熱損失을 고려한 白熱電球의 설계

The Design of Incandescent Lamps considering the Heat Loss

지 철근* · 강 기호**

(Chee, Chol Kon · Kang, Ki Ho)

요 약

백열등의 POWER 평형 방정식을 통한 동작중의 필라멘트 온도와 광속 그리고 수명을 예측할 수 있는 이론이 제시되었다.

POWER 평형 방정식을 설정함으로써 백열등 시스템을 모형화 하고, 방정식을 풀므로써 예측할 수 있는 형식으로 예측이론은 구성되었다. 가정용 110V 전구에 대해 실측치와 비교해 본 결과, 충분한 타당성이 있음이 입증되었다.

본 이론의 정확한 예측성은 백열등을 해석, 이해하는 데 열쇠가 되는 제특성치를 제공할 수 있었는데 아직 생산단계에 있지 않은 크립톤가스등의 특성치도 추정할 수 있었다.

또 하나의 유용성은 백열등 설계를 위한 제 매개변수간의 관계식을 본이론으로부터 도출하여 기존의 경험에만 의존하는 설계법을 보완하고 새로운 설계시 최적설계치를 선정할 수 있다는 점에 있다.

1. Introduction

Since the light-emitting principle of incandescent lamps (abbreviated to I / L) is temperature radiation, color characteristics and color rendering of I/L are better than those of discharge lamps whose light-emitting principle is luminescence. On the other hand, its low luminous efficiency and short lamp life are its fatal weak points.

Luminous efficiency is related with the heat loss of I / L and lamp life with the mass loss of the filament. However, since the mechanism is very complicated, it is difficult to examine the relationship closely. The traditional method of design for I / L is often a lengthy process involving a series of lamp trials, which is limited by the bounds of the designers experience and intuition.

This paper describes a prediction theory of power balance equation in I / L which is able to predict the filament temperature and as a straightforward result, the luminous flux.

The theory is composed of modeling of I / L system which mainly prescribes the power balance equation, and solving the equation by numerical method.

In applying the calculation results to lamp design, a useful relationship equation is derived which enables to overcome the limit of trial and error method or modify the given lamp specification for a special purpose one.

*서울대학교 전기공학과 교수

**서울대학교 전기공학과 대학원

2. Modeling I / L System

2.1. Langmuir's sheath model

In a gas-filled incandescent lamp, temperature profile should be known to analyze the heat loss and the mass loss of filament. Langmuir has derived the profile under the assumptions that the transport of energy in the neighbourhood of the wire will be mainly by pure conduction of heat and not by convection.[1)]

Hence the temperature profile is derived as follows

$$T(r)=T_f-(T_f-T_w)\frac{\ln(2r/a)}{\ln(b/a)},\quad \left(\frac{1}{2}a<r<\frac{1}{2}b\right) \tag{1}$$

Thus the temperature curve from the wire to the wall of the lamp is as indicated in Fig. 2-1.

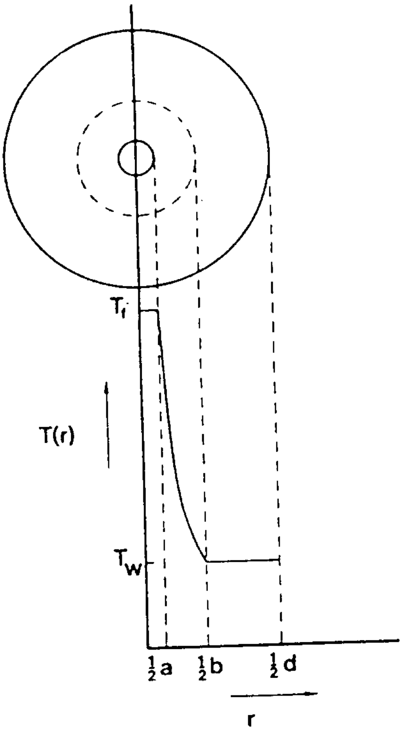


Fig.2.1. Temperature profile
(d=diameter of the wall
b=Langmuir sheath diameter
T_f=filament temperature)

2.2. Heat loss and efficacy

In a gas filled lamp, the input power is balanced by the sum of the radiated power W_{RAD}, the conducted power to the gas W_{GAS} and the conducted power to the lead and support wires, W_C.

$$W_{IN}=W_{RAD}+W_{GAS}+W_C$$

To closely examine the relationship of heat loss and efficacy (i.e. luminous efficiency) the mean filament temperature at which power balance occurs should be determined by solving the power balance equation. Once the filament temperature has been found, the light output may be calulated in a given heat loss(W_C) for a filling gas state.

The terms of the power balance equation are to be prescribed as follows.

2.2.1 The electrical input power

Assuming no resistive losses in the leadwires and negligible deformation of the filament wire in the coiling process, the electrical input power is given by:

$$W_{IN}=\frac{V^2}{R_F}=\frac{\pi V^2 r^2}{R(T_f)L}\,[W]$$

V = supply voltage
R_F = filament resistance(Ω)
r = wire radius(m)
T_f = mean filament temperature(K)
$R(T_f)$ = resistivity of wire material at T_f(Ω m)
L = length of wire in filament coil (m)

For tungsten, tabulated values of the bulk electrical resistivity versus temperature, have been reduced to a polynomial of the form:, $R(T_f)=\alpha T_f^2+\beta T_f+\gamma$ giving errors not greater than ±0.05 percent over the range 2200-3200Kwhen compared with the tabulated values.

2.2.2. The power loss to the gas filling

The heat flow in the sheath is by conduction only and outside the sheath by convection only.

Hence the heat flow per unit length per second is equal to:

$$Q=-2\pi r\lambda(\overline{T})\frac{d}{dr}T(r)$$
$$=\frac{-2\pi\lambda(\overline{T})}{\ln(b/a)}\,T_f-T_w),\quad (\overline{T}\fallingdotseq\frac{1}{2}(T_f+T_w)) \tag{2}$$

Q can be determined if b is known. To determine b, we are to be aided by General Theory of Heat Dissipation by Nusselt.[2)]

According to the theory,

$$b\ln\left(\frac{b}{a}\right)=\frac{2c\,\eta^{2/3}}{g^{1/3}\rho^{2/3}\beta^{1/3}\theta_w^{1/3}P_r^{1/3}} \tag{3}$$

where

Pr(Prantl no.) = (q + 2) / (q + 9 / 2)

g = 9.18(m . S^{-2})

q = freedom of molecule

f = density of gas (kg. m^{-3})

$\theta w = T_f - T_w$,

= viscosity of gas (N.S.m^{-2})

Here, P(density of gas)must be represented as filling pressure (atm.) Po(pressure of gas for T=To) because the pressure in the operation P is very difficult to know the distribution . However, the problem is solved, F.H.R. Almer and J.de Ridder's theory applied to it.[3] According to the theory:

$$P = T_{eff} \frac{P_o}{T_o} \qquad (4)$$

Where

$$T_{eff} = d^2 \left\{ \frac{d^2 - b^2}{T_w} + \frac{a^2}{T_f} + 8 \int_{a/2}^{b/2} \frac{rdr}{T(r)} \right\}^{-1} \qquad (5)$$

Therefore for an ideal filling gas the concentration as a function of r will be given by

$$n_2 \{T(r)\} = \frac{P}{kT(r)} = \frac{T_{eff}}{T(r)} n_2(T_o) = \frac{T_{eff} P_o}{kT_o T(r)}$$

Since $n_2 >> n_1$,

$$\rho \cong m_2 n_2 (\overline{T}) = m_2 n_2 \left(\frac{T_f + T_w}{2} \right) = \frac{2m_2 P_o T_{eff}}{kT_o (T_f + T_w)} \qquad (6)$$

Now ρ can be written as a function of Po.
Applying (6) to (3)

$$b \ln(b/a) = 2c \left\{ \frac{kT_o (T_f + T_w)}{2m_2 P_o} \eta \right\}^{2/3} \cdot \left\{ \frac{T_w}{(T_f - T_w) g Pr} \right\}^{1/3} \cdot T_{eff}^{-2/3}$$

Now diameter of Langmuir sheath b has been able to be solved numerically for a given Po, and from eqn. (2), Q (for entire length ℓ) has been calculated as follows.

$$Q = 2\pi l (T_f - T_w) \lambda(\overline{T}) / \ln(b/a) = \frac{2\pi l}{\ln(b/a)} \cdot \int_{Tw}^{Tf} \lambda(T) dT \qquad (7)$$

Derivation of the thermal conductivity (T) is not trivial, and not particularly accurate.

However, Muller[4] has shown that thermal conductivity and viscosity of certain gases can be represented inthe form of a simple exponential eqn., and this particular aspect has been studied in detail for mixtures of two gases by J.R.Coaton[5] and for mixtures of three gases by the present author in this paper.

It was found that the viscosity and thermal conductivity of the common lamp-making gases, and of a wide of mixtures containing these, can be expressed in the form.

$$\left. \begin{array}{l} \lambda(T) = D(\lambda)\ T^{N(\lambda)} \\ \eta(T) = D(\eta)\ T^{N(\eta)} \end{array} \right\} \qquad (8)$$

ղ ղ ղ ղ ղ

where D and N are constants and exponents re-

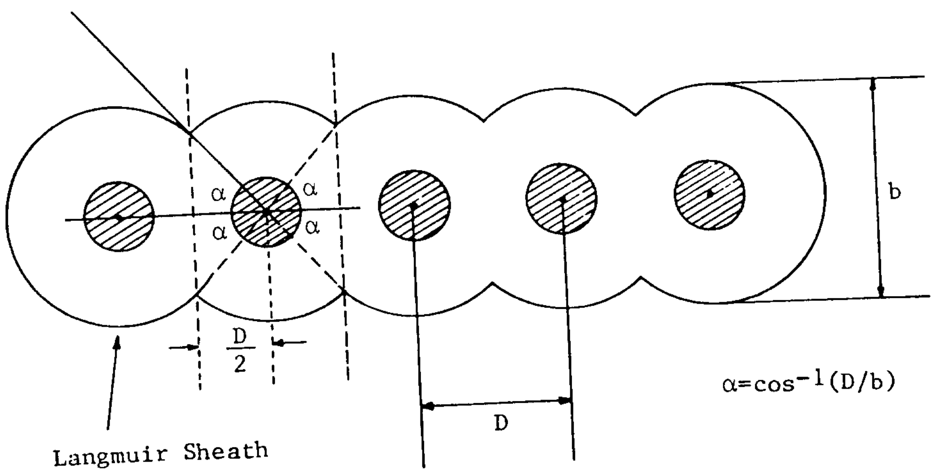


Fig.2.2. Plan view of planar filament configuration

spectively, which have a discrete value for each transport coefficient and gas (or mixture), and (T) is the viscosity at the temperature T deg K.

Now applying(8) to (7),

$$Q=\frac{2\pi l}{\ln(b/a)}\cdot\frac{D(\lambda)}{1+N(\lambda)}\cdot\left[T_f^{1+N(\lambda)}-T_w^{1+N(\lambda)}\right] \tag{9}$$

On the other hand, in cases where the Langmuir sheaths of adjacent parallel filament sections overlap, (Fig.2-2), the power loss to the gas is reduced by an amount which depends on the section spacing D and the Langmuir sheath diameter b.

For sections not at ends of the array the angle θ into which heat is directed towards a neighbouring sheath is given by

$$\theta=4\alpha=4\cos^{-1}\left(\frac{D}{b}\right) \tag{10}$$

Consequently, the power loss to the gas from these sections is reduced by the factor $(1-\theta/2\pi)$ approximately. If there are N sections, the overall gas power loss reduction factor is given by:

When

β =Power reduction factor,

$$\beta=\frac{1}{N}\left\{(N-2)\left(1-\frac{\theta}{2\pi}\right)+2\left(1-\frac{\theta}{4\pi}\right)\right\}$$
$$=1-\frac{1}{2\pi}\left(1-\frac{1}{N}\right)\cos^{-1}\left(\frac{D}{b}\right) \tag{11}$$

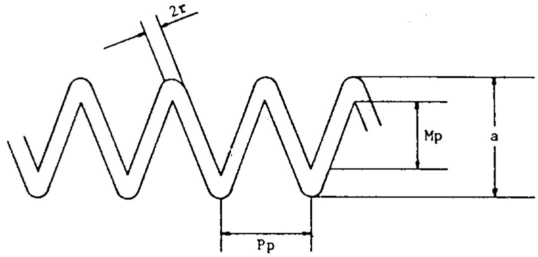
Now, the power loss to the gas filling W_{GAS} has had a complete methematical form.

$$W_{GAS}=\beta\cdot Q$$
$$=\beta\cdot\frac{2\pi l}{\ln(b/a)}\cdot\frac{D(\lambda)}{1+N(\lambda)}\cdot\left[T_f^{1+N(\lambda)}-T_w^{1+N(\lambda)}\right] \tag{12}$$

2.2.3. The radiated power-single coil filament

A useful parameter for comparing the various theories is the "power reduction factor" δ_1^1, which is the ratio of the power radiated from a coil compared with the power which would radiate at the same temperature if straightened out.

Thus, using stefan's radiation law and ignoring radiation received from the lamp envelope



Mp: PRIMARY MANDREL DIAMETER

Pp: PRIMARY COIL PITCH

Fig.2.3. Coving parameters for a single coil filament

$$W_{RAD}=\delta_1^1\,\epsilon_T(T_f)\,A_S\,\sigma\,T_f^4$$

where

$\epsilon_T(T_f)$ = total emissivity of filament material at temperature T_f K

σ = Stefan's constant $(5.67\times10^{-8}\ W^{-2}K^{-4})$

As = areas of filament wire (m²)

In the calculations for single coil filaments reported in this paper, the results of the theory of Vukcevich[5] have been selected since it gives satisfactory results in practice.

According to Vukcevich:

$$\delta_1^1=\frac{\delta_{g_1}}{1-(1-\epsilon_T(T_f))(1-\delta_{g_1})} \tag{14}$$

where

δ_{g_1} = "geometric shadow factor"

= fraction of radiation leaving coil directly without interreflections

and

$$\delta_{g_1}=\frac{1}{\pi}\left\{\tan^{-1}\sqrt{4P_1^2-1}+\tan^{-1}\sqrt{P_1^2-1}-\frac{2}{1+M_1}\cdot\sqrt{1-\frac{1}{P_1^2}}\right\}$$

where

$$P_1=\frac{P_p}{2_r}=\text{pitch ratio},$$
$$M_1=\frac{M_p}{2_r}=\text{Mandrel ratio}.$$

Tabulated values of the temperature dependence of the total emissivity of tungsten[6] have been reduced to a second order polynomial which gives values agreeing with the tabulated values to better than ±0.6 percent over the temperature range 2500-3200K:

$$\epsilon_T(T_f) = aT_f^2 + bT_f + c$$

However, when a filament is coiled, the total emissivity increases because of "black body effect", that is

$$\epsilon_T(T_f)_{coil} = \epsilon_T(T_f)_{flat} + \triangle\epsilon_T$$

where $0 < \triangle\epsilon_T < 0.3$, and $\triangle\epsilon_T$ is dependent primarily on pitch ratio of the filament.

2.2.4. The filament support and lead wire power loss

Observations of temperature gradients in leadwires of 40-200W lamps infer a power loss of approximately $0.015W_{IN} \sim 0.016\ W_{IN}$[7]. Since the conpucted heat losses are such a small part of the overall power balance equation, approximate experimental determinations are adequate. Thus, for household lamps,

$$W_C = 0.015W_{IN} \tag{15}$$

From above-mentioned, the terms of power balance eqn. has been modeled as mathematical forms, which defines a function of T_f.

$$\begin{aligned} f(T_f) &\triangleq W_{IN} - W_{GAS} - W_{RAD} - W_C \\ &= \frac{\pi V^2 r^2}{R(T_f)\cdot L} - \beta\cdot\frac{2\pi l}{\ln(b/a)}\cdot\frac{D(\lambda)}{1+N(\lambda)}\cdot \\ &\quad [T_f^{1+N(\lambda)} - T_w^{1+N(\lambda)}] \\ &\quad - \frac{\delta_{g_1}}{1-(1-\epsilon_T)(1-\delta_{g_1})}\,\epsilon_T(T_f)A_s\sigma T_f^4 \\ &\quad - 0.015\frac{\pi V^2 r^2}{R(T_f)\cdot L} \end{aligned} \tag{16}$$

If the solution which $f(T_f) = 0$ is known by a computer program, the absorbed power, the luminous and the mass loss are calculated.

3. Calculations and discussion

3.1. Derivation of model constants

3.1.1. The thermal conductivity and viscosity

The values for mixtures of three gases by the author for a mixture of nitrogen (N_2), argon (Ar) and krypton(Kr), the values can be expressed as table 3-1.

Table 3.1. D and N for mixtures of three gases

Composition (%)			Viscosity (Ns/m^2)		Thermal conductivity (mW/mdeg)	
N_2	Ar	Kr	D(η)	N(η)	D(λ)	N(λ)
10	20	70	.507253	.696555	.411748	.620330
10	30	60	.464862	.705209	.434152	.619453
10	40	50	.426161	.714578	.454221	.619764
10	60	30	.348519	.735874	.278906	.693257
10	80	10	.271616	.761651	.528865	.621693

3.2.2 Calculations of model program

Redefining equation (5) as g(b), eqn.(5) is given by g(b),

$$g(b) \triangleq b\ln(b/a) - 2c\cdot\left\{\frac{KT_o(T_f+T_w)}{2m_2\ P_o}\cdot D(\eta)\cdot\left(\frac{T_f+T_w}{2}\right)N(\eta)\right\}^{2/3}\cdot\left\{\frac{T_w}{(T_f-T_w)gP_r}\right\}^{1/3}\cdot T_{eff}^{-2/3}$$

In the vertical case, $c = 1/8.45$.

If b is known which $g(b) = 0$, T_f which $f(T_f) = 0$ also can be known by numerical method. Once T_f is known, luminous flux, efficacy and lamp life are to be known straightforwardly. Luminous flux is defined generally as follows.

$$F(T_f) \triangleq 681\cdot\int v(\lambda)\cdot\epsilon_\lambda(T_f)\cdot W_\lambda(T_f)A_s d\lambda \tag{17}$$

where

$V(\lambda)$ = eye sensitivity

$\epsilon_\lambda(T_f)$ = spectral emissivity of tungsten at T_f

$W_\lambda(T_f)$ = spectral luminous emittance $= 37415\cdot\lambda^{-5}\{\exp(14388|\lambda T_f) - 1\}\ [\omega/cm^2]$

But since the radiation body is filament coil in a I/L, luminous flux is modified by considering the coiling effect and the "black body effect". Therefore modified equation is given by

$$F(T_f) \triangleq 681\cdot\int v(\lambda)\ \epsilon_\lambda(T_f)\ \delta_1^1(\lambda)A_s\ W_\lambda(T_f)d\lambda$$

where

$\delta_1^1(\lambda)$ = "spectral power reduction factor"

$= \frac{\delta_{g_1}}{1-(1-\epsilon_\lambda(T_f))(1-\delta_{g_1})}$ = , as used by

Vukcevich $\epsilon_\lambda(T_f) = \epsilon_\lambda(T_f)$ flat tungsten $+\triangle\epsilon_\lambda$, as used by total emissivity

In the calculation of the program, $\in(Tf)$ is approximated in the form of fourth polynomial and $V(\lambda)$ is fed into the program datum by datum.

The flow chart of the algorithm is expressed as Fig 3-1. The procedure of $g(b) = 0$ is bisectional method and of $f(T_f) = 0$ is logarithmic method.

3.3. The Results and discussion

3.3.1 Accuracy check of the theory with measurement Input data for household 110V lamps are as follows.

The computed results together with average experimental data are presented in table 3-2, which are derived by a optical high-temperature meter and corresponding IES Handbook data, i.e., T_f corresponding to measured Tc(color temp.)

The results in Table 3-2 demonstrate the good predictive accuracy of the theory despite the use of many simplifying assumptions. The calculated values of luminous flux are within the limits of error of experimental measurements. Teh observed filament operating temperatures and luminous flux are, on average, slightly higher than those predicted. The discrepancy, which is of little practical consequence, may result from two causes :

i) The use of approximations in the derivation of δ_{g_1} and the adaptation of a less-accurate $\triangle\epsilon_\lambda$ and $\triangle\epsilon_T$

ii) Variations of the measured in themselves.

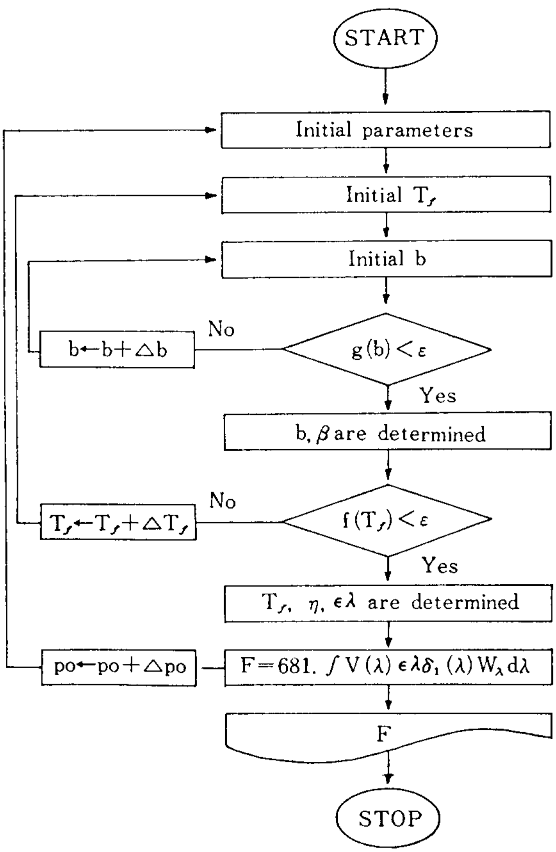


Fig.3. 1. Flow chart of $F(T_f)$

Table 3.2. Predicted and measured PO= 0.8

Lamp Rating[w]	Luminous Flux (lumen) Predicted	Luminous Flux (lumen) Experimental	Filament Temp (K) Predicted	Filament Temp (K) Experimental
40	363. 96	2570	388	2582
60	680. 89	2665	672	2676
100	1275. 00	2702	1280	2714
200	3028. 00	2804	3040	2817

＊ Average error in luminous flux prediction : -1%

＊ Average error in filament temerature prediction : -1%

3.3.2. Use of theory for the calculations with filling gas variation

Because of the very high order of accuracy attainable with the theory, we can calculate characteristic values with variation of gas filling. In figures

(1) always means that gas filling state is N_2: Ar = 1:9, and

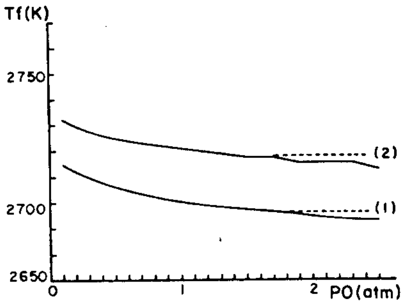
(2) always means that gas filling state is N_2 : Ar:Kr = 1:2:7.

The results say that

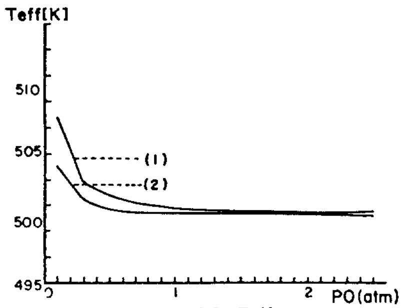
i) The filament temperature decreases slowly with the pressure of filling gas for both the(1) and (2), which simultaneously means increasing of input power and decreasing of luminous flux. (Fig. 3-2)

ii) Teff is nearly constant over the pressure of 1 atm., which means that the operation pressure is proportional to the design pressure over the pressure. (Fig. 3-3)

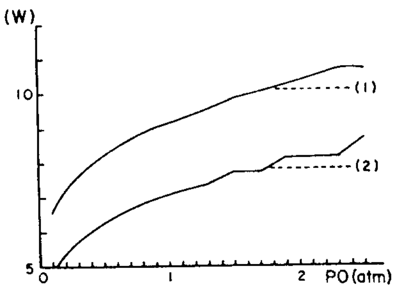
iii) The coduction power loss increases rapidly under about 2 atm., and does slowly over that with the pressure of filling gas.(Fig. 3-4)



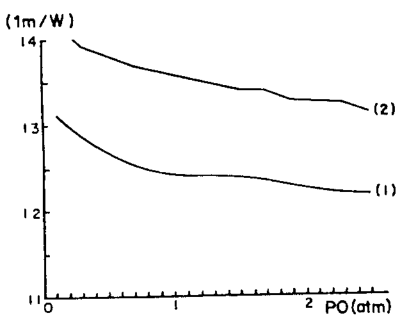
PO-Tf



PO-Teff



PO-conduction



PO-Efficacy

iv) The luminous flux decreases rapidly initially, and then does slowly over 1 atm. At 1 atm the luminous flux of (2) is greater than that of (1) by 10 percent. (Fig. 3-5)

3.5. Application to the design.

A modification to the program enables filament design exponents to be calculated automatically. For example, if the relationship between primary pitch P1 and lamp power W_{IN} for fixed coil length, wire diameter, etc. is required, it is assumed to be of the form:

$W_{IN} = \text{constant} \times p_p nl$

Calculation of is then readily derived from the theoretically predicted relationship between input power and primary pitch, all other being held constant.

Some typical results including other wattage lamps are given in Table 3-3.

Table 3.3 Calculated single coil 110 volt household lamp design exponents

Rating (W)	Pitch ratio vs. power n_1	Mandrel ratio vs. power n_2	Coil length vs. power n_3	Applied voltage vs. power n_4
40	.638	−0.450	−0.524	1.500
60	.643	−0.483	−0.559	1.539
100	.628	−0.466	−0.559	1.550
200	.637	−0.445	−0.563	1.556

Then

$$\frac{\text{Power 2}}{\text{Power 1}} = \frac{(\text{Pitch Ratio 2})^{n1}}{(\text{Pitch Ratio 1})} \times \frac{(\text{Mandrel Ratio 2})^{n2}}{(\text{Manderel Ratio 1})} \times \frac{(\text{coil length 2})^{n3}}{(\text{coil length 1})} \times \frac{(\text{volt 2})^{n4}}{(\text{volt 1})}$$

These exponents have been used to expedite the design of modified filaments. Another application has been in the design of filament and Mandrel size watching schemes which ensure that variations in lamp performance due to material tolerance may be minimized.

4. Conclusions

From the study following conclusions are to be said.

(1) The problem of accurately predicting the lu-

minous flux and mean filament operating temperature in an I / L from the physical dimensions of the tungsten of the filament, the compositon and pressure of the filling gas and the applied voltage has been solved through the study.

(2) Accuracy of the theory with the measurements has given us various calculations of I / L system including krypton gasfilled lamps not yet produced in industry, which are keys to the analysis and understanding of it.

(3) Another validity of the theory is derivation of the equation of inter-paramenters for optimal lamp design, which enables to overcome the limit of trial and error method or modify the given lamp specificaton for another new one.

5. References

1) Langmuir, I., "Physical Review", Vol. 34, 1912. p.40

2) Quoted in Elenvass, W., "Light Sources", p.209-217, (MacMillan, 1972)

3) F.H.R.Almer and J.de Ridder, "Calculation of the Radial tungsten transport in cylindrical gas-filled incandescent lamps", Lighting Research and Technology, Vol.8 No.1 1976, pp31-35.

4) Muller, R., Chemie-Ingr. Tech., Vol.40, 1968, p.344.

5) Coaton,J.R., "Calculations of power loss to the gas filing of incandescent lamps", Lighting Research and Technology, Vol. 3, NO.2, 1971, p.163

6) Quoted in Baker, C., "The effect of coil pitch and wire diameter variations on incandesent filament behavior," Journal of the illuminating engineering society, Vol. 5, October 1975, p.3.

7) "American Institute of Physics Handbook,"p.6-168, (McGraw Hill, Second Edition, 1963).

8) 지철근,"전기응용", p.22 (문운당, 1987)

9)"Janaf Thermochemical Tables," (August 1967). V2(G12)