모래의 振動다짐에 영향을 주는 因子에 대한 研究

A Study of Vibratory Densification of Sands

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摘 要

效果的인 모래의 다짐 方法으로 振動에 의한 方法이 널리 使用되고 있다. 이 研究의 目的은 모래의 振動 다짐시 영향을 주는 因子의 個別的인 效果를 分析하고 各 因子들이 複合되어 振動다짐에 주는 效果를 분석하는 것이다. 이 研究에서는 振動時間, 加速度, 過載荷重, 含水比, 試料의 다짐전 密度등을 變化시켜가며 振動테이블 위에서 제작된 試料를 다짐하였고 試驗結果는 相對密度의 變化에 의하여 分析하였다.

振動다짐의 效果는 다짐加速度 振動數 및 모래의 粒度에 따라 變하며 過載荷重이나 振巾은 比較的 적은 영향을 주는 因子였다. 加速度가 1g 보다 큰 경우 다짐效果는 振動이 시작되는 初期에 發生하였고 그후는 緩漫하게 振動效果가 나타났다. 效果的인 다짐을 위하여는 最適加速度와 最適 過載荷重의 選擇이 重要하며 最適値보다 큰 경우는 다짐의 效果가 減少한다. 乾燥된 모래와 飽和된 모래 모두 같은 最適加速度에서 最大의 다짐效果를 나타냈다.

각 因子 즉 加速度, 振動數, 單位重量, 含水比, 均等係數, 有效粒徑, 過載荷重 및 振動時間 등의 效果를 複合하여 振動다짐의 效果를 相對密度의 變化로 나타낼 경우 $\Delta D_R = 38.0\,\Delta^{0.14}$의 關係를 얻었다.

I. Introduction

Vibratory densification of sand has been commonly employed in soil engineering, even though the mechanisms which control the behavior of granular materials during vibration have not been completely understood. When the effects of various parameters affecting vibratory densification and their influence on packing of particles are fully understood, the most effective means of compaction by vibration can be devised. It is also possible to predict the settlement of a foundation resting on a granular soil resulting from ground vibration.

Although many studies have been made in the past of the behavior of granular materials subjected to vibration, there is no universally agreed upon procedure for vibratory compaction yet because of many variables associated with the process of vibratory densification.(3,4,6) A thorough evaluation of individual and/or combined parameters affecting the mechanism of vibratory densification of sand, therefore, would bring about a better comprehension of the problem.

The utilization of a vibratory method is an ef-

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fective means of compacting sand. Although there have been studies pertaining to the densification and liquefaction of sands, no comprehensive and systematic study has been conducted to isolate and evaluate the relative effects of the different parameters associated with vibratory densification of sand. Thus, the primary objective of the present research is to present the results of a theoretical and an extensive laboratory test program directed toward evaluating the effect of various parameters, both individually and combined, on the densification and liquefaction of sand with an aim toward obtaining the most effective means of densification by vibration. In an effort to better understand how vibration variables and material properties affect the compaction process, a dimensional analysis is carried out.

Ⅱ. Theoretical Background

A descriptive variable term the "change in relative density" is used to study the effectiveness of vibratory densification and to represent the natural condition of a cohesionless soil. This is the ratio, expressed in percentage, of the relative densities before and after densificaton. Relative density would be an appropriate means to define the looseness and denseness of cohesionless soils in a meaningful way, because important properties such as shear strength and consolidation characteristics could be correlated quite well by relative density. Relative density has also found to be useful in liquefaction and other seismic studies for sand and gravel soils subjected to earthquake or other vibrational conditions.

The variables which affect the densification of granular materials include particle size, gradation, angularity, surface roughness, density condition and intensity of deposition, momentum and impact velocities of particles during deposition, presense or lack of lubrication, vibratory characteristics such as acceleration, amplitude, frequency and natural frequency of the system, external pressure, ambient temperature, and humidity. Because of the many factors involved, the roles of the individual variables are generally obscured.

In an effort to better understand the interrelationship of these variables and how they affect the densification process, a demensional analysis of the Pi-theorem has been used. The Pi-theorem first developed by Buckingham.$^{(2)}$ This provides an excellent tool by which these quantities can be organized into the smallest number of sigificant, dimensionless groupings, from which an equation can be evaluated.

For the initial analysis, the following seven variables, which are considered to be most significant, were selected.

ρ =mass density of material (FT^2L^{-4})
σ =surcharge (FL^{-2})
t =time of vibration (T)
a =acceleration (LT^{-2})
ω =frequency (T^{-1})
D_{35}=effective diameter (L), and
Cu=uniformity coefficient (none).

The four dimensionless π terms obtained were $a^2\rho/\omega^2\sigma$, $D_{35}\omega^2/a$, Cu, ωt.

Since $\rho = \sigma/g$ where γ is the density of the material, and g is the gravitational acceleration.

$$\rho = \sigma/g = \gamma_d\,(1+w)/g$$

where γ_d =dry density and w=moisture of the material. The following functional equation for the change in relative density ΔD_R can be obtained :

$$\Delta D_R = f\left(\frac{a}{g}, \frac{\gamma D_{35}}{\sigma Cu}, \frac{t}{T}\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

where f=unknown function of the dimensionless groups of independent varialbles.

Now, designating ;

$$\frac{\gamma D_{35}}{\sigma Cu} = \text{mass ratio} = \beta$$

$$\frac{a}{g} = \text{acceleration ratio} = \alpha$$

$\frac{t}{T}$=time factor= τ

Eq(1) can be shown in terms of combined parameters as :

$$D_R=f(\alpha,\ \beta,\ \tau)=f(\Lambda) \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

where $\Lambda=\alpha\cdot\beta\cdot\tau$

The above equations indicate the dependence of densification upon the mass factor, acceleration ratio and time factor, or the combination thereof. The dimensional analysis forms a basis for the design of the experimental program for the investigation of the functional relationships among the various terms illustrated in the above equations.

Ⅲ. Vibratory Densification

A series of laboratory tests were conducted on a uniform sand and an ideally graded sand to investigate the relative effects of various parameters associated with vibratory densification.

Material—A brief description of the engineering properties is given here. The specific gravity of the sand was 2.65. The ideally graded sand was artificially mixed with a definite amount for each size based on Weymouth's theory of particles interference.⁽⁵⁾ The uniform sand used in the tests passed sieve No.40 and was retained all on the sieve No.60. The Uniformity Coefficients were 8.9 for the ideally graded sand 1.4 for the uniform sand. The maximum density for the uniform and the ideally graded sands, obtained by a series of tests, were 12.90 and 15.47 KN/m^3, respectively.

Experimental Setup and Procedure—An overall experimental setup is shown in Fig.2. It consists of a lucite mold, the vibration table, the oscilloscope and control console, the pressure supply and the hydraulic power supply which serves to drive the vibration table. The mold was fastaned to the table of a Model 840 Servohydraulic Vibration Test System made by MTS System Corporation. A Model 308A Quartz Accelerometer was mounted upright on the upper plate of the mold for the response measurement. Nitrogen cylinder was used to apply confining pressure through a rod connected to the piston. This rod extended through

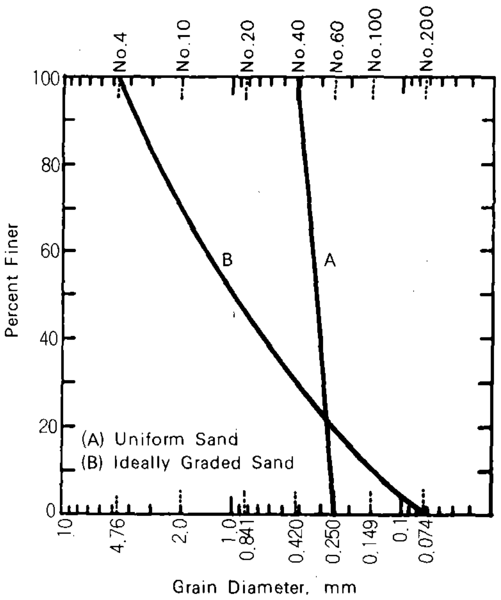

Fig. 1. Gradation Curves of Material Tested

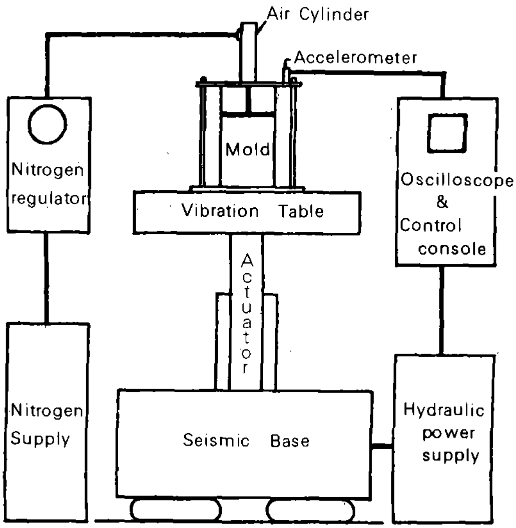

Fig. 2. Schematic Diagram of Experimental Set-up

an opening in the upper plate and acted on an aluminium plate which distributed the load to sand specimens.

After a specimen was prepared in a loose initial condition for each test, the mold was transfered to the vibration table and clamped in place with care in order to minimize any disturbance. Vibration of the system was initiated and readings of settlement using an Ames'dial gauge were taken at prescribed time intervals.

Test Schedule—The effect of vibration for the uniform and the ideally graded sands was determined by running a series of tests with a surcharge of 0.28, 6.89, and 13.79 KPa. The samples of sand were tested at frequencies of 10, 15, 20, 25, and 30 Hz and at five levels of acceleration 1.0, 2.0, 3.0, 4.0, and 5.0g. Settlement measurements were taken after 1.0, 2.0, 3.0, 4.0, 8.0, 15.0, and 20.0 minutes of vibration.

Ⅳ. Presentation and Discussion of Test Results

Presentation and Discussion of Test Results

The purpose of this experimental program was to evaluate the effects of various parameters, independently or combined, on the vibratory densification of sand with the aim of finding the most effective means of vibratory densification in the field. The test parameters studied were time duration, acceleration, frequency, confining pressure, moisture content, amplitude, velocity, and a combination of these parameters. An average density of the mold was used in this test. A computer program was used to analyze the test data.

The mechanism of densification of sand particles can be explained in terms of "air lubrication", analogous to liquefaction, which is defined as the state in which air cannot escape from the voids between particles at a sufficiently fast rate when soil is reducing in volume due to vibration (1). Thus, significant pore air pressure may have developed, with ensuing reduction in shear

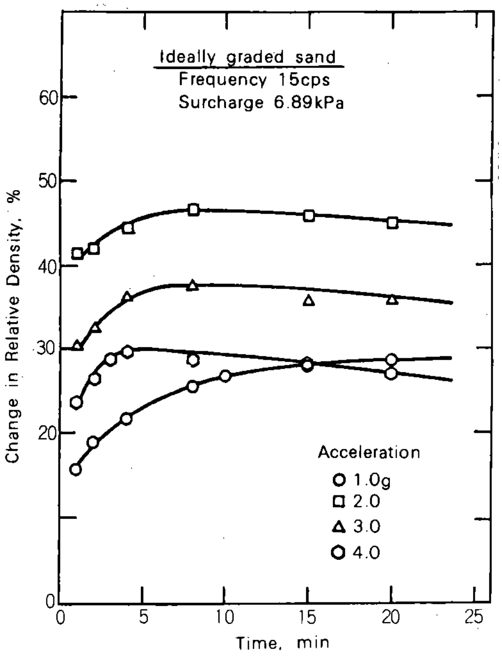

Fig. 3. Effect of Time Duration (Ideally Graded Sand) on Change in Relative Density

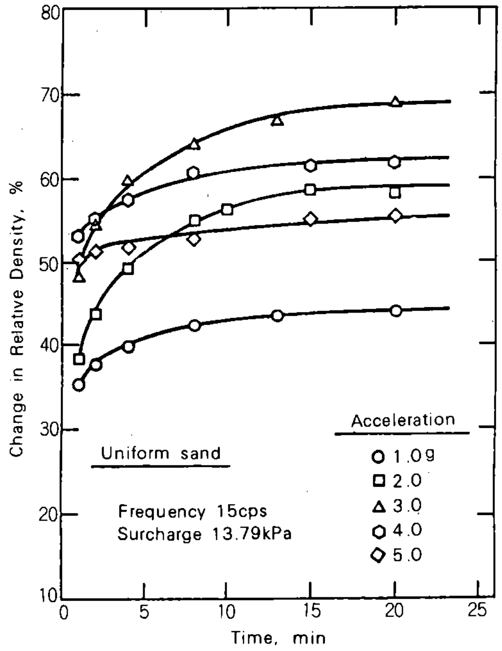

Fig. 4. Effect of Time Duration (Uniform Sand) on Change in Relative Density

resistance of the material. The ability of the sand particles to re-orient themselves is then increased. When subjected to air lubrication, soil particles rebound during the period of vibration and settle by gravity when the vibration ceases resulting in a volume reduction. The possibility of the occurrence of this phenomenon will depend upon the relative density or void ratio and the intensity of vibration, that is, acceleration or frequency.

Effect of Time of Vibration—The effect of time of vibration on the change in relative density for various test condition is shown in Figs.3 and 4 for the two types of sand. It is observed that the change in relative density, from the initial loose state, takes place immediately after vibration is initiated, but the rate of the change in densification gradually decreases with time of vibration. It can be seen that a limited change in relative density or a small further increase was obtained for uniform sand after a certain period of time. It is also found that with the ideally graded sand a decrease in relative density occurred after the peak was achieved. This may be explained by the fact that an overvibration which is defined as loosening of packing of soil particles due to high energy applied in the absence of sufficient confining pressure, took place. It is of interest to note that the highest change in relative density was obtained at an acceleration of 2 g for the ideally graded sand and 3 g for the uniform sand. This indicates that specimen subjected to high acceleration than the optimum can develop overvibration so that the effectiveness of densification is somewhat reduced.

Effect of Acceleration—Figs.5 and 6 demonstrate a summary plot of all tests preformed with 0.28 KPa surcharge for the uniform and the ideally graded sands. Each point on the curve was obtained by taking the average value for the change in relative density due to vibration at all frequencies tested. Thus, these curves indicate the overall effect of acceleration on the change in relative density due to vibration. The peak val-

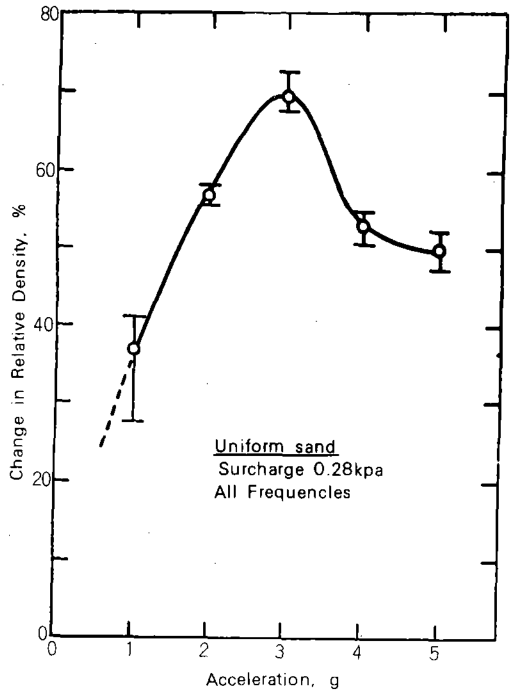

Fig. 5. Effect of Acceleration (Summary) on Uniform Sand

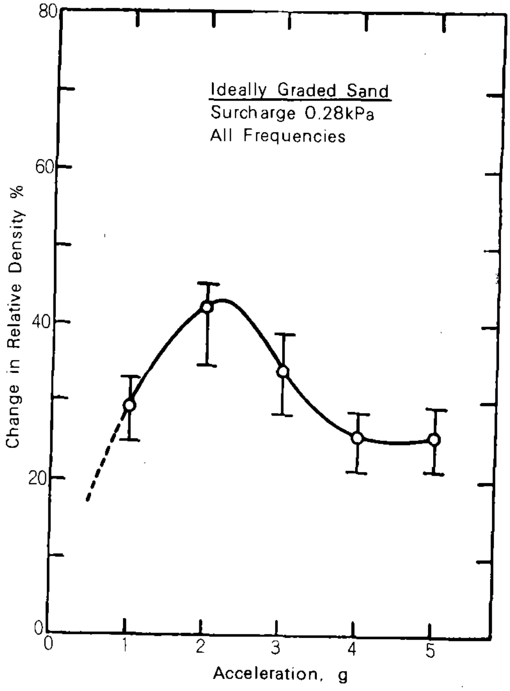

Fig. 6. Effect of Acceleration (Summary) on Ideally Graded Sand

ues are obtained at the 3 g and 2 g acceleration levels for the uniform and the ideally graded sands, respectively. The degree of densification increases rapidly as the acceleration is increased to this point, and then gradually decreases to an equilibrium level with a further increase in acceleration. The fact that the change in relative density decreases beyond the optimum acceleration level with a further increase in acceleration can be explained by greater energy when higher accelerations than optimum are applied. In other word, the soil particles can no longer occupy their most favorable position at higher accelerations than optimum when greater energy is imparted to the soil medium. This means that a maximum densification of sands will occur at an appropriate acceleration which is sufficient to overcome the shear stress between the particles. Similar results were obtained. with 6.89 and 13.79kPa surcharge for the uniform and the ideally graded sands.

A similar observation can be made with regard to Fig. 7 as well in which the results for the saturated sand subjected to an increase in acceleration are shown. The change in relative density increases until it reaches an optimum value at an acceleration level of approximate 3 and 2 g for the uniform and the ideally graded sands, respectively. The relative density then gradually decreases to an equilibrium value with a further increase in acceleration.

It should be noted that both the dry and saturated soils possess the same optimum acceleration at which the change in relative density becomes maximum. This implies that soil when saturated with air or water will behave sim-ilarly under vibration, and thus the behavior may be explained by a similar mechanism : air lubrication for dry soil and liquefaction for saturated soil. Liquefaction is defined as a condition when a soil will undergo continued deformation at a constant low residual stress or with low resistance, due to build

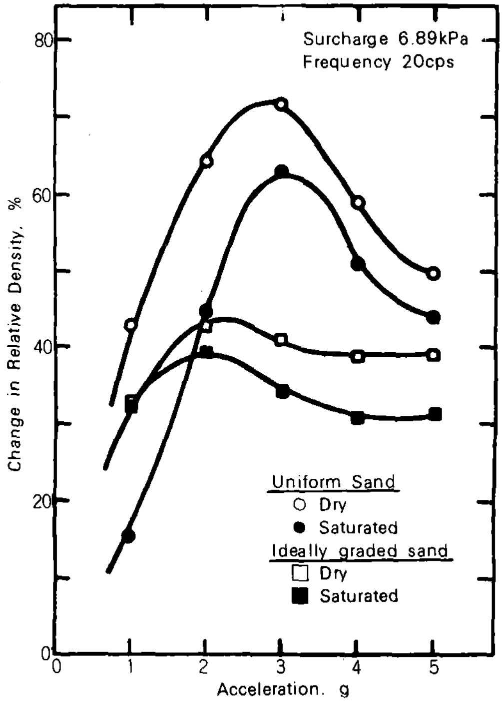

Fig. 7. Effect of Acceleration on Saturated Sand

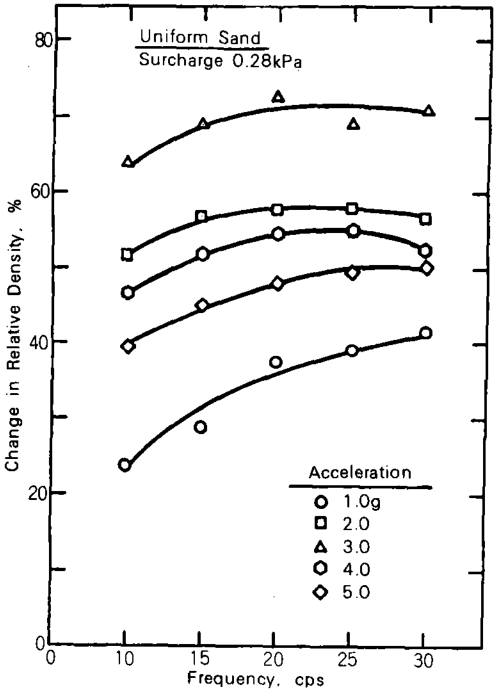

Fig. 8. Effect of Frequency on Change in Relative Density (Uniform Sand)

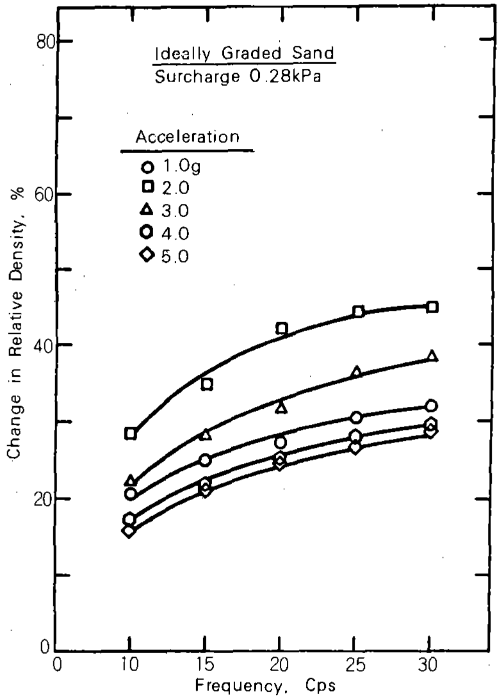

Fig. 9. Effect of Frequency on Change in Relative Density (Ideally Graded Sand)

to build-up and maintenance of high pore water pressure which reduces the effective confining pressures to a very low value ; pore water pressure build-up, therefore, leads to liquefaction.

At low acceleration levels, such as 1 g or less, the change in relative density is small because the intensity of vibration is not sufficient to overcome the shear stress between the particles.

Effect of Frequency—For any given acceleration level, the density generally increases as the frequency is increased, and then the rate of increase of the density gradually decreases with a continued increase in frequency. It is observed in Figs. 8 and 9 that for the uniform sand there is practically little effect of frequency, except at 1 g level, on the change in relative density. However, for the ideally graded sand the effect of frequency is more pronounced at all acceleration levels. It is evident from these figures that for the ideally graded sand acceleration and frequency are equally effective in the vibratory compaction of granular soils within the ranges tested. For the uniform sand, acceleration is more effective than frequency, and thus is the most significant and controlling parameter in the vibratory densification.

Effect of Surcharge—In the case of vibration with a surcharge, the change in relative density increases generally with increase in surcharge at all acceleration levels. Doubling of surcharge from 6.89 to 13.79 kPa, however, increases the change in relative density by a small amount. It is seen that the change in relative density appears to reach a maximum at approximately 10 kPa, after which it starts to decrease. This behavior may be explained in terms of intergranular pressure. In the case of sand vibrated under a low surcharge, the individual particles were relatively free to move. That is, an increase in the effective stress on the sample beyond a certain value reduces the capacity of the sand to undergo volume change when subjected to vibration. This shows that beyond a certain optimum value the higher the effective stress before vibration, the more difficult it is for vibration to reduce the intergranular stresses sufficiently to permit the particle to move. thus the application of an excessive amount of surcharge pressure reduces the amount of volume change possible in sand due to vibration. It is shown that a greater change in relative density occurs for the ideally graded sand at 2 or 3 g acceleration level, regardless of the surcharge applied. This result is explained by the fact that the confining stress causes a greater amount of intergranular stresses between the sand particles. however, at the higher acceleration, for example 4 and 5 g, sand particles can not be densified to the densest state due to greater energy than optimum.

Effect of Amplitude—Within the range of amplitude studied the change in relative density assumes a peak value at a very low amplitude and decrease with in amplitude (decreasing frequency). The change in relative density of the ideally graded dry sand as a function of amplitude for a given surcharge of 0.28 kPa and for

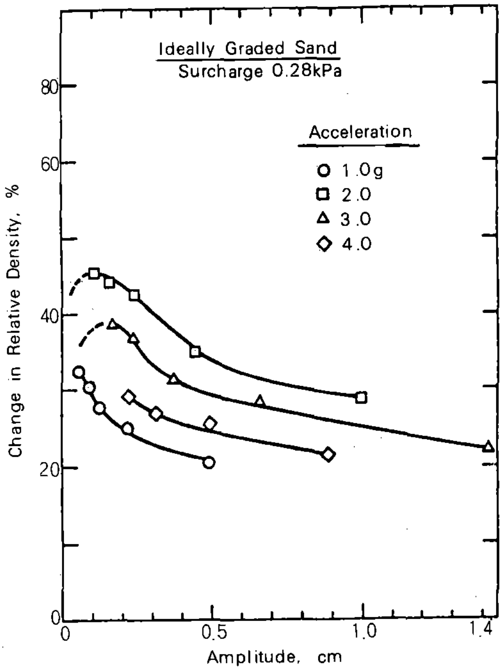

Fig. 10. Effect of Amplitude on Accelerations (Ideally Graded Sand)

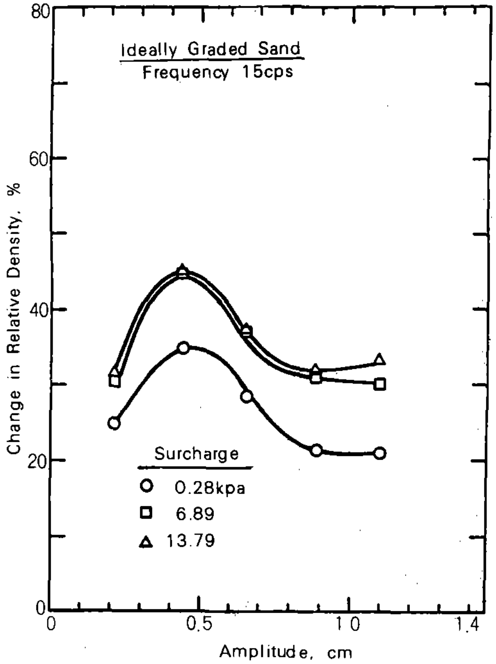

Fig. 11. Effect of Amplitude on Surcharges (Ideally Graded Sand)

is subjected to higher than an optimum acceleration levels. Similar results are obtained for acceleration levels of 1, 2, 3, and 4 g is presented in Fig.10. Once again, the change in relative density decreases with increasing amplitude. The peak value of the change in relative density in Fig.11 occurs at an amplitude of approximately 0.5 cm (acceleration of 2 g). In the case of higher amplitudes at a constant frequency (higher acceleration), the change in relative density is lower because at a high acceleration a large amount of energy is released to the sand particles, thus causing them to assume a packing which is not favorable to the densest. Therfore, the effectiveness of densification by vibration will be reduced when sand the uniform sand. It is evident that the change in relative density is affected by amplitude in as much as it is affected by acceleration and frequency, since these three parameters are interrelated and functions of each other.

Effect of Moisture Content—The effect of moisture content on the change in relative density may be observed from Fig.7. As previously stated, the behavior of saturated sand during vibration is quite similar to that of dry sand. The change in relative density increases until it reaches an optimum value at an acceleration level of approximate 3 and 2 g for the uniform and the ideally graded sands, respectively. The relative density then gradually decreases to an equilibrium value with a further increase in acceleration. It should be noted that both the dry and the saturated soils possess the same optimum acceleration at which the change in relative density becomes maximum. Thus, a mechanism of densification of saturated sand can be explained by a similar mechanism of densification of dry sand : air lubrication for dry sand and liquefation for saturated sand.

Effect of Combined Paramenter—The experimental results discussed so far were based on typical relationships between each of the individual vibratory test parameters and the resulting change in relative density. These results do not, however, provided an overall relationship among the change in relative de-

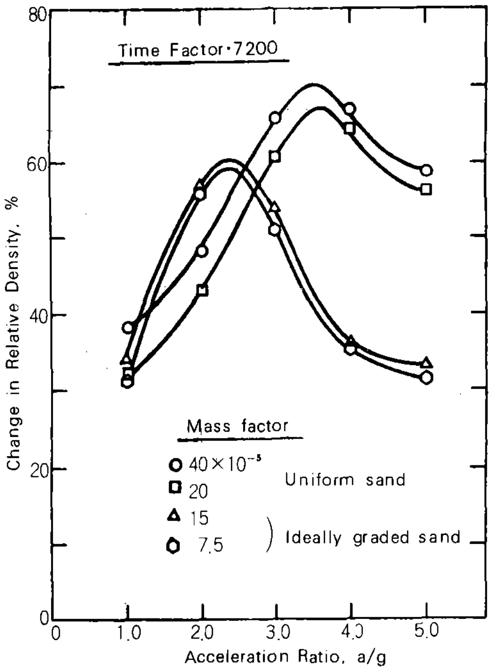

Fig. 12. Effect of Acceleration Ratio

nsity and the various parameters involved. This is because the effect of each individual parameter on densification is dependent upon other parameters. Thus, when the effect of one particular parameter is discussed, other parameters must be specified. Non-dimensional functional relationships were derived through the use of dimensional analysis considering the variables affecting the vibratory densification of sand(Eq. 1)

In the Fig. 12 are plotted the change in relative density ΔD_R as a function of the acceleration ratio α for two constant mass factors of 40.0×10^{-5}, 20.0×10^{-5} for the uniform sand and 15×10^{-5} and 7.5×10^{-5} for the ideally graded sand, and at time factor, 7200. The change in relative density increases with an increase in acceleration up to a peak point, and then decreases with an further increases in acceleration ratio. It is of interest to note that for a given type of soil, uniform or ideally graded, the change in relative density is not significantly affected by the mass factor.

It is seen that the change in relative density slightly increases with increase of time factor. As discussed previously, the time of vibration and amplitude are not significant to the densification of sand. Thus, the change in relative density is not significantly affected by the time factor because the time factor is a product of time of vibration and frequency.

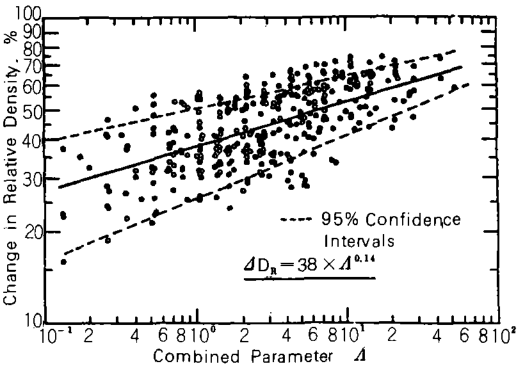

Fig. 13. Effect of Combined Paramenter

In order to obtain the relationship between the change in relative density and the combined parameter Λ, data points are plotted in Fig.13. The circles in the figure represent all of the experimental data obtained with the surcharge of 6.89 and 13.79 kPa. The relationship between these two quantites may be expressed based on linear regression method, as follows :

$$\Delta D_R = 38.0\,\Lambda^{0.14} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3)$$

Eq. 3 is represented in Fig. 13 as a solid line. The dotted lines indicate the 95 percent confidence interval. The computer program for the regression analysis is used.It is observed in Fig. 12 that the change in relative density increases, in general, with greater Λ values. It may be concluded that there is a good correlation between the change in relative density and the theoretically derived combined parameter Λ which incorporates all of the affecting variables such as acceleration, frequency, surcharge, unit weight of sand, gradation, and time of vibration. The level of vibration required to achieved a desired degree of densification may be determined from the relationship obtained.

In the design for vibratory compaction of sand, Eq.3 is useful. For the amount of compaction required (i.e., the change in relative density), the value of Λ can be obtained from Eq.3. Having this value will allow the selection of an acceleration ratio value for a given sand properties (γ, Cu, D_{35}) and surcharge (σ) under a specified value such that the required compaction may be realized.

V. Conclusion

1. The change in relative density is profoundly affected about equally by acceleration and frquency, and also by gradation of sand ; secondary factors include ampliteude and surcharge.

2. Most of the vibratory densification of sand takes place very rapidly (2 minutes for the ideally graded sand and 4 minutes for the uniform sand). The time rate of densification is non-uniform and decreases with increasing time.

3. There is an optimum acceleration at which a maximum change in relative density is achieved, after which a further increase in acceleration causes a decrease in change in relative density due to the effect of a greater amount of energy when higher accelerations than an optimum value are applied. The optimum acceleration varies with grain size distribution. It was 2 g for the ideally graded sand and 3 g for the uniform sand tested in the present study. Thus, the optimum acceleration must be determined for a given type of soil.

4. Acceleration and frequency exert a substantially greater influence on densification than displacement amplitude. Within the range of frequencies and acceleration tested, therefore, acceleration and frequency are equally effective and thus they are controlling parameters in the vibratory densification of the ideally graded sand. For uniform sand, however, acceleration plays a more dominant role than frequency and thus is the most significant parameter. Since acceleration, freequency and amplitude are interdependent, the ffect of each of these parameters must be examined in light of the two remaining parameters.

5. Surcharge increases the level of densification that can be obtained by vibration to an optimum surcharge value, and then it starts to decrease due to a greater amount of intergranular stress within the sand particles which serves to increase the friction between the particles, requiring supplemental energy for overcoming it.

6. For any given condition, both the dry and the saturated sands possess the same optimum acceleration at which the change in relative density becomes maximum. This implies that sand when saturated with air or water will behave similarly under vibration.

7. It has been shown analytically and experimentally that one functional relationship combining the variables of vibration (acceleration and frequency), the variables of the materials (unit weight, water content, uniformity coeffeicent and effective diameter) along with surcharge and time of vibration, may be a tool in determining the amount of the change in relative density obtainable by vibration.

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