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BAYESIAN SHRINKAGE ESTIMATION OF THE RELIABILITY FUNCTION FOR THE LEFT TRUNCATED EXPONENTIAL DISTRIBUTION

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1. Introduction.

Consider the two parameters exponential distribution with positivity constraint on the truncation parameter defined by the probability density function,

$$(1)\quad f(x|\theta, \lambda)=\lambda^{-1}\exp[-\lambda^{-1}(x-\theta)], \ x>\theta, \ \lambda>0,$$

where $\theta>0$ for the density to be left truncated.
The model (1) will be referred to as the left truncated exponential distribution.

It is well-known that the left truncated exponential distribution is really appropriate as a lifetime distribution model for reliability and life-testing.

Evans and Nigm (1980) investigated that the use of the two parameters exponential distribution with no positivity constraint on the truncation parameter as a lifetime distribution model is unrealistic and may lead to inefficient inferences and prediction.

Both classical and Bayesian estimation of the reliability function for the two parameters exponential distribution with or with no positivity constraint on the truncation parameter have studied by many authors, including Basu (1964), Varde(1969), Grubbs(1971), Pierce(1973), Sinha and Guttman(1976), Sinha and Kale(1980), Martz and Waller (1982), Trader(1985) and so on.

Trader(1985) considered the truncated normal distribution as a prior distribution for the truncation parameter in the left truncated exponential distribution.

The shrinkage estimation techniques have been advocated by a number of authors as a procedure for lowering the mean squared-error (MSE) of the minimum variance unbiased estimator (MVUE) or maximum likelihood estimator(MLE) [see Thompson(1968), Mehta and Srinivasan(1971), Pandey and Singh(1980), Pandey and Srivastava(1985) and so on].

In recent year, the use of Bayes shrinkage estimation of the parameters for binomial, Poisson and normal distributions were considered by Lemmer (1981) at first. But he has been little paid attention to lifetime distributions-exponential, Weibull etc.

Pandey and Upadhyay(1985) considered the Bayes shrinkage estimation of the parameters for Weibull distribution, and discussed the relative s-efficiencies of these Bayes shrinkage estimators with respect to the unbiased estimators of Engelhardt and Bain(1977) on the basis of a Monte Carlo study of 500 random samples.

In this study, we will consider some Bayes shrinkage estimators of the reliability function in the left truncated exponential distribution.

First, we will give the MVUE and Bayes estimators of the reliability function with the noninformative and conjugate prior distributions in this model.

Next, using the Bayes estimator instead of the guessed value which is close to the true unknown value, such as that given by Pandey(1985), we will propose some Bayes shrinkage estimators of the reliability function in this model.

Finally, we will compare the relative s-efficiencies of the Bayes shrinkage estimators with respect to the MVUE by the Monte Carlo simulation and numerical evaluation technique in the sense of MSE.

2. MVUE and Bayes estimators of the reliability function.

Let $x_{(1)} \leqq x_{(2)} \leqq \cdots \leqq x_{(r)}$ be the first r ordered observations of n failure times form the left truncated exponential distibution(1) under test without replacement.

For a given time t, the reliability function, the probability that survival until time t, is given by

$$(2) \quad R_t = 1 - F(t) = \int_t^\infty f(x)dx = \exp\left(-\frac{t-\theta}{\lambda}\right), \quad t \geqq \theta,$$

where F is the cummulative distribution function of the failure time x.

Basu(1964) obtained the MVUE of the reliability function at time t to be

$$(3) \quad \hat{R}_t = \begin{cases} \frac{n-1}{n}\left(1 - \frac{t-x_{(1)}}{y_r}\right)^{r-2} & , \quad x_{(1)} < t < x_{(1)} + y_r \\ 1 & , \quad x_{(1)} > t \\ 0 & , \quad t > x_{(1)} + y_r \end{cases} ,$$

where $y_r = \sum_{i=1}^{r} (x_{(i)} - x_{(1)}) + (n-r)(x_{(r)} - x_{(1)})$.

The likelihood function is given by

$$(4)\quad L(\theta,\lambda|x_{(1)},\ x_{(2)},\cdots,\ x_{(r)})\propto\lambda^{-r}\exp\{-\lambda^{-1}[y_r+n(x_{(1)}-\theta)]\},$$
$$0<\theta<x_{(1)},\ \lambda>0.$$

The noninformative joint prior distribution of θ and λ is taken as [Sinha and Guttman(1976)]

$$(5)\quad g_1(\theta,\lambda)=\begin{cases}(1-b) & ,\ \text{if } \theta=\theta_0,\ \lambda=\lambda_0\\ b/\lambda^a & ,\ \text{otherwise,}\end{cases}$$

where $a>0$, $0\leq b\leq 1$, $\lambda>0$, $0<\theta<x_{(1)}$, θ_0, λ_0 are the prior values in the vicinities of the true values θ and λ, respectively, and the prior distribution has weight $(1-b)$ in the prior values and weight b in the rest intervals.

We obtain the joint posterior distribution of θ and λ as

$$(6)\quad g_1(\theta,\lambda|x_{(1)},x_{(2)},\cdots,x_{(r)})$$
$$=\frac{L(\theta,\lambda|x_{(1)},\ x_{(2)},\cdots,\ x_{(r)})g_1(\theta,\ \lambda)}{\int_0^{x(1)}\int_0^\infty L(\theta,\ \lambda|x_{(1)},\ x_{(2)},\cdots,\ x_{(r)})g_1(\theta,\lambda)\,d\lambda d\theta}.$$

Therefore, from (4), (5) and (6), the Bayes estimator of the reliability function with the noninformative prior distribution under the squared-error loss can be written as

$$(7)\quad R^*_{t1}=E[R_t|x_{(1)},\ x_{(2)},\cdots,\ x_{(r)}]$$
$$=\int_0^{x(1)}\int_0^\infty R_t\, g_1(\theta,\lambda|x_{(1)},\ x_{(2)},\cdots,\ x_{(r)})\,d\lambda d\theta$$
$$=\frac{P_{r+a-2}(p_1+1,\ p_2+t-x_{(1)},\ p_3+t)+Q(q_1+t-\theta_0)}{P_{r+a-2}(p_1,\ p_2,\ p_3)+Q(q_1)}$$

where

$$P_{r+a-2}(p_1,\ p_2,\ p_3)=b\ \frac{\Gamma(r+a-2)}{p_1}\ \frac{1}{p_2^{\,r+a-2}}\ \left(1-\left(\frac{p_2}{p_3}\right)^{r+a-2}\right),$$

$$Q(q_1)=(1-b)\lambda_0^{-r}\exp\left(-\frac{q_1}{\lambda_0}\right),$$

$$p_1=n,\ \ p_2=y_r,\ \ p_3=y_r+nx_{(1)},\ \ q_1=y_r+nx_{(1)}-n\theta_0.$$

Also, we can use the conjugate joint prior distribution of θ and λ as [Evans and Nigm(1980)]

$$(8)\quad g_2(\theta,\lambda)=\begin{cases}(1-b) & ,\ \text{if } \theta=\theta_0\ ,\ \lambda=\lambda_0\\ b\dfrac{1}{\lambda^c}\exp\left\{-\dfrac{d+h(m-\theta)}{\lambda}\right\}, & \text{otherwise,}\end{cases}$$

where to be a proper prior distribution we must have $c>2$, $d>0$, $h>0$, and $0<\theta<m$, $\lambda>0$.

From (8), the joint posterior distribution of θ and λ becomes

$$(9)\quad g_2(\theta,\lambda|x_{(1)},x_{(2)},\cdots,x_{(r)})$$
$$=\frac{L(\theta,\lambda|x_{(1)},x_{(2)},\cdots,x_{(r)})\,g_2(\theta,\lambda)}{\int_0^A\int_0^\infty L(\theta,\lambda|x_{(1)},x_{(2)},\cdots,x_{(r)})g_2(\theta,\lambda)d\lambda d\theta},$$

where $A=\min(m,x_{(1)})$.

Therefore, from (4), (8) and (9), the Bayes estimator of the reliability function with the conjugate prior distribution under the squared-error loss can be written as

$$(10)\quad R^*_{t2}=E(R_t|x_{(1)},x_{(2)},\cdots,x_{(r)})$$
$$=\int_0^A\int_0^\infty R_t\,g_2(\theta,\lambda|x_{(1)},x_{(2)},\cdots,x_{(r)})d\lambda d\theta$$
$$=\frac{\begin{array}{c}P_{r+c-2}(p_1+h+1,\ p_2+nx_{(1)}+d+hm+t-\\(n+h+1)A,\ p_3+d+hm+t)+Q(q_1+t-\theta_0))\end{array}}{\begin{array}{c}P_{r+c-2}(p_1+h,\ p_2+nx_{(1)}+d+hm-(n+h)A,\\p_3+d+hm)+Q(q_1)\end{array}}$$

3. Bayes shrinkage estimators of the reliability function.

Let us consider a shrunken estimator of the form

$$T = k(\hat{\theta} - \theta_0) + \theta_0 ,$$

where $0 \leq k \leq 1$, $\hat{\theta}$ is the MVUE of θ and θ_0 is a prior value which is close to the true unknown value θ.

This shrinkage estimator for θ was considered by Thompson (1968) at first, and showed that T is more efficient than MVUE for mean parameter in the sense of MSE when sample size is small and θ_0 is in the vicinity of true value θ in the normal, Poisson, binomial and gamma population.

Now we propose two classes of the Bayes shrinkage estimator of the reliability function:

(12) $$T_{Rt1} = k_1(\hat{R}_t - R^*_{t1}) + R^*_{t1} = k_1\hat{R}_t + (1-k_1)R^*_{t1},$$

(13) $$T_{Rt2} = k_2(\hat{R}_t - R^*_{t2}) + R^*_{t2} = k_2\hat{R}_t + (1-k_2)R^*_{t2},$$

where $0 \leq k_1, k_2 \leq 1$, $\hat{R}_t$ is the MVUE of R_t, R^*_{t1} is the Bayes estimator of R_t with the noninformative prior distribution and R^*_{t2} is the Bayes estimator of R_t with the conjugate prior distribution.

4. Comparisons of the relative s-efficiencies of the Bayes shrinkage estimators with respect to the MVUE.

The relative s-efficiencies of the Bayes shrinkage estimators with respect to the MVUE of the reliability function are given by

(14) $\mathrm{REF}_1(T_{Rt1}, \hat{R}_t) = \mathrm{Var}(\hat{R}_t)/\mathrm{MSE}(T_{Rt1})$,

(15) $\mathrm{REF}_2(T_{Rt2}, \hat{R}_t) = \mathrm{Var}(\hat{R}_t)/\mathrm{MSE}(T_{Rt2})$,

where $\hat{R}_t$ is the MVUE of R_t, T_{Rt1} is the Bayes shrinkage estimator of R_t with the noninformative prior distribution (5) and T_{Rt2} is the Bayes shrinkage estimator with the conjugate prior distribution (8).

A Monte Carlo study has been performed on the relative s-efficiencies of the proposed Bayes shrinkage estimators with respect to the MVUE and the numerical values of the relative s-efficiencies of proposed Bayes shrinkage estimators with respect to the MVUE have been evaluated by use of the computer system.

The Monte Carlo simulation on the relative s-efficiencies of T_{Rt1} and T_{Rt2} with respect to $\hat{R}_t$ has been performed as the following four parts.

Part 1: 500 random samples of the first r ordered failure times were generated from the left truncated exponential distribution (1) with the parameters λ and θ such that θ/θ_0 is fixed at 1 and λ/λ_0: 0.50(0.25) 1.75 varies with (n, r), and REF_1 $(T_{Rt1}, \hat{R}_t)$ were evaluated for $a=1, b=0.2$ (0.2) 0.8 and $k_1=0.2$ (0.2) 0.8 to avoid complexity on the table 1.

Part 2: 500 random samples of the first r ordered times were generated from the left truncated exponential distribution (1) with the parameters λ and θ such that λ/λ_0 is fixed at 1 and θ/θ_0: 0.50(0.25) 1.75 varies with (n, r), and $\text{REF}_1(T_{Rt1}, \hat{R}_t)$ were evaluated for $a=2$, $b=0.2(0.2)$ 0.8 and $k_1=0.2(0.2)$ 0.8 on the table 2.

Part 3: 500 radom samples of the first r ordered failure times were generated from the left truncated exponential distribution (1) with the parameters λ

and θ such that θ/θ_0 is fixed at1 and λ/λ_0: 0.50 (0.25) 1.75 varies with (n, r), and REF_2 (T_{Rt2}, $\hat{R}_t$) were evaluated for $c=4$, $d=2$, $h=1$, $m=2$, $b=0.2(0.2)$ 0.8 and $k_2=0.2(0.2)$ 0.8 on the table 3.

Part 4: 500 *random samples of the first* r ordered failure times were generated from the left truncated exponential distribution (1) with the parameters λ and θ such that λ/λ_0 is fixed at 1 and θ/θ_0: 0.50(0.25) 1.75 varies with (n, r), and $\text{REF}_2(T_{Rt2}, \hat{R}_t)$ were evaluated for $c=5$, $d=2$, $h=1$, $m=2$, $b=0.2(0.2)$ 0.8 and $k_2=0.2(0.2)$ 0.8 on the table 4.

Throughout the table 1-4, we obtain the following results:

(a) T_{Rt1} is more efficient than MVUE $\hat{R}_t$ in the sense of MSE for all possible values of n, r, a, b and k_1 contained the effective interval which is in the vicinity of true value λ or θ.

(b) T_{Rt2} is also much more efficient than MVUE $\hat{R}_t$ in the sense of MSE for all possible values of n, r, c, d, h, m, b and k_2 contained the effective interval which is in the vicinity of true value λ or θ.

(c) When the guessed value λ_0 is true, that is λ/λ_0 is 1, T_{Rt1} and T_{Rt2} are most efficient in the sense of MSE.

(d) T_{Rt2} is more efficient than T_{Rt1} in the sense of MSE.

Table 1. Relative s-efficiencies of T_{Rt1} with respect to $\hat{R}_t$

(a=1)

λ/λ_0	n	r	k_1 = 0.2, b = 0.2	0.2, 0.4	0.2, 0.6	0.2, 0.8	0.4, 0.2	0.4, 0.4	0.4, 0.6	0.4, 0.8	0.6, 0.2	0.6, 0.4	0.6, 0.6	0.6, 0.8	0.8, 0.2	0.8, 0.4	0.8, 0.6	0.8, 0.8
0.50	10	7	0.889	1.041	1.117	1.163	0.969	1.066	1.113	1.141	1.020	1.067	1.090	1.103	1.032	1.044	1.052	1.057
	20	15	0.785	0.950	1.036	1.039	0.856	0.979	1.040	1.077	0.920	0.998	1.035	1.057	0.970	1.005	1.021	1.031
0.75	10	7	1.566	1.385	1.292	1.234	1.477	1.316	1.239	1.192	1.333	1.221	1.168	1.136	1.166	1.112	1.087	1.071
	20	15	1.228	1.198	1.168	1.144	1.247	1.181	1.144	1.119	1.207	1.138	1.106	1.086	1.118	1.076	1.057	1.046
1.00	10	7	2.084	1.587	1.384	1.269	1.727	1.424	1.291	1.212	1.431	1.269	1.193	1.146	1.192	1.127	1.095	1.074
	20	15	2.166	1.588	1.351	1.216	1.767	1.417	1.260	1.166	1.448	1.261	1.170	1.113	1.197	1.123	1.083	1.057
1.25	10	7	1.659	1.426	1.313	1.244	1.461	1.309	1.236	1.192	1.286	1.198	1.157	1.132	1.132	1.095	1.077	1.067
	20	15	1.366	1.272	1.210	1.163	1.278	1.201	1.157	1.126	1.185	1.132	1.104	1.036	1.091	1.065	1.052	1.044
1.50	10	7	1.297	1.210	1.234	1.207	1.222	1.186	1.173	1.165	1.146	1.123	1.117	1.115	1.072	1.061	1.059	1.059
	20	15	0.988	1.011	1.075	1.102	0.999	1.034	1.060	1.032	1.005	1.025	1.042	1.053	1.005	1.013	1.022	1.030
1.75	10	7	1.126	1.152	1.170	1.184	1.033	1.118	1.135	1.143	1.069	1.031	1.093	1.105	1.035	1.041	1.048	1.054
	20	15	0.830	0.975	1.034	1.033	0.919	0.934	1.031	1.039	0.947	0.992	1.021	1.030	0.974	0.997	1.013	1.027

Table 2. Relative s-efficiencies of $T_{R t1}$ with respect to $\hat{R}_t$

(a=2)

θ/θ_0	n	r	k_1 = 0.2, b = 0.2	0.2, 0.4	0.2, 0.6	0.2, 0.8	0.4, 0.2	0.4, 0.4	0.4, 0.6	0.4, 0.8	0.6, 0.2	0.6, 0.4	0.6, 0.6	0.6, 0.8	0.8, 0.2	0.8, 0.4	0.8, 0.6	0.8, 0.8
0.50	10	7	2.055	2.087	2.109	2.113	2.530	2.483	2.406	2.254	2.220	2.155	2.067	1.917	1.538	1.508	1.474	1.414
	20	15	0.930	0.930	0.930	0.932	1.466	1.466	1.467	1.468	1.857	1.857	1.857	1.856	1.551	1.551	1.551	1.550
0.75	10	7	5.136	3.578	2.704	2.061	3.188	2.498	2.053	1.692	2.051	1.784	1.585	1.404	1.396	1.315	1.247	1.179
	20	15	8.086	6.790	5.326	3.603	4.795	4.143	3.436	2.576	2.612	2.411	2.170	1.833	1.546	1.498	1.434	1.334
1.00	10	7	2.887	2.146	1.815	1.614	2.138	1.740	1.544	1.419	1.620	1.428	1.325	1.256	1.259	1.188	1.147	1.118
	20	15	3.017	2.029	1.623	1.384	2.190	1.672	1.426	1.271	1.640	1.393	1.260	1.170	1.264	1.174	1.120	1.080
1.25	10	7	2.126	1.763	1.613	1.528	1.732	1.513	1.419	1.364	1.426	1.309	1.256	1.224	1.187	1.140	1.118	1.104
	20	15	1.540	1.305	1.279	1.241	1.375	1.248	1.199	1.173	1.231	1.157	1.127	1.111	1.108	1.074	1.061	1.053
1.50	10	7	1.849	1.632	1.548	1.502	1.568	1.432	1.377	1.348	1.339	1.263	1.232	1.215	1.153	1.121	1.107	1.100
	20	15	1.311	1.254	1.234	1.223	1.222	1.182	1.168	1.161	1.141	1.116	1.108	1.103	1.067	1.056	1.052	1.050
1.75	10	7	1.722	1.576	1.520	1.491	1.489	1.395	1.360	1.341	1.296	1.242	1.222	1.211	1.135	1.112	1.103	1.098
	20	15	1.254	1.231	1.223	1.219	1.183	1.167	1.161	1.158	1.117	1.107	1.103	1.102	1.056	1.051	1.050	1.049

Table 3. Relative s-efficiencies of T_{Rt}, with respect to $\hat{R}_t$

(c=4)

λ/λ_0	n	r \ k_2, b	0.2: 0.2	0.2: 0.4	0.2: 0.6	0.2: 0.8	0.4: 0.2	0.4: 0.4	0.4: 0.6	0.4: 0.8	0.6: 0.2	0.6: 0.4	0.6: 0.6	0.6: 0.8	0.8: 0.2	0.8: 0.4	0.8: 0.6	0.8: 0.8
0.50	10	7	0.459	0.563	0.699	0.946	0.651	0.731	0.832	1.002	0.890	0.908	0.949	1.032	1.058	1.021	1.014	1.031
	20	15	0.272	0.358	0.472	0.670	0.391	0.478	0.585	0.753	0.576	0.416	0.721	0.839	0.821	0.836	0.869	0.924
0.75	10	7	2.234	2.258	2.252	2.201	2.321	2.146	1.998	1.943	1.933	1.765	1.633	1.509	1.414	1.343	1.285	1.227
	20	15	1.106	1.188	1.256	1.332	1.460	1.420	1.376	1.333	1.592	1.460	1.356	1.264	1.364	1.276	1.207	1.144
1.00	10	7	11.442	7.537	5.311	3.636	4.653	3.810	3.139	2.485	2.442	2.221	2.011	1.768	1.490	1.436	1.380	1.307
	20	15	13.637	8.848	5.803	3.529	4.983	4.091	3.270	2.423	2.517	2.293	2.048	1.738	1.506	1.454	1.390	1.297
1.25	10	7	3.723	3.362	3.001	2.581	2.941	2.639	2.363	2.061	2.058	1.911	1.772	1.613	1.417	1.369	1.322	1.263
	20	15	2.023	1.907	1.799	1.685	1.992	1.822	1.686	1.548	1.698	1.572	1.471	1.367	1.325	1.271	1.227	1.177
1.50	10	7	1.872	1.864	1.862	1.862	1.869	1.779	1.714	1.651	1.631	1.544	1.479	1.417	1.302	1.260	1.227	1.194
	20	15	0.909	0.956	1.017	1.110	1.053	1.062	1.084	1.126	1.134	1.112	1.106	1.110	1.112	1.089	1.075	1.066
1.75	10	7	1.365	1.421	1.486	1.582	1.431	1.420	1.426	1.453	1.366	1.326	1.307	1.301	1.202	1.173	1.156	1.146
	20	15	0.723	0.802	0.884	1.001	0.840	0.890	0.944	1.022	0.942	0.961	0.987	1.029	1.002	1.000	1.006	1.021

Table 4. Relative s-efficiencies of T_{R12} with respect to $\hat{R}_t$

(c=5)

θ/θ_0	n	r	k_2=0.2, b=0.2	0.2, 0.4	0.2, 0.6	0.2, 0.8	0.4, 0.2	0.4, 0.4	0.4, 0.6	0.4, 0.8	0.6, 0.2	0.6, 0.4	0.6, 0.6	0.6, 0.8	0.8, 0.2	0.8, 0.4	0.8, 0.6	0.8, 0.8
0.50	10	7	2.035	2.015	2.062	2.096	2.592	2.593	2.593	2.598	2.297	2.292	2.282	2.256	1.562	1.560	1.555	1.544
	20	15	0.930	0.930	0.930	0.930	1.465	1.463	1.466	1.465	1.857	1.857	1.857	1.857	1.552	1.552	1.552	1.552
0.75	10	7	12.821	10.956	8.918	6.507	5.435	4.946	4.389	3.639	2.681	2.560	2.413	2.192	1.550	1.523	1.489	1.433
	20	15	9.253	9.199	9.049	8.628	5.470	5.401	5.278	4 998	2.803	2.784	2.747	2.663	1.590	1.585	1.577	1.557
1.00	10	7	10.763	7.203	5.207	3.653	4.580	3.786	3.171	2.560	2.431	2.225	2.036	1.815	1.488	1.439	1.390	1.326
	20	15	14.303	9.671	6.540	4.035	5.127	4.325	3.550	2.682	2.553	2.363	2.147	1.852	1.515	1.472	1.418	1.335
1.25	10	7	6.326	4.469	3.442	2.651	3.624	2.932	2.503	2.099	2.195	1.981	1.808	1.631	1.435	1.378	1.327	1.270
	20	15	4.842	3.765	2.432	1.844	3.103	2.405	1.959	1.603	2.028	1.761	1.558	1.375	1.391	1.311	1.242	1.173
1.50	10	7	4.581	3.419	2.733	2.282	3.071	2.536	2.191	2.901	2.034	1.831	1.681	1.541	1.396	1.337	1.289	1.239
	20	15	2.643	1.991	1.678	1.475	2.116	1.715	1.504	1.361	1.648	1.443	1.324	1.239	1.278	1.202	1.153	1.117
1.75	10	7	3.759	2.921	2.459	2.113	2.743	2.297	2.022	1.802	1.924	1.736	1.606	1.463	1.367	1.303	1.264	1.222
	20	15	1.943	1.617	1.470	1.332	1.699	1.468	1.361	1.296	1.441	1.306	1.240	1.199	1.203	1.147	1.118	1.099

5. Conclusions.

In the comparisons of the Monte Carlo relative s-efficiencies of the proposed Bayes shrinkage estimators for the reliability function with respect to the MVUE in the left truncated exponential distribution based on type II censoring, the proposed estimators are more efficient than MVUE in the sense of MSE for all possible values of $n, r, a, b, c, d, h, m, k_1$ and k_2 if λ/λ_0 and/or θ/θ_0 approach 1. Also, the Bayes shrinkage estimator with the conjugate prior distribution is more efficient than the Bayes shrinkage estimator with the noninformative prior distribution.

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