

# 입력보상 및 최적출력 PIM제어를 적용한 스위칭 직류변환기의 제어기 설계

## A Controller Design for Switching Regulator Using an Optimal Output PIM Control with Feedforward Comensation

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### Abstract

This paper describes a design method for the buck type switching regulator to improve transient and steady state performances. Necessary design considerations on the power stage are given before designing the controller to obtain better transient responses with less control effort and a feedforward compensation is also given to effectively improve the steady state performance. In the design of the controller, a PIM (proportional-integral-measurable) control method with optimized constant feedback gains is presented to get better transient and steady state performances without complicating the implementation of controller. Computer simulations and experimental results are given to show the usefulness of the presented technique.

### 1. Introduction

In the past decade, a great deal of researches on the switched mode DC to DC converter (switching regulator) has been done to obtain a practical tool for analysis (1)~(7) and to improve its performances as the increasing demands of the switching power supply. In these researches, the results are mainly based on the conventional frequency domain analysis (7)~(16). However, these methods can not directly handle the design of the feedback compensator with optimized feedback gains. Furthermore, it is rather difficult to predict the closed loop output responses in the time domain, especially when

the output transient performances are important design specifications. In the design of the controller, proportional control method of the output error with proper compensation in the frequency domain has been widely used for the simplicity of implementation (12)~(14). But these methods generally result in the deficiencies of performances. In recent researches, a current mode control method has been introduced to improve performances of the system (15)~(16). However, in the practical applications, there still exist some problems such as the complexity of the controller and difficulties in the current sensing.

Among the several topologies of the switching regulators, buck (step down) type topology has some advantages over the other topologies due to the typical characteristics of high efficiency, low output ripple, and simple structure easy to control. In this paper, therefore, a design method of the optimal output PIM (proportional-integral-measurable) controller for the buck type switching regulator is presented to satisfy the dy-

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dynamic performances in the time domain without complicating the implementation of the controller and a feed-forward control method is also given to improve the steady state performance under the situation of the nonideal DC input to the switching regulator. Furthermore, necessary design considerations on the power stage are given to obtain a better closed-loop transient responses with less control effort under the constraint of the permissible switching ripple. In the optimization of the controller parameters, a design technique using the time weighted quadratic performance index is proposed to obtain a better transient output responses rather than those based on the conventional quadratic performance index. The computer simulations and experimental results are also given to show the usefulness of the presented techniques.

## 2. Power Stage Design

### 2.1 Design problem formulation

To satisfy the steady state performance requirements of the buck type switching regulator, both the characteristics of the power stage and that of the controller should be considered. Since the inherent switching ripple components of the output voltage can not be generally removed by the feedback controller, small bandwidth of the low pass filter (LPF) is required to reduce the switching ripple component of the output voltage. However, small bandwidth of the LPF will result in the poor dynamic characteristics, thus we are faced with the trade off problem between the steady state switching ripple magnitude and the transient performance of the output voltage. In the design of the power stage parameters, the following design problems can be formulated to satisfy the conditions discussed above.

- 1) Bandwidth of the LPF should be selected as wide as possible under the constraint of the permissible switching ripple.
- 2) Care must be taken to prevent the series resonant frequencies from coinciding with the significant frequencies contained in the input DC voltage source inherently.
- 3) In the possible operating duty ratio, the converter should operate on the continuous conduction mode.

### 2.2 Characteristics of the power stage

In order to satisfy the power stage design problems formulated in the previous section, the characteristics of the power stage should be investigated. As shown in Fig. 1, 4th order power stage gives wider bandwidth than 2nd order power stage under the condition of the same attenuation ratio at the switching frequency. In the following, 4th order LPF scheme is chosen and analyzed for the power stage of the buck type switching regulator.

The magnitude of the transfer function of the power stage in the frequency domain is calculated easily as follows:

$$|H(j\omega)| = 1/[p(\omega)]^{1/2} = 1/[p_1^2(\omega) + p_2^2(\omega)]^{1/2} \quad (1)$$

where,

$$p_1(\omega) = a\omega^4 - c\omega^2 + 1, \quad p_2(\omega) = \omega(d - b\omega^2)$$

$$a = C_1' C_2' L_1' L_2', \quad b = C_1' L_1' L_2'$$

$$c = C_1' L_1' + C_2' L_1' + C_2' L_2', \quad d = L_1' + L_2'$$

In the above equation,  $\omega$  denotes angular frequency and  $C_1', C_2', L_1'$  and  $L_2'$  are normalized parameters with respect to load resistor  $R$  as follows:

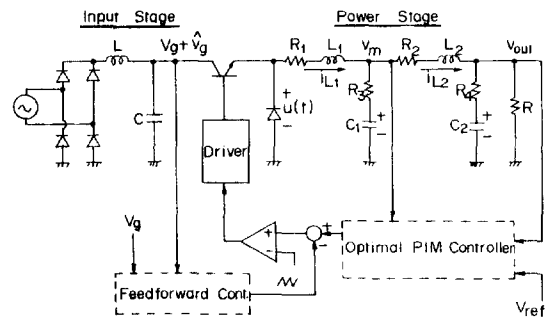


Fig. 1 Circuit diagram of the buck type switching regulator ( $R_1, R_2, R_3, R_4$ : Effective series resistance of inductor and capacitor.)

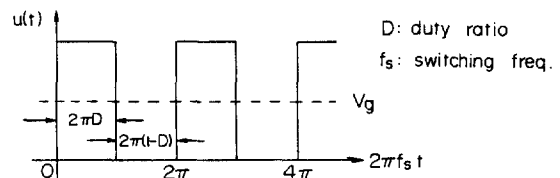


Fig. 2 Waveform of the input voltage to the power stage (in case of continuous conduction mode operation)

$$C'_1=C_1R, C'_2=C_2R, L'_1=L_1/R, L'_2=L_2/R \quad (2)$$

Since there is the degree of the freedom in the determination of the parameters, some constraints are given on the normalized parameters as follows:

$$C'_1=C'_2=C', L'_1 \geq L'_2. \quad (3)$$

To find resonant frequencies, one can obtain the following significant frequencies by equating  $p_1(w), p_2(w), \dot{p}_1(w), \dot{p}_2(w)$  to zero respectively:

$$w_1 \cong \sqrt{1/[C'(2L'_1+L'_2)]} \quad (4)$$

$$w_2 \cong \sqrt{(2L'_1+L'_2)/(C'L'_1L'_2)} \quad (5)$$

$$w_3 = \sqrt{(L'_1+L'_2)/(C'L'_1L'_2)} \quad (6)$$

$$w_4 = \sqrt{(2L'_1+L'_2)/(2C'L'_1L'_2)} \quad (7)$$

$$w_5 = \sqrt{(L'_1+L'_2)/(3C'L'_1L'_2)}. \quad (8)$$

From (4) to (7), the following inequalities can easily be derived:

$$w_1 < w_5 < w_4 < w_3 < w_2. \quad (9)$$

In order to find the ranges of resonant frequencies at which  $p(w)$  locally minimized, the derivatives of the  $p(w)$  for  $w_1$  through  $w_5$  are calculated using the relation of  $\dot{p}(w) = 2p_1(w)\dot{p}_1(w) + 2p_2(w)\dot{p}_2(w)$  as follows:

$$\dot{p}(0) = 0 \quad (9-a)$$

$$\dot{p}(w_1) > 0 \quad (9-b)$$

$$\dot{p}(w_2) > 0 \quad (9-c)$$

$$\dot{p}(w_3) < 0 \quad (9-d)$$

$$\dot{p}(w_4) < 0 \quad (9-e)$$

$$\dot{p}(w_5) > 0. \quad (9-f)$$

It is easily known that one of the local minimum frequencies exists between  $w_3$  and  $w_2$ . To find one more local minimum frequency left, the values of  $p(0)$  and  $p(w_i)$  is calculated as:

$$p(0) = 1 \quad (10-a)$$

$$p(w_1) = p^{\dagger}(w_1) + p^{\ddagger}(w_1) = p^{\ddagger}(w_1) = w_1^{\dagger}(d - bw_1^{\dagger})^2 \quad (10-b)$$

In the above equations,  $p(w_1)$  is nearly zero provided that the following condition holds. This condition is believed to be appropriate except for the case of extremely heavy-duty loads;

$$C' >> 3L'_1. \quad (11)$$

Since  $p(w)$  is a function of 8th order with even symmetry, the resonant frequencies ( $w_{r1}, w_{r2}$ ) of the power stage are obtained from (8), (9) and (10) as follows:

$$w_3 < w_{r1} < w_2 \quad (12-a)$$

$$w_{r2} \approx w_1 \quad (12-b)$$

Next, in the following, an important condition on the operation mode of the converter is to be considered. The input waveform to the power stage shown in Fig. 2 is valid in case of the continuous conduction mode. However, a practical converter may operate on the discontinuous conduction mode unintentionally, which generally causes severe ripple problem. To prevent the converter from operating on this mode, it is necessary to satisfy the following condition for continuous conduction mode 3)~4);

$$L_1 > R(1-D)/(2f_s) \quad (13)$$

where  $D$  and  $f_s$  denote steady state duty ratio and switching frequency, respectively.

### 2.3 Design procedure.

Using the resonant frequencies and the condition for the continuous conduction mode, the following inequalities are obtained to satisfy the design problems;

$$w_{r1, \max} = w_2 = \sqrt{(2L'_1+L'_2)/(C'L'_1L'_2)} < 2\pi f_1^* \quad (14)$$

$$w_{r2} \approx w_1 = \sqrt{1/[C'(2L'_1+L'_2)]} > 2\pi f_2^* \quad (15)$$

$$(1-D)/(2f_s) < L_1 < 1/(2f_s) \quad (16)$$

In (14),  $w_{r1, \max}$  denotes maximum possible value of the resonant frequency and in the following,  $f_1^*$  can be obtained to satisfy the design problem 1) discussed previously. The maximum peak to peak switching ripple magnitude is derived using the peak to peak magnitude of the fundamental component of  $u(t)$  and transfer property of the power stage as follows:

$$V_{s, pp, \max} = \frac{4V_a}{\pi} |H(j2\pi f_s)| = \frac{4V_a}{\pi} (f_1^*/f_s)^4 \quad (17)$$

and the permissible switching ripple is also given by multiplying the constant  $r$  to the total output ripple desired as follows:

$$V_{s,per.} = r \cdot (\% r, pple) \cdot D \cdot V_g / 100. \quad (18)$$

By equating (17) and (18),  $f_1^*$  is determined as follows:

$$f_1^* = [r \cdot \pi \cdot (\% ripple) \cdot D / 400]^{\frac{1}{4}} f_s \quad (19)$$

where, 0.1~0.5 is used as the value of  $r$  which depends on the designer's choice as a rule of thumb. In (15),  $f_1^*$  is selected to meet the design problem 2) as follows:

$$f_2^* = \alpha \beta N f \quad (20)$$

where, generally 2~5 is used as the value of  $\alpha$  to prevent significant frequency contained in the rectified DC input from coinciding with resonant frequencies,  $N$  is the number of phase ( $N=1$  for single phase,  $N=3$  for three phase),  $f$  is the frequency of AC input, and  $\beta$  is 1 for the half wave rectifier and 2 for full wave rectifier. Finally, in (16), the upper range of  $L_1'$  is obtained by setting the steady state duty ratio zero for the possible operating range of the switching regulator. The permissible ranges of the power stage parameters are shown in Fig. 3 as the shaded area. Furthermore, for design purpose, the maximum permissible ranges of  $L_2'$  and  $C'$  are approximately given from (3), (14) and (15) when  $L_1'$  is predetermined by (16) as follows:

$$3 / (4\pi^2 f_1^{*2} L_1') < C' < 1 / (8\pi^2 f_2^{*2} L_1') \quad (21)$$

$$4L_1' / [(f_1^* / f_2^*)^2 - 2] < L_2' \leq L_1' \quad (22)$$

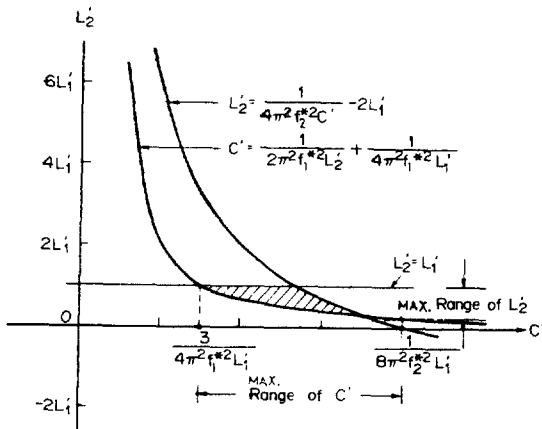


Fig. 3 Permissible ranges of the parameters of the power stage

### 3. Controller Design

#### 3.1 Modelling of the buck type switching regulator

It is assumed that the converter shown in Fig. 1 operates in the continuous conduction mode with the parameters chosen as previously stated. To obtain an analytical tool for analysis, the state space averaging concept has been adopted 1)~2). With the choice of the state vector,

$$x(t) = [V_m(t) \ V_{out}(t) \ i_{L1}(t) \ i_{L2}(t)]^T \quad (23-a)$$

the state equations are obtained by

$$\dot{x}(t) = A x(t) + B u(t) \quad (23-b)$$

$$y(t) = C x(t) \quad (23-c)$$

$$y_m(t) = C_m x(t) \quad (23-d)$$

where

$$A = \begin{bmatrix} -R_3(1/L_1 + 1/L_2) & R_3/L_2 \\ RR_4/[L_2(R+R_4)] & -\frac{R(R_4/L_2 + 1/RC_2)}{(R+R_4)} \\ -1/L_1 & 0 \\ 1/L_2 & -1/L_2 \\ 1/C_1 - R_1R_3/L_1 & R_2R_3/L_2 - 1/C_1 \\ 0 & \frac{R(1/C_2 - R_2R_4/L_2)}{(R+R_4)} \\ -R_1/L_1 & 0 \\ 0 & -R_2/L_2 \end{bmatrix}$$

$$B = [R_3/L_1 \ 0 \ 1/L_1 \ 0]^T$$

$$C = [0 \ 1 \ 0 \ 0]$$

$$C_m = [1 \ 0 \ 0 \ 0]$$

the state vector, where,  $y(t)$  and  $y_m(t)$  denote voltage and measurable output voltage respectively and  $A, B, C,$  and  $C_m$  have the proper dimensions as described above.

#### 3.2 Feedforward compensation

In this section, necessary design considerations on the improvement of the steady state performance (output ripple) are given in the following. The ripple component contained in the practically nonideal DC input source can be thought as disturbance term and must be reduced effectively. Although a great majority of the disturbance can be reduced by the effect of the feedback controller provided that the closed loop dynamics is much faster than the unknown disturbance, the magnitude of the

reduced ripple component contained in the output voltage, in the practical applications, may often be considered unsatisfactory. An effective compensation technique, therefore, is required to obtain the improved steady state performances. In order to do so, a feedforward control concept is now introduced as shown in Fig. 4. In this figure, the disturbance ( $\hat{v}_g$ ) can be cancelled out by subtracting the effective mangnitude from the control signal entering the input terminal of the pulse width modulation (PWM) amplifier. This type of the feedforward compensator is easily implemented using the operational amplifier and passive elements.

### 3.3 Control problem formulation

When a step reference voltage  $y_s (=V_{ref})$  is added to the buck type switching regulator, the output voltage  $y (=V_{out})$  is desired to track  $y_s$  with no steady state error and the minimum time is also desired to reach the steady state condition with a reasonable overshoot under the limitation of bounded input voltage. In order to meet both the performance requirements and the ease of implementation effectively, we consider the following PIM controller:

$$u(t) = k_i z(t) - k_p [y(t) - y_s(t)] - k_m y_m(t) \quad (24)$$

where the subscript s denotes steady state,  $z(t) = y(t) - y_s(t)$ ,  $z(0) = 0$ , and  $k_i$ ,  $k_p$ , and  $k_m$  are constant feedback gains to be determined. This type of controller provides no steady state error for a step response by the integral action of the output error and better dynamic response is also expected using the measurable variable. We take the time weighted quadratic performance index of the form,

$$J = \int_0^\infty [t^k (y - y_s)' Q (y - y_s) + (u - u_s)' r_1 (u - u_s)] dt \quad (25)$$

to emphasize both the transient responses and the practical limitation of the bounded input problem. In (25), prime denotes transpose, Q and  $r_1$  are the symmetric positive definite matrices, and  $u_s$  and  $y_s$  the steady state input to the power stage and the steady state desired output ( $V_{ref}$ ), respectively. The time weighted quadratic performance index is of prime importance rather than the conventional quadratic performance index ( $k=0$  case in (25)) especially when the sustained error of the out-

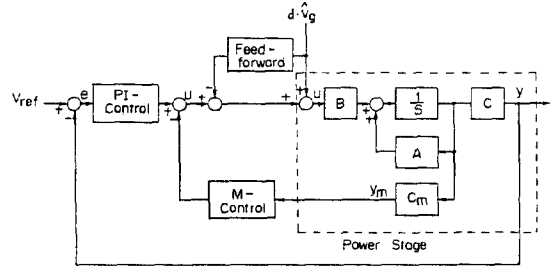


Fig. 4 Block diagram of the closed loop system

put are expected. The control problem is now to be formulated as finding the constant feedback gains of the output PIM control law which minimizes the time weighted quadratic performance index J under the situation of no disturbance in the input by the feedforward compensation.

### 3.4 Optimal constant feedback gains

In the steady state with no output error, steady state values of the states and input are derived from

$$\begin{bmatrix} \bar{x}_s \\ \bar{u}_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ y_s \end{bmatrix} \quad (26)$$

where  $u_s = -k_i z_s - k_m y_{ms}$ . Using the variables  $\bar{x} = x - x_s$ ,  $\bar{u} = u - u_s$ ,  $\bar{y} = y - y_s$ ,  $\bar{y}_m = y_m - y_{ms}$ ,  $\bar{z} = z - z_s$ , we can obtain the following equations:

$$\dot{\bar{x}} = A\bar{x} + B\bar{u}, \quad \bar{x}(0) = -x_s \quad (27-a)$$

$$\bar{y} = C\bar{x} \quad (27-b)$$

$$\dot{\bar{y}}_m = C_m \bar{x} \quad (27-c)$$

$$\bar{z} = \bar{y} = C\bar{x}, \quad \bar{z}(0) = -z_s \quad (27-d)$$

where

$$z_s = -k_i^{-1} u_s - k_i^{-1} k_m y_{ms} \quad (28)$$

Now, we define the new control input v and the augmented state vector w,

$$w = [\bar{z}^T \quad \bar{x}^T]^T, \quad v = \bar{u} \quad (29)$$

Then, the augmented system is described as

$$\dot{w} = \bar{A}w + \bar{B}v, \quad (30)$$

$$v = -Kcw \quad (31)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, w(0) = \begin{bmatrix} -z_s \\ -x_s \end{bmatrix}, \\ K &= [k_i \ k_p \ k_m], \\ \bar{C} &= \begin{bmatrix} I_{r \times r} & 0 \\ 0 & C \\ 0 & C_m \end{bmatrix} \end{aligned}$$

and the performance index is expressed as follows

$$J = \int_0^\infty (t^k w^T \bar{Q} w + w^T \bar{C}^T K^T r_1 K C w) dt \quad (32)$$

where

$$\bar{Q} = \text{diag}(O_{r \times r}, C^T Q C).$$

Using the procedure presented in 17)~19), the necessary conditions for feedback gain matrix  $K$  to be optimal with respect to the performance index (32) is given by

$$\begin{aligned} dJ/dK &= 2 \left[ -\bar{B}^T \sum_{i=1}^{k+1} P_i L_i + r_1 K \bar{C} L_{k+1} \right] \bar{C}^T \\ &- 2 k_i^{-1} \left[ (I_{r \times r}, O_{r \times n}) P_{k+1} \begin{bmatrix} z_s & z_s^T \\ x_s & z_s^T \end{bmatrix}, O_{r \times r}, (I_{r \times r}, \right. \\ &\left. O_{r \times n}) P_{k+1} \begin{bmatrix} z_s & x_s^T \\ x_s & x_s^T \end{bmatrix} C_m^T \right] = 0, \end{aligned} \quad (33)$$

where  $P_i$  and  $L_i$  satisfy the following equations:

$$\begin{aligned} (\bar{A} - \bar{B}K\bar{C})^T P_i + P_i (\bar{A} - \bar{B}K\bar{C}) + \bar{Q} &= 0 \\ (\bar{A} - \bar{B}K\bar{C})^T P_{i+1} + P_{i+1} (\bar{A} - \bar{B}K\bar{C}) + P_i &= 0, \quad (i=1, 2, \dots, k-1) \\ (\bar{A} - \bar{B}K\bar{C})^T P_{k+1} + P_{k+1} (\bar{A} - \bar{B}K\bar{C}) + P_k + \bar{C}^T K^T r_1 \\ K \bar{C} &= 0 \\ (\bar{A} - \bar{B}K\bar{C}) L_i + L_i (\bar{A} - \bar{B}K\bar{C})^T + L_{i+1} &= 0, \quad (i=1, 2, \dots, k) \\ (\bar{A} - \bar{B}K\bar{C}) L_{k+1} + L_{k+1} (\bar{A} - \bar{B}K\bar{C})^T + w(0) w(0)^T &= 0. \end{aligned}$$

Then, the final cost becomes

$$J = w(0)^T P_{k+1} w(0).$$

In the above equation (33), state vector  $x$ , input vector  $u$ , output vector  $y$ , and measurable output vector  $y_m$  are assumed as  $n$ ,  $r$ ,  $r$  and  $p$  dimensional vector, respectively.

#### 4. A design example and experimental results

Using the same nomenclature previously stated, it is assumed that the design specifications are given as follows

$f_s = 15\text{kHz}$ , %ripple=0.05,  $R = 10\Omega$ ,  $D = 0.5$ , full wave rectified input stage, Ac input: 60Hz, single phase.

If we choose  $\alpha = 2.5$  and  $r = 0.1$ , then  $f_1^* = 999\text{Hz}$ ,  $f_2^* = 300\text{Hz}$  from (19) and (20) respectively. From (2) and (16), the range of  $L_1$  is obtained as

$$167\mu\text{H} < L_1 < 333\mu\text{H}$$

and maximum ranges of  $L_2$  and  $C$  are determined from (2), (19) and (20) with  $L_1$  being  $300\mu\text{H}$  as follows:

$$254\mu\text{F} < C < 469\mu\text{F}.$$

These ranges of parameters satisfy the condition (11) sufficiently. The frequency response of the power stage for the selected parameters is simulated and shown in Fig. 5 where it is noted that the equation (12) is well satisfied.

In the computer simulations of the converter using the PIM controller, the following performance index is used,

$$J = \int_0^\infty [(t/\text{tr})^k \bar{y}^2 + r_1 \bar{u}^2] dt \quad (34)$$

where  $k$  is positive integer. Then, the weighting matrix  $\bar{Q}$  becomes as

$$\bar{Q} = \text{diag}(0, 0, (1/\text{tr})^k, 0, 0) \quad (35)$$

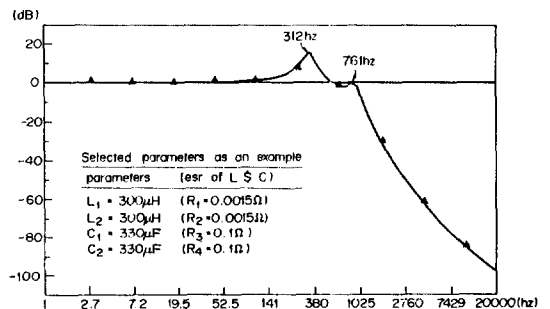


Fig. 5 Frequency response of the power stage for the parameters selected as an example

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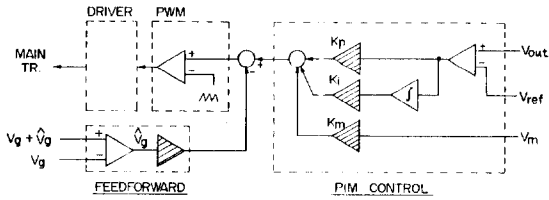


Fig. 6 Circuit diagram of the controller

where  $t_r$  in (34) is used to give relatively larger weighting on the sustained output error square after rising time expected. The circuit diagram of the PIM controller with feedforward compensator is given in Fig. 6. In this figure, the controlled signal activates main switching transistor through the pulse width modulator and transistor driving circuit. According to the optimal gains derived from the proposed algorithm as shown in Table 1, the transient and steady state responses of the output

are simulated and experimented as shown in Fig. 8 through 10. As shown in Table 1, 5% settling time of output response using the time weighted quadratic performance

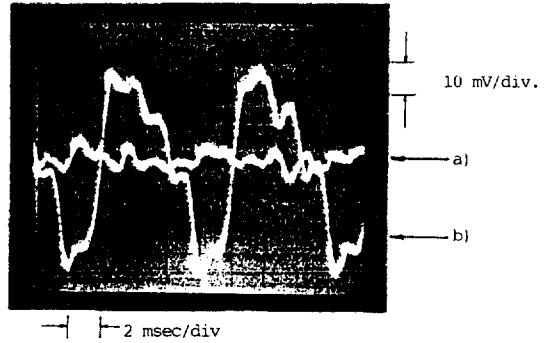


Fig. 7 Steady state output responses (ripple)  
a) with feedforward compensation  
b) without feedforward compensation

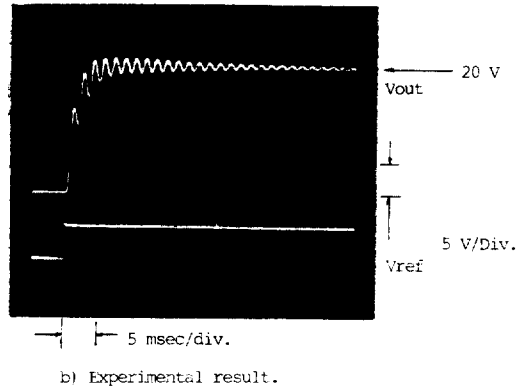
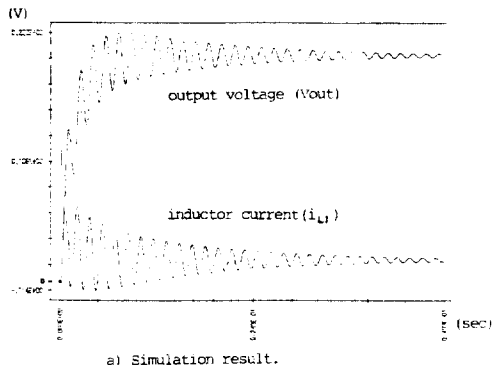


Fig. 8 Transient responses of the closed loop system (in case of  $k=0$ )

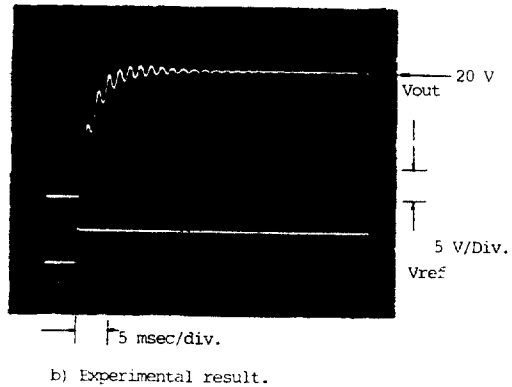
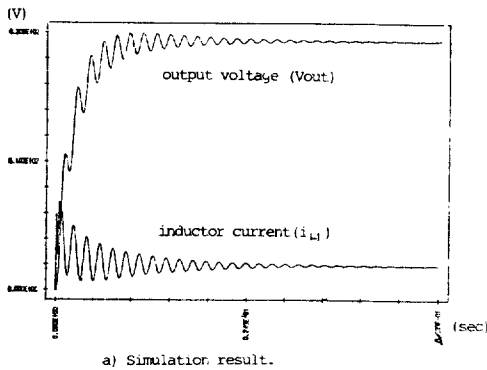


Fig. 9 Transient responses of the closed loop system (in case of  $k=2$ )

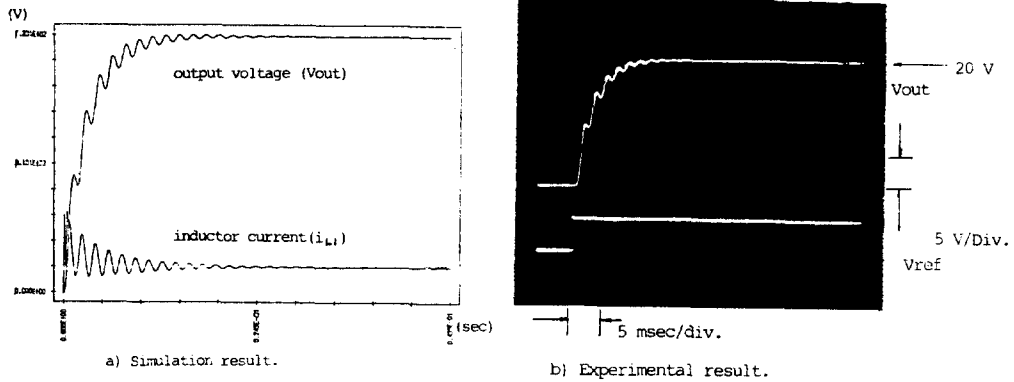


Fig. 10 Transient responses of the closed loop system (in case of  $k=5$ )

index ( $k=2, 5$ ) is shorter than that of using the conventional quadratic performance index ( $k=0$ ) due to the oscillatory response. Furthermore, from the controlled inductor waveforms, it can be considered that the continuous conduction mode operation is well guaranteed as already expected in the power stage design. The effect of feedforward controller is also presented experimentally as shown in Fig. 7 where the steady state output ripple is shown to be reduced significantly. Thus, it is shown that the experimental results well satisfy the theoretical buck type switching regulator.

Table 1. Optimal constant gains and settling times

time weighting index (k)	tr	r	optimal gains			settling time (msec)
			ki	kp	km	
0	0.005	0.2	7211	1.91	11.7	22
2	0.005	0.2	8931	1.00	18.3	10
5	0.005	0.2	6827	1.12	18.8	9.3

### 5. Conclusion

This paper has investigated a design method of the buck type switching regulator to improve transient and steady state performances. The parameters of the power stage has been analytically determined before designing the controller to get better transient responses with less control effort. A design method of an output PIM controller with optimized constant feedback gains has been presented without complicating the controller by using the measurable variable, and a feedforward compensation has also been given to improve steady state

performance in a simple manner. As shown in the design example, steady state output ripple has also significantly been reduced from 60 msec to 10 msec using the feedforward compensation, and 5% settling times of output responses are 9.3 msec for  $k=5$ , 10 msec for  $k=2$ , and 22 msec for  $k=0$  respectively. Thus, it is believed that better transient responses have been obtained using the time weighted quadratic performance index and the usefulness of the proposed design method which covers the design of controller as well as power stage has been proved by the computer simulations and experimental results.

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