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Analysis of Wave Velocity for Temperature Propagation in a Mechanical Face Seal

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기계평면 시일에서 온도전파를 위한 파속도의 이론적해석

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요 약

미끄럼 운동에 의하여 미세한 시일링간극에 열이 발생할 때 어떠한 빠르기로 발생된 열이 재질 내부로 전파되는가를 속도의 개념으로 이론적 해석을 하였다. 발생된 마찰열이 시일 재질 내부로 전파되는 속도는 불안정한 온도의 파장에 의하여 커다란 영향을 받고 있다.

1. Introduction

Mechanical face seals are widely used to liquids and gases in industrial applications such as gas turbines, hydraulic actuators, pumps, and reactor fueling systems. The sealing function of mechanical face seal is achieved by two primary seal rings with faces to minimize or prevent leakage. One of the primary seal rings is attached to the housing.

The sliding contact between two seal rings is continuously maintained by forces acting in axial direction and as a result heat in the vicinity of the interface is generated. This phenomenon was investigated by Barber[1, 2]. This heat may lead to the destruction of the interface contact areas. The frictional heating due to the irregularity of the contacting surfaces may be concentrated on particular regions of the interface. These regions expanded above the level of the surrounding surface and reduced the area of real contact, thereby concentrating the contact and elevating the local temperature still futher. This process is a thermoelastic instability. Thermoelastic instability has been studied in detail by Burton, et al. [3-5]. To analyze this phenomenon it may be useful to predict the speed of temperature propagation into the body.

In order to predict the wave velocity for temperature propagation into the body, classical equation of heat flow has been applied to a seal-like configuration with one face having a sinusoidally varying temperature distribution. This analytical method may be useful to explain the thermoelastic instability phenomenon in the sliding contact.

2. Analysis

A thermally conductive plate slides on a thermal insulator. The conductor moves against a stationed insulator at velocity U along the xaxis. The problem of a semi-infinite blade geometry is shown in Fig. 1.

We assume that the heat generated by viscous friction between the parallel plates is transferred into the solid. The face geometry with a sinusoidal waviness will cause the non-uniform heating. This may be led to the thermoelastic deformation in the interface. A problem on the conduction of heat of non-steady state and moving temperature

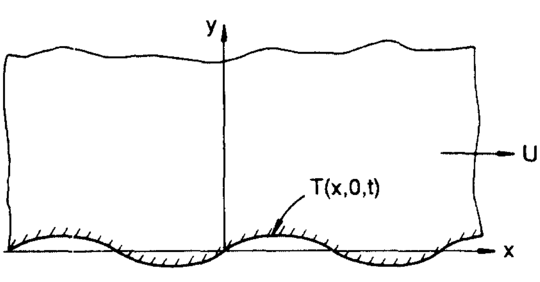

Fig. 1 Semi-infinite blade with a sine variation in the surface temperature

disturbance will be considered.

To simplify the equation of heat flow, we assume the width z of the blade to be small. It is assumed that the thermal diffusivity, α_m within the metal does not vary with temperature. The governing differential equation can then be written.

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha_m} \cdot \frac{\partial T}{\partial t} \tag{1}$$

where T is the temperature distribution in the body and t is time. Eq. (1) may be solved using the following boundary conditions. The temperature variations relative to the fixed surface are assumed as follow :

$$T(x, 0, t) = |T_i| e^{\beta t} \sin[\varkappa\{x - (c+U)t\}] \tag{2a}$$

$$T(x, y, t) = 0 \quad \text{as } y \rightarrow \infty \tag{2b}$$

where $|T_i|$ is a constant amplitude of temperature, β represents the exponent of growth of temperature wave, c denotes the transversal velocity of temperature wave, and $\varkappa$ is the wave number defined as

$$\varkappa = 2\pi/\lambda \tag{3}$$

where λ is a wavelength.

The corresponding S problem of Eq.(1) is defined by

$$\frac{\partial^2 S}{\partial y^2} = \frac{1}{\alpha_m} \frac{\partial S}{\partial t} \tag{4}$$

with the boundary conditions

$$S(x, 0, t) = |Ti| e^{\beta t} \cos[\varkappa\{x - (c+U)t\}] \tag{5a}$$

$$S(x, y, t) = 0 \quad \text{as } y \rightarrow \infty \tag{5b}$$

If we introduce the complex combination $\widetilde{T} = S + i \cdot T$, it is constructed by multiplying Eq. (1) to Eq. (2b) by i and adding them to Eq. (4) to Eq. (5b), respectively. The modified equation for $\widetilde{T}$ is then given by

$$\frac{\partial^2 \tilde{T}}{\partial y^2} = \frac{1}{\alpha_m} \frac{\partial \tilde{T}}{\partial t} \tag{6}$$

with the boundary conditions

$$\tilde{T}(x, 0, t) = |T_i| e\, i \{\varkappa x - [\varkappa(c+U) + i\beta] t\} \tag{7a}$$
$$\tilde{T}(x, y, t) = 0 \quad \text{as } y \rightarrow \infty \tag{7b}$$

The solution form of the modified equation (6) may be written as

$$\tilde{T} = Y(y) e^{i\{\varkappa x - [\varkappa(c+U) + i\beta] t\}} \tag{8}$$

where the function $Y(y)$ is determined so that the heat transfer equation (6) and its boundary conditions (7a, b) must be satisfied. Substituting Eq. (8) into Eq. (6) gives

$$Y'' + i\frac{[\varkappa(c+U) + i\beta]}{\alpha_m} Y = 0 \tag{9}$$

Substituting Eq. (8) into the boundary conditions (7a, b) yields

$$Y(0) = |T_i| \tag{10a}$$
$$Y(y) = 0 \quad y \rightarrow \infty \tag{10b}$$

Therefore the ordinary differential equation (9) can be solved as

$$Y = B_1 e^{i\sqrt{i\xi}\, y} + B_2 e^{-i\sqrt{i\xi}\, y} \tag{11}$$

where

$$\xi = \frac{\varkappa(c+U) + i\beta}{\alpha_m} \tag{12}$$

Consider the complex relationship for Eq. (11) and substitute it into Eq. (8). Then

$$\tilde{T} = B_1 e^{-\sqrt{\frac{\xi}{2}} y + i\{\varkappa x - [\varkappa(c+U) + i\beta] t + \sqrt{\frac{\xi}{2}} y\}} + B_2 e^{\sqrt{\frac{\xi}{2}} y + i\{\varkappa x - [\varkappa(c+U) + i\beta] t - \sqrt{\frac{\xi}{2}} y\}} \tag{13}$$

Using the first boundary condition (7a), the unknown coefficient B_1 of Eq. (13) is obtained as, $B_1 = |T_i|$. Since the temperature disturbance should be finite as y becomes infinite, B_2 is zero. Thus, the solution of Eq. (6) becomes

$$\tilde{T} = |T_i| \{\cos[\varkappa(x - (c+U)t) + (a-b)y] + i \sin[\varkappa\{x - (c+U)t\} + (a-b)y]\} e^{-(a+b)y + \beta t} \tag{14}$$

where

$$a = \frac{\beta}{4 b \alpha_m} \tag{15a}$$

$$b = \left(\frac{-\varkappa(c+U) + [\varkappa^2 (c+U)^2 + \beta^2]^{1/2}}{4 \alpha_m} \right)^{1/2} \tag{15b}$$

The negative case of Eq. (15b) will be discarded because the temperature should be bounded as y goes to infinite. Since $\tilde{T} = S + i \cdot T$, the soultion of the temperature perturbation in the body may be found

$$T = |T_i| e^{-(a+b)y + \beta t} \sin\{\varkappa[x - (c+U)t] + (a-b)y\} \tag{16}$$

The temperature fluctuations due to the frictional heating on the edge of the body are propagated into the body with the wave velocity c given by [6]

$$c = \{2\alpha_m [\varkappa(c+U)]\}^{1/2} \tag{17}$$

We may consider limiting case; non-moving plate, i. e., $U = 0$. The temperature wave can propagate into the solid even though the body does not move. Thus we have to discard the negative case. The wave equation of Eq. (17) may be rearranged as

$$c = \frac{2\pi\alpha_m}{\lambda} [1 + (1 + \frac{\lambda U}{\pi \alpha_m})^{1/2}] \tag{18}$$

This equation indicates an importance of the wavelength to the wave velocity into the body.

3. Conclusion

Fig. 2 shows the distributions of wave velocity c with the sliding velocity U of the conductor. Curves are plotted for various values of the wavelength. As the sliding speed increases, the distribution of the wave velocity increases with approximately half of a parabolic shape. At low value of the wavelength, the wave velocity is much higher than the long wavelength.

Equation (18) serves to provide the estimate of the wave velocity into the body as a function of material property, wavelength and speed of the blade. The wavelength of temperature disturbance appears to be an important factor to predict the wave velocity when the heat transfers to the body. The wave velocity expression (18) may be essential to understand the thermoelastic instability phenomenon in frictionally heated sliding contact.

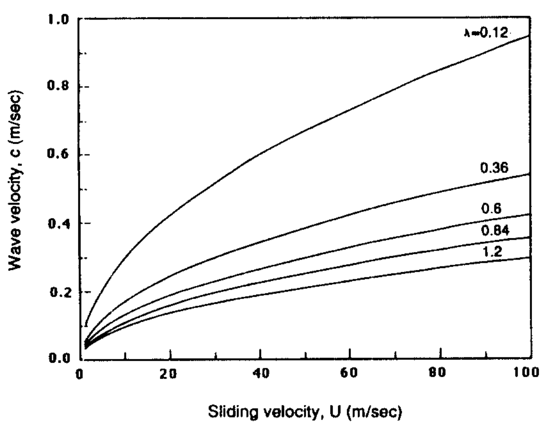

Fig. 2. Relationship between the wave velocity and the sliding speed of body with various values of the wavelength.

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