of the form:

$$dp_{ab}=\frac{3p_0(C_n+\eta_{0t}-3)d\eta}{C_n(3-\eta)^2} \tag{13}$$

Substituting Eq. (13) into Eq. (11) and combining with Eq. (10), the volumetric strain increment caused by the stress increment from 'b' to 'c' will be given by:

$$(dv)_{bc}=\frac{\lambda dp}{p(1+e)}-\frac{3\lambda p_0(C_n+\eta_{0t}-3)d\eta}{p(1+e)C_n(3-\eta)^2} \tag{14}$$

(3) Shear Strain Increments in Constant-η Path

The experimental observations from a series of five anisotropic consolidation tests give strain paths as shown in Fig. 8. The constant stress ratios used in this investigation were 0.4, 0.7, 1.0, 1.2 and 1.5. This figure shows that the slope $(d\varepsilon/dv)$, observed during anisotropic consolidation (constant-η path) is only a function of the stress ratio. The strain increment ratios, $(dv/d\varepsilon)_\eta$, estimated from Fig. 8 are reproduced in Fig. 9 as a function of the stress ratio. But it should be recalled that the strain paths in (ε, v) plane are bilinear, passing through the false origin for all stress ratios. This trend is somewhat similar to the volumetric yield locus caused by an apparent overconsolidation. This figure gives the two curves in $(\eta, (dv/d\varepsilon)_\eta)$ plane; one is for stress state inside the yield locus while the other is for stress state outside the yield locus. The relationship between η and $(dv/d\varepsilon)_\eta$ for these two curves can be modelled by fitting into the actual test data in a similar form to that derived in the Modified Theory as follows:

For stress state inside the yield locus (Curve 1 in Fig. 9).

$$(dv/d\varepsilon)_\eta = 2.30(M^2-\eta^2)/\eta \tag{15}$$

For stress state outside the yield locus (Curve 2 in Fig. 9).

$$(dv/d\varepsilon)_\eta = 1.13(M^2-\eta^2)/\eta \tag{16}$$

In the same figure the flow rules associated with different theories are also compared; these are the Modified Theory[7], the Pender's model[5] and the Cam-Clay (or Granta Gravel) Theory[8]. The flow rule adopted in the Pender's Model gives a nearly similar figure as the one derived from the Modified Theory, but both flow rules seem to deviate from the actual curves for rockfills. The strain increment ratio, $(dv/d\varepsilon)_\eta$, at any particular stress ratio is underestimated by the flow rule in the Modified Theory. The flow rule in the Granta-Gravel (or Cam-Clay) Theory is not realistic to simulate the rockfill behaviour. The incremental shear strain during anisotropic consolidation (along the constant-η stress path) can thus be evaluated from Eqs. (15) and (16).

Then dv and $d\varepsilon$ for the drained stress paths can be determined from the following relationships:

$$(dv)_{ac} = (dv)_{ab} + (dv)_{bc} = (dv)_{bc} \tag{17}$$

$$(d\varepsilon)_{ac} = (d\varepsilon)_{ab} + (d\varepsilon)_{bc} \tag{18}$$

4. Determination of Model Parameters

The parameters required in the proposed model are summarized in Table 1. The type of tests to obtain the specific parameters is also described.

Table1. Summary of Procedures to obtain Basic Parameters

Model Parameters	To be obtained from
λ	slopes of the (e-ln p) plot from isotropic and anisotropic consolidation tests
κ	slope of the (e-ln p) line during isotropic swelling tests
C_i	slopes of the pore pressure vs, stress ratio plot before and after transition respectively
η_t	stress ratio at the transition point in pore pressure vs. stress ratio plot from the CIU tests
η_0 & η_{0t}	intercept of the (u-η) line at η-axis before and after transition respectively
$M(\eta_p)$	peak stress ratio
a_i & b_i	intercept and slope of the transformed hyperbolic plots between (ε/η) and ε from CIU tests

5. Comparison between Experimental and Predicted Values

The drained stress-strain behaviour followed during conventional triaxial tests (at two confining stresses of 20 and 140t/m^2) are compared with the model predictions in Figs. 10 and 11.

In general, the prediicted values are in good agreement with the experimental observations. On the other hand the volumetric strains are more or less underestimated and the disagreement becomes obvious as the shear strain increases. Also the deviation is larger with an increase in the pre-shear consolidation stress, whereas the general trends are within a reasonable degree of accuracy. It can be concluded, therefore, that the incremental stress-strain theory can be used to calculate the strains of compacted rockfills sheared under drained conditions.

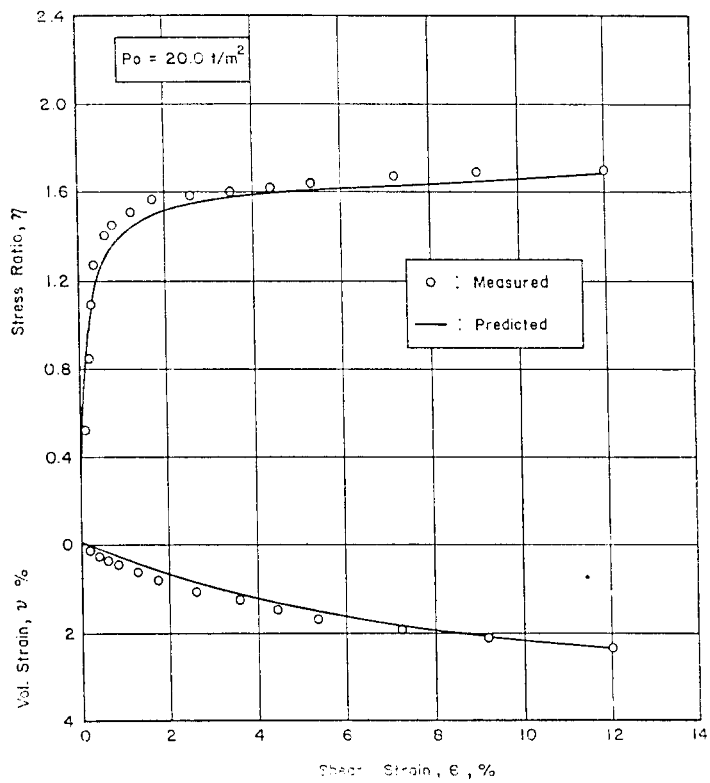

Fig. 10. Measured and Predicted Strains of Rockfill During Triaxial Drained Test (p_0=20.0t/m^2)

6. Conclusion

An incremental stress-strain theory for rockfill is proposed within the framework of the theories developed at Cambridge[2),7)] ; the volumetric strain due to a stress increment ($d\eta, dp$) is the same as the increment due to an undrained component followed by an increment with a constant-η path. The main modifications that are necessary for the application of this theory for rockfill compared to normally consolidated clays are:

(a) The undrained stress path keeps on changing in shape and position in the stress space as the pre-shear consolidation stress increases. Thus this variation needs to be accomodated in the incremental stress-strain theory.

(b) The shear strain during undrained tests is dependent both on the stress ratio and the mean normal stress.

(c) The parameter, λ, is not a constant for rockfill. λ-values depend on the stress ratio and increase with an increase in the stress ratio.

Based on the relationship between the pore pressure and the stress ratio, the equation of the undrained stress path is successfully formulated in the author's theory to account for change in shape and in position.

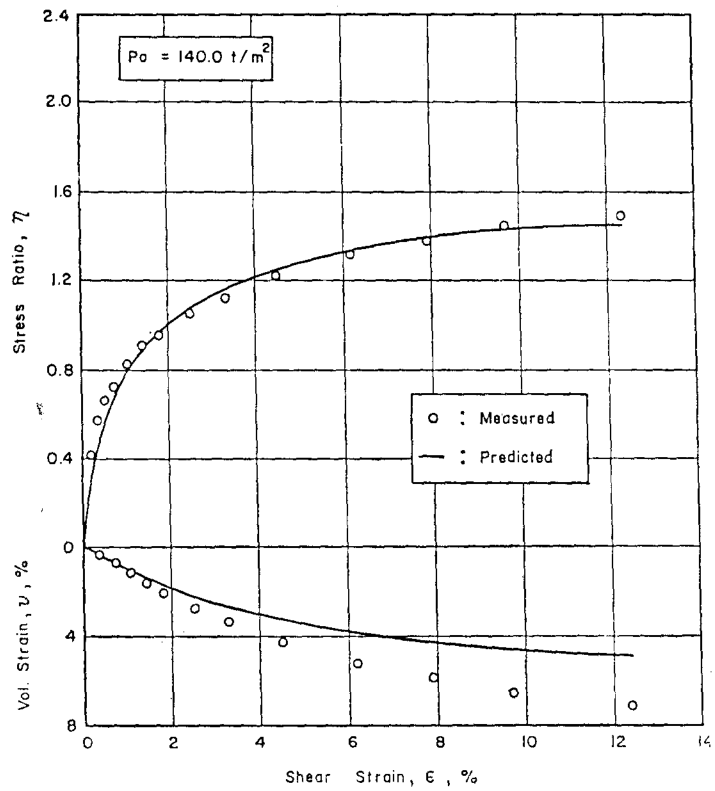

Fig. 11. Measured and Predicted Strains of Rockfill During Triaxial Drained Test ($p_0=140.0\text{t/m}^2$)

In general the predicted values are in good agreement with the experimental observations. The volumetric strains are somewhat underestimated. The deviation of the volumetric strains becomes obvious with an increase in p_0, whereas the general trend is within a reasonable degree of accuracy.

It should be noted, however, that the shape of undrained stress path of rockfill is strongly influenced by the maximum particle size as well as the stress level. Hence the application of this proposed theory is limited only to the given conditions of the maximum particle size and the stress level. Further development of this theory is recommended with the help of exclusive testing program.

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要 旨

흙의 應力一變形 關係를 解析할 수 있는 理論들이 캠브리지그룹에 依해 여러가지 形態로 提案되었다. 그러나 흙이 彈塑性體라고 假定한 이 理論들은 단지 正規壓密土의 特性만을 考慮한 것이어서 過密土에 適用하는 데는 많은 制約條件이 따른다. 따라서 本 硏究에서는 다짐된 砂礫材料의 應力一變形 理論을 提案하였으며 그 展開過程은 正規壓密土에 대한 理論과 恰似하나 過壓密土의 特性을 함께 考慮하였다. 특히 非排水 三軸壓縮時의 有效應力經路는 測定된 間隙水壓과 應力比의 關係式에서 誘導되었다.

砂礫材料의 非排水 三軸壓縮實驗과 非等方 壓縮實驗에서 얻어진 結果를 바탕으로 排水條件에서의 應力一變形關係를 豫測하고 이를 實測値와 比較檢討하였다. 豫測値는 實測値와 比較的 잘 一致하였고, 이 傾向은 砂礫材料의 應力一變形解析을 爲해 彈塑性概念을 導入할 수 있음을 示唆한다.

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