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A NOTE ON FUZZY TOPOLOGY, FUZZY GROUPS AND FUZZY TOPOLOGICAL GROUPS

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1. Introduction

Zadeh's introduction [17] of the notion of a fuzzy set in a universe could generalize and extend main concepts and structures of the presentday mathematics into the framework of fuzzy sets. Goguen [6] has studied and generalized the work of Zadeh. The most generalization was the consideration of order structures beyond the unit closed interval. The concept of a fuzzy topological space and some of its basic notions have been studied by Chang [4] as one of applications of concepts of a fuzzy set. In the development of a parallel theory based on fuzzy sets, many interesting phenomena have been observed. For example, the concept of compact fuzzy topological spaces introduced in the literature by Chang holds only for finite products. The next compactness results by Goguen are an Alexander Subbase Theorem and a Tychonov Theorem for finite products and he was the first to point out a deficiency in Chang's compactness. Weiss [15] and Lowen [8] introduced new definitions of compactness in a fuzzy topological space which is available for the infinite products. However, the definition of a fuzzy topological space by them has been pointed out that it has the great deficiency that its definition is not the generalization of an ordinary topological space.

In [13] Resenfield has used the notion of a fuzzy set to develop theory of fuzzy groups. In [1] authors have point out a deficiency in Rosenfield's definition of fuzzy groups. They have used a different structure to define a fuzzy group. The structure is one of stronger conditions than Rosenfield's. Foster [5] has introduced a fuzzy topological group by use of definitions of Lowen's and Rosenfield's. In [11] authors defined a fuzzy topological group by use of the concept of Q-neighborhood introduced in [12] and showed that thier definition and Foster's definition are equivalent under some condition.

Let X be an ordinary nonempty set which we will call the *universe.* A fuzzy set A in X is a function on X into the closed unit interval $[0,1]$, assigning each x in X to its grade of membership $A(x)$ in A. The grade of membership function is often called a *generalized characteristic function.* Fuzzy set operations; inclusion, union, intersection, generalized union and intersection, complement; are defined by use of $\leq$, max, min, sup and inf, $1-$, similarly to the corresponding notions in ordinary set operations, respectively. It is one of important problems that it was shown that in Zadeh's structure of fuzzy set theories the class of generalized characteristic functions is a distributive but noncomplementary lattice and it is just a Brouwerian lattice. Roughly to speak, $A \cap A' = \phi$ does not hold in the fuzzy structure, where A' denotes the complement of A. In fuzzy structure there are problems left, for example, a fuzzy point, compactness in fuzzy topological spaces, fuzzy neighborhoods problems.

In this paper we will use definitions which, we think, are most suitable in the presentday publications. We can find definition without mentions in the available references.

2. Fuzzy points, fuzzy topologies and fuzzy neighborhoods.

How to define a fuzzy point reasonably in a fuzzy set is one of fundamental problems in fuzzy structures. In [12, 16, 17] a fuzzy point was defined in different ways. We will follow the definition in [12, 17], named a fuzzy point instead of a fuzzy singleton in [6].

DEFINITION 2.1. A fuzzy set in X is called a *fuzzy point* if it takes the value 0 for all point y in X except one, say x in X. If its value at x is k in $(0, 1]$, then we will denote the fuzzy point by a lowercase letter $x(k)$.

DEFINITION 2.2. Let $x(k)$ be a fuzzy point and A a fuzzy set in a universe X. Then $x(k)$ is said to *be in* A or A *contains* $x(k)$, denoted by $x(k) \in A$ (or simply $x(k)$ in A), if $k \leq A(x)$ all x in X.

Evidently every fuzzy set A can be expressed as a union of all fuzzy points which belong to A. As we will know later on, the concept of fuzzy points is very important for the construction of fuzzy neighborhood in fuzzy topological spaces. When a mapping between universes is defined, the inverse image and image of fuzzy sets in them were defined almost similarly to those in ordinary sets.

DEFINITION 2.3. Let f: $X \rightarrow Y$ be a mapping of universe X

into universe Y, and A and B fuzzy sets in X and Y, respectively. Then the *inverse image* of B, $f^{-1}(B)$, is the fuzzy set in X with membership given by $f^{-1}(B)(x)=B(f(x))$ for all x in X and the *image* of A, $f(A)$, is the fuzzy set in X with membership given by

$$f(A)(y)=\sup_{z\in f^{-1}(y)} A(z), \text{ if } f^{-1}(y)\neq\phi$$
$$= \quad 0 \qquad\qquad , \text{ otherwise}$$

for all y in Y, where $f^{-1}(y)=\{x|f(x)=y\}$.

DEFINITION 2.4. Let A and B be fuzzy sets in universes X and Y, respectively. The *fuzzy product* $A\times B$ of A and B is defined as the fuzzy set in the usual set product $X\times Y$ with the membership given by $A\times B(x,y)=\min\ (A(x), B(x))$ for all (x,y) in $X\times Y$.

PROPOSITION 2.5. Let p_i be the projection of $X_1\times X_2$ into X_i, for $i=1,2$, and $A=A_1\times A_2$ a fuzzy product in $X_1\times X_2$. Then $p_i(A)\subset A_i$, for each $i=1,2$.

PROOF. If $i=1$, $p_1(A)(x_1)=\sup_{(z_1,z_2)\in p_1^{-1}(x_1)} A(z_1,z_2)$

$$=\sup_{(z_1,z_2)\in p_1^{-1}(x_1)} \min(A_1(z_1), A_2(z_2))$$

$$=\min(A_1(x_1),\ {}_{z_2}\ \sup_{X_2} A_2(z_2))$$

for all x_1 in X_1. Similarly we can prove the case of $i=2$.

Let X be a universe. Then a family $\mathcal{T}$ of fuzzy sets in X is called a *fuzzy topology* on X if (i) $\phi, X\in\mathcal{T}$, where ϕ is a fuzzy empty set (ii) If $A,B\in\mathcal{T}$, then $A\cap B\in\mathcal{T}$ (iii)

If $A_i \in \mathcal{T}$ for all $i \in I$, then $\bigcup_{i \in I} A_i \in \mathcal{T}$. The pair $(X, \mathcal{T})$ is called a *fuzzy topological space*(for short, fts) and the members of $\mathcal{T}$ are called *$\mathcal{T}$-open fuzzy set*. The complement of a $\mathcal{T}$-open fuzzy set is called a *$\mathcal{T}$-closed fuzzy set*. We will drop $\mathcal{T}$ without confusions. If $\mathcal{T}_1$ and $\mathcal{T}_2$ are fuzzy topologies on a universe X and $\mathcal{T}_1 \subset \mathcal{T}_2$, then we say that $\mathcal{T}_2$ is *finer* than $\mathcal{T}_1$ or $\mathcal{T}_1$ is *coarser* than $\mathcal{T}_2$. A *base* and a *subbase* of an fts were defined as the similar way in an ordinary topological space and the *interior* of a fuzzy set is defined as the largest open fuzzy set contained in the fuzzy set and the *closure* of a fuzzy set is defined as the smallest closed fuzzy set containing the fuzzy set. The properties of the interior and the closure are like those in the usual topological spaces.

The neighborhood of a fuzzy point in an fts has been defined in different manners [4, 6, 8, 12, 15, 16]. A fuzzy set N in an fts $(X, \mathcal{T})$ is called a *neighborhood*(for short, nbd) of fuzzy point $x(k)$ if there is an O in $\mathcal{T}$ such that $x(k) \in O \subset A$. In [12], corresponding to this, authors have defined a more reasonable definition by use of a new concept. We will use it.

DEFINITION 2.6. A fuzzy point $x(k)$ is said to be *quasi-coincident* with a fuzzy set A, denoted by $x(k)\ q\ A$, if $k > A'(x)$ or $k + A(x) > 1$. The quasi-coincident with two fuzzy sets A and B, denoted by $A\ q\ B$, means that there exists x in X such that $A(\mathrm{x}) > B'(x)$, or $A(x) + B(x) > 1$.

DEFINITION 2.7. A fuzzy set N in an fts $(X, \mathcal{T})$ is called a *Q-nbd* of $x(k)$ if there is an O in $\mathcal{T}$ such that $x(k)$ $q\ O \subset N$; a nbd N is said to be *open* iff N is open.

It was shown in [10] that $A\subset B$ iff A and B' are not quasi-coincident; $x(k)$ is in a fuzzy set A iff $x(k)$ is not quasi-coincident with A'. From the fact, the substitute for the fact that A and A' do not intersect in general topology is the fact that A and A' are not quasi-coincident in fuzzy topology. This means much suitability for definition, while in Zadeh's theory the class of generalized characteristic fuctions is just a Brouwerian lattice.

Let $(X, \mathcal{T})$ be an fts and A a fuzzy set in X. It is easy to prove that the family $\mathcal{T}_A=\{A\cap U|U\in\mathcal{T}\}$ is a fuzzy topology on A. Thus we say that the pair $(A, \mathcal{T}_A)$ is called a *fuzzy subspace* of $(X, \mathcal{T})$. A mapping f of an fts $(X, \mathcal{T})$ into an fts $(Y, \mathcal{U})$ is said to be *fuzzy continuous* (for short, F-continuous) if, for each B in $\mathcal{U}$, $f^{-1}(B)$ is in $\mathcal{T}$. We will denote a mapping f an fts $(X, \mathcal{T})$ into an fts $(Y, \mathcal{U})$ by f: $(X, \mathcal{T})\to(Y, \mathcal{U})$.

PROROSITION 2.8. Let f: $(X, \mathcal{T})\to(Y, \mathcal{U})$ be a mapping. Then the following are equivalent:

(1) f is F-continuous.

(2) For each $\mathcal{U}$-open fuzzy set V, $f^{-1}(V)$ is $\mathcal{T}$-open fuzzy set.

(3) For any nbd V of $f(x(k))$, there exists a nbd U of $x(k)$ such that $f(U)\subset V$.

(4) For each fuzzy point $x(k)$ in X and each Q-nbd V of $f(x(k))$, there exists a Q-nbd U of $x(k)$ such that $f(U)\subset V$.

PROOF. We will prove (1)$\Longleftrightarrow$(4) and leave the remainder for readers.

Let $f(x)=y$. Then $f(x(k))=y(k)$ from definitions. Since V is a Q-nbd of $f(x(k))$, there is W in $\mathcal{U}$ such that $W \subset V$ and $W(y)+k>1$. Let $f^{-1}(W)=U$. Then U is in $\mathcal{T}$ and $U(x)+k=W(y)+k>1$. Thus U is a Q-nbd of $x(k)$ and $f(U)=ff^{-1}(W) \subset V$. The converse is obvious.

REMARK 2.9. It is easy to prove that the restriction of a mapping f of $(X,\mathcal{T})$ to $(Y,\mathcal{U})$ and the composition $g\circ f$ of f and g is F-continuous if f and g are F-continuous and we can get some theorem for complete condition to be F-continuous by means of interior, closure and so on.

DEFINITION 2.10. Let $(A,\mathcal{T}_A)$ and $(B,\mathcal{U}_B)$ be fuzzy subspaces of fts's $(X, \mathcal{T})$ and $(Y, \mathcal{U})$, respectively. Then a mapping f: $(A,\mathcal{T}_A)\to(B,\mathcal{U}_B)$ is said to be *relatively fuzzy continuous* (for short, RF-continuous) if, for each W in $\mathcal{U}_B$, $f^{-1}(W)\cap A$ is in $\mathcal{T}_A$.

PROPOSITION 2.11. Let f: $(X,\mathcal{T})\to(Y,\mathcal{U})$ be fuzzy continuous and $(A, \mathcal{T}_A)$, $(B,\mathcal{T}_B)$ fuzzy subspaces, respectively. If $f(A) \subset B$, then f:$(A,\mathcal{T}_A)\to(B,\mathcal{U}_B)$ is RF-continuous.

PROOF. To apply Proposition 2.8, let $a \in A$ and N a Q-nbd of $f(a)$ and $f(a)=b$. Then $f(a(k))=b(k)$. Since N is a Q-nbd of $f(a(k))$, there is a V in $\mathcal{U}_B$ such that $b(k)$ $qV\subset N$, that is, $V\subset N$, $V(y)+k>1$. Since V is in $\mathcal{U}_B$, there is an $O\in\mathcal{U}$ such that $V=B\cap O$. Thus $f^{-1}(V)=$ $f^{-1}(B\cup O)=A\cup f^{-1}(O)$. So we have $f^{-1}(V)\in\mathcal{T}_A$ because $f^{-1}(O)\in\mathcal{T}$ from the F-continuity of f. Let $U=f^{-1}(V)$. Then U is a Q-nbd of $a(k)$ such that $f(U)\subset V$.

DEFINITION 2.12. Let $\{(X_i,\mathcal{T}_i)|i \in I\}$ be a family of fts's and $X=\prod_{i\in I} X_i$ the usual set product. Then $(X, \mathcal{T})$ is called the *product fts* if $\mathcal{T}$ is the coarsest fuzzy topology on X such the projection p_i of X onto X_i is fuzzy continuous for each $i \in I$. The fuzzy topology $\mathcal{T}$ is called the *product fuzzy* on X.

REMARK 2.13. Since the concept of the product fts is almost similar to that of the product space in the usual sense, Thus it can be shown that the product fuzzy topology $\mathcal{T}$ on X has the fuzzy set of the form $p^{-1}(U_i)$ as a subbase where U_i is in $\mathcal{T}_i$, $i\in I$. Therefore, the base for $\mathcal{T}$ is the form of finite intersection of $\{p_i^{-1}(U_i) | U_i \in \mathcal{T}_i\}$.

PROPOSITION 2.14. Let $\{(X_i, \mathcal{T}_i)\}$, $i\in I$, be a family of fts's, $(X, \mathcal{T})$ the product fts and $f:(Y, \mathcal{U}) \to (X, \mathcal{T})$ a mapping. Then f is F-continuous iff $p_i \circ f$ is F-continuous for each i.

PROOF. Let $B\in\mathcal{T}_i$. Then $(p_i \circ f)^{-1}(B) = (f^{-1} \circ p_i^{-1})(B)$ is in $\mathcal{U}$. Hence $\{(f^{-1}[p_i^{-1}(B)]) | i \in I\}$ is a family of $\mathcal{U}$-open fuzzy set in Y. Since f^{-1} preserves union and intersection in fuzzy sets as well as in ordinary sets, f is F-continuous. The converse is trivial.

PROPOSITION 2.15. Let $(X, \mathcal{T})$ be the product fts of $\{(X_i, \mathcal{T}_i) | i=1, 2, \cdots, n\}$. Let each A_i be fuzzy set in X_i and A a product fuzzy set in X. Let B be a fuzzy set in a fts $(Y, \mathcal{U})$ and f: $(B, \mathcal{U}_B) \to (A, \mathcal{T}_A)$ a mapping. Then f is RF-continuous iff $p_i \circ f$ is RF-continuous for each $i \in I$.

PROOF. Apply Propositions 2.5, 2.11 and Remark 2.13.

PROPOSITION 2.16. Let A and B be fuzzy sets and C the product fuzzy set of fts's $(X,\mathcal{T})$, $(Y,\mathcal{U})$ and $(Z,\mathcal{V})$ the product fts, respectively. Then for each $a \in X$ such that $A(a) \geq B(y)$ for all y in Y, the mapping $i: y \to (a, y)$ of $(B,\mathcal{U}_B)$ into $(C,\mathcal{V}_C)$ is RF-continuous.

PROOF. We have $i(B) \subset C$ from the membership function of $i(B)$ and the concept of product fuzzy set. It is shown that the identity and constant mappings are F-continuous. From this and the F-continuity of composition, we apply Proposition 2.14. Let $i_1: y \to a$ and $i_2: y \to y$ be mappings. Then $i = i_1 \circ i_2$. Using Proposition, the proof is complete.

3. Fuzzy groups and Fuzzy topological groups.

Rosenfield [13] has defined a fuzzy groups to extend and to generalize the notion of groups structures; let X be a universal group and G a fuzzy set in X with the grade of membership $G(x)$ for all x in X. Then G is called a *fuzzy group* in X if, for every x, y in X, (i) $G(xy) \geq \min(G(x), G(y))$ (ii) $G(x^{-1}) \geq G(x)$. In [1] authors pointed out a deficiency in it and gave examples which are groups in usual sense, but not fuzzy groups in sense of Rosenfield. This means that a fuzzy group may be not a generalization of a groups. To get rid of the default, they have used different operator, so called t-norm, to define a fuzzy group.

DEFINITION 3.1. A *t-norm* is a function $T: [0,1] \times [0,1] \to [0,1]$ satisfying; for each a, b, c, d in $[0,1]$,

(1) $T(0,0)=0$, $T(a,1)=a=T(1,a)$ (boundary conditions)

(2) $T(a,b) \leq T(c,d)$ whenever $a \leq c, b \leq d$ (monotonicity)

(3) $T(a,b) = T(b,a)$ (symmetric)

(4) $T(T(a,b),c) = T(a, T(b,c))$ (associativity)

EXAMPLE 3.2. T_W is defined by the boundary conditions and $T_W(a,b)=0$ for each (a,b) in $[0,1)\times[0,1)$, $\min(a,b)$, $T_m = \max(0, a+b-1)$ and $\mathrm{Prod}(a,b)=ab$ are t-norms.

REMARK 3.3. The t-norms in Example 3.2 hold obviously;

(1) $T_W(a,b) \leq T_m(a,b) \leq \mathrm{Prod}(a,b) \leq \min(a,b)$

(2) For any t-norm T, $T_W(a,b) \leq T(a,b) \leq \min(a,b)$

DEFINITION 3.4. Let X be a universal group and G a fuzzy set in X. Then G is called a *fuzzy group* in X if, for each x, y in X, (i) $G(xy) \geq T(G(x), G(y))$ (ii) $G(x^{-1}) \geq G(x)$, where T is a t-norm defined.

PROPOSITION 3.5. G is a fuzzy group in X iff, for every x, y in X, $G(xy^{-1}) \geq T(G(x), G(y))$.

PROOF. Let G be a fuzzy group in X. Then $G(xy^{-1}) \geq T(G(x), G(y^{-1})) \geq T(G(x), G(y))$ because $G(x^{-1}) \geq G(x)$ for all x in X and from the monotonicity of T. The converse follows from [13, 5.6] because we can replace min by T from Remark 3.3 and is thus omitted.

PROPOSITION 3.6. Let f be a homomorphism of group X into group Y in usual sense G a fuzzy group in Y. Then the inverse image $f^{-1}(G)$ of G is a fuzzy group in X.

PROOF. For all x, y in X, applying Proposition 3.5, $f^{-1}(G)(xy^{-1}) = G(f(xy^{-1})) = G(f(x)(f(y))^{-1}) \geq T(G(f(x)), G(f(y)^{-1}) \geq T(G(f(x)), G(f(y)))$.

REMARK 3.7. Similarly, it can be shown without much difficulty that the image $f(G)$ of a fuzzy group G under the homomorphism f(in usual sense) is a fuzzy group.

PROPOSITION 3.8. Let G be a fuzzy group in a group X and e the identity in X. Then $G(x^{-1})=G(x)$ and $G(x)\leq G(e)$.

PROOF. $G(x)=G((x^{-1})^{-1})\geq G(x^{-1})\geq G(x)$ for all x in X. The remainders can be shown similary to Proposition 3.5 and are thus omitted.

Kaufmann [7] has introduced the concept of ordinary subset of level t of a fuzzy set to decompose a fuzzy set into a ordinary set. Lat A be a fuzzy set of X. Then the ordinary set $A_t=\{x\in X|A(x)\geq t\}$ is called a *level subset* of fuzzy set A. He has shown that every fuzzy set can be decomposed as products of ordinary susets (i.e., the level subsets) and a number in [0,1]. Thus some questions will be arised; what a level subset of a fuzzy group of a universal group will be a subgroup of the group? One of the answer is, so-called, level subgroup, the level subset $A_t=\{x\in X|t\leq A(e)$, $t\in[0,1]$ and e is the identity in $X\}$. It can prove that the A_t is a subgroup in X; The number of such level subgroups in X may depend on t. Since $t\in[0,1]$, there can be an infinite number of level subgroups in X although X is finite. However it means a contradiction because the number of all subsets of a finite group must be finite. Thus we have a question: when level subgroups of a fuzzy groups are equal each other?

Let G be a fuzzy group X and $G_e=\{x\in X|G(x)=G(e)\}$.

Then G_e is one of level subgroups in X and for $a\in X$, let r_a: $x\to xa$ and l_a: $x\to ax$ denote, respectively, right and left translations of X into itself.

PROPOSITION 3.9. Let G be a fuzzy group in a group X. Then for all $a\in G_e$, $r_a(G)=l_a(G)=G$.

PROOF: Let $a\in G_e$. Then $a^{-1}\in G_e$ since G_e is a subgroup. Since $G(e)=1$ and $T(k,1)=k$, $l_a(G(x))=G(xa^{-1})\geq T(G(x), G(e))=G(x)=G(xa^{-1}a)\geq T(G(xa^{-1}), G(e))=G(xa^{-1})=l_a(G(x))$ for all x in X. The proof for r_a is similar to this.

We will study properties of fuzzy topological groups(for short, ftg) from now on. Foster [5] has defined an ftg by means of the fts in sense of Lowen and the fuzzy group in sense of Rosenfield. In [11] authors have defined it by use of Q-nbd of fuzzy points and showed that their concept is equivalent to Foster's. We will apply the definitions in this note to define an ftg.

Let X be a universal group and A, B fuzzy sets in X. We define AB and A^{-1} by the respective formulas; $AB(x)=\sup_{yz=x}\min(A(y), A(z))$ and $A^{-1}(x)=A(x^{-1})$ for each x in X.

DEFINITION 3.10. Let X be a universal group and $(X,\mathcal{T})$ an fts. Let G be a fuzzy group and $(G,\mathcal{T}_G)$ a fuzzy subspace of $(X,\mathcal{T})$. Then G is called an *ftg* if

(i) The mapping g: $(x,y)\to xy$ of $(G,\mathcal{T}_G)\times(G,\mathcal{T}_G)$ into $(G,\mathcal{T}_G)$ is RF-continuous.

(ii) The mapping h:$x\to x^{-1}$ of $(G,\mathcal{T}_G)$ into itself is RF-

continuous.

EXAMPLE 3.11. Let $X=(R,+)$ be the usual topological group. Let $\mathcal{L}$ be a family of all lower semicontinuous functions of X into $[0,1]$. Then $(X,\mathcal{L})$ is ftg because $\mathcal{L}$ is a fuzzy topology on X.

PROPOSITION 3.12. Let X be a universal group and $(X,\mathcal{T})$ an fts. Then a fuzzy group G in X is an ftg iff the mapping f: $(x,y)\to xy^{-1}$ of $(G,\mathcal{T}_G)\times(G,\mathcal{T}_G)$ into $(G,\mathcal{T}_G)$ is RF-continuous.

PROOF. We can get from Proposition 2.15 that the mapping f is RF-continuous. Since the composition of RF-continuous mappings is RF-continuous, the composition $(x,y)\to(x,y^{-1})\to xy^{-1}$ is RF-continuous. Conversely, let e be the identity in X, then we have $G(x)\leq G(e)$ for all x in X from Proposition 3.8. Let i be a mapping of $(G,\mathcal{T}_G)\times(G,\mathcal{T}_G)$ such that $i:y\to(e,y)$. Then i is RF-continuous from Proposition 2.16 and $h:x\to x^{-1}$ is a composition of $x\to(e,x)\to ex^{-1}$. Hence h is RF-continuous. Similarly, $g:(x,y)\to(x,y^{-1})\to x(y^{-1})^{-1}$ is RF-continuous.

PROPOSITION 3.13. Let X be a topological group. Then X is an ftg iff for any Q-nbd W of $ab^{-1}(k)$, there are Q-nbds U of $a(k)$ and V of $b(k)$ such that $UV^{-1}\subset W$.

PROOF. It is similar to the proof in general topological groups and so is omitted.

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