

A Study of Computer Simulation of Back-and-Forth Patrol

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Abstract

A patroller is to protect a patrol area with a certain length front D by proceeding at constant speed on courses parallel to the patrol front, traveling back and forth between area boundaries and reversing course at each area boundary. Transitors enter the area uniformly distributed across the patrol front on a course perpendicular to the patrol front. Any transitor that closes the patroller to within his sweep radius R is detected. This paper use plane trigonometry to derive the theoretical probabilities of detection and develop a Monte Carlo computer simulation Model.

I. INTRODUCTION

In the modern naval generation, detecting the enemy has become not only one of the most important functions of any naval operation but has acquired the stature of a science with which the student of naval warfare must be familiar. Any action against an enemy must be preceded by knowledge of his presence and position. The problem of detection is dependent on three major aspects such as the physical characteristics of the instrument of detection and of the target, the path and location of the search unit. The direction and deployment of naval forces.

The science of detection as a branch of naval tactics, seeks solutions to problems of contacting and tracking hostile forces presumed to be nonfriendly. An understanding of the essentials of target

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detection will make possible the development of efficient searches, patrols and screens used in many types of naval operations. The measures of effectiveness for detection can be used in conjunction with cost, reliability, and other factors to compare the effectiveness of different detection systems and to determine how to use these systems most efficiently (Ref 1: p. 50).

This paper will use plane trigonometry to derive the theoretical probabilities of detection using back-and-forth searches and develop for a patroller reversing course at the patrol area boundary and a Monte Carlo computer simulation model.

II. THEORETICAL DETECTION PROBABILITY

A. MODEL DESCRIPTION

Refer to Figure 2-1, a patroller is to protect a patrol area with a certain length front D by proceeding at constant speed on courses parallel to the patrol front, traveling back and forth between area boundaries and reversing course at each area boundary. Transitors enter the area uniformly distributed across the patrol front on a course perpendicular to the patrol front.

Any transitor that closes the patroller to within his sweep radius R is detected. It is convenient in this section to work with R , one-half the sweep width W .

In this approach to detection probability determination, patrollers are assumed to have the capability of "sweeping" a patrol area, detecting all targets within the swept area and failing to detect all those outside. This is equivalent to assuming the definite range law of detection, where range, or sweep radius, is obtained (from actual at sea results) by integrating under the lateral range curve and calculating the range corresponding to an equal area under a curve of probability one. This is mathematically easy to handle, and the resulting detection probabilities for back-and-forth search, while not exact, are sufficiently accurate (Ref 1: p. 155).

B. THE METHOD OF DETECTION PROBABILITY

The method used here for the kinematics between transitor and patroller is to keep the transitor position fixed with only the patroller and its detection circle moving relatively to the transitor. The relative areas swept after several cycles of back-and-forth search will appear as in Figure 2-2, for patrollers reversing course at the boundary. The angle is determined by transitor and patroller speeds, i.e., $\theta = \tan(u/v)$, and Figures 2-2 depict the case where $u \cong v$.

In Figure 2-3 the portion of the shaded area that lies within the patrol area D represents the relative area swept for one patroller trip between boundaries. The angle establishes the direction of the relative motion between patroller and transitor, and $\tan \theta = u/v$. A brief inspection of Figure 2-3 will reveal that consideration of one or any number of patroller search legs will result in the same effectiveness for search.

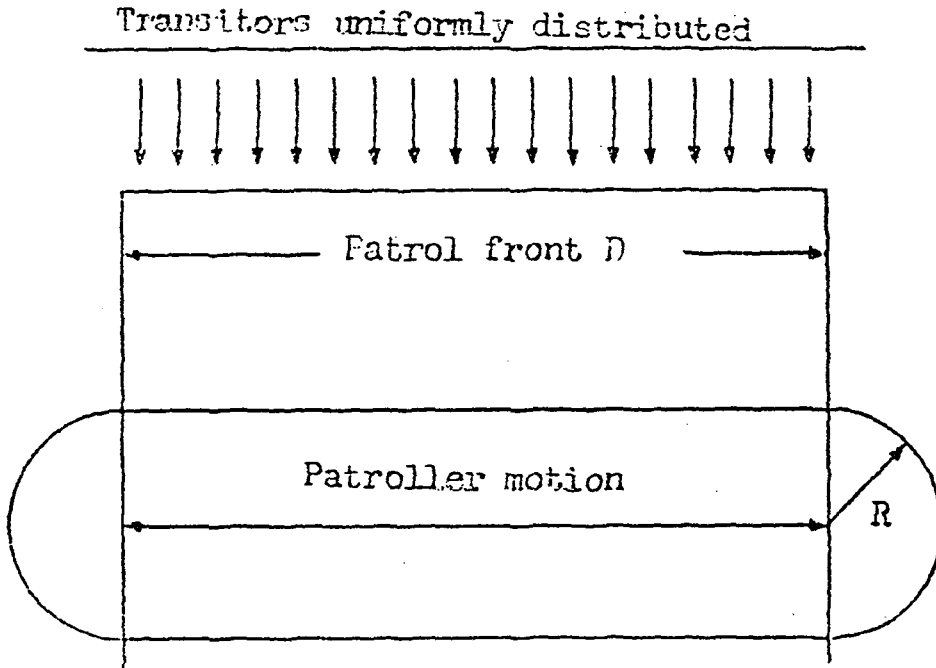


Figure 2-1. True Areas Covered by Back-and-Forth Search for Patroller Reversing at Area Boundary

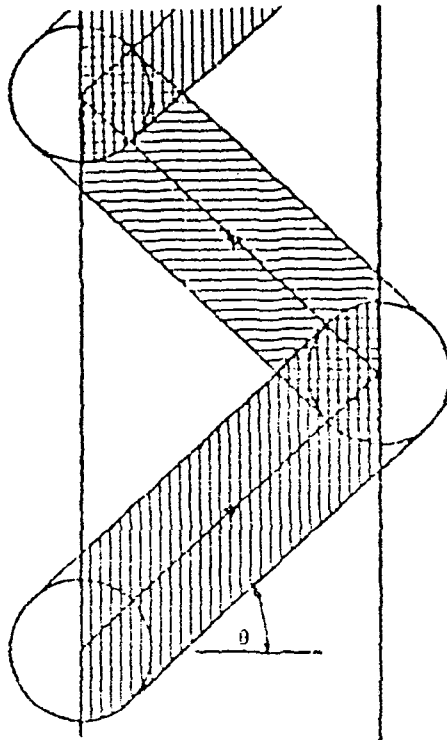


Figure 2-2. Relative Area Swept for Patroller Reversing Course at Area Boundaries.

Referring back to Figure 2-3, the probability of detection can be seen to be the ratio of the swept area (within the patrol area) to the total relative area for the leg. The method used here is to determine the shaded area (parallelogram minus the areas of the triangle that fall outside the patrol area) and divide by the total relative area.

The area within the parallelogram of Figure 2-3 is base times height. Referring to Figure 2-4:
 $\sin \theta = R / (\text{half the base})$,

but: $\tan \theta = \frac{u}{v}$

so: $\sin \theta = \frac{u}{\sqrt{u^2 + v^2}} = \frac{1}{\sqrt{1 + (\frac{v}{u})^2}}$

then: half the parallelogram base = $R / (\sin \theta) = R \sqrt{1 + (\frac{v}{u})^2}$

and the whole parallelogram base is $2R \sqrt{1 + (\frac{v}{u})^2}$

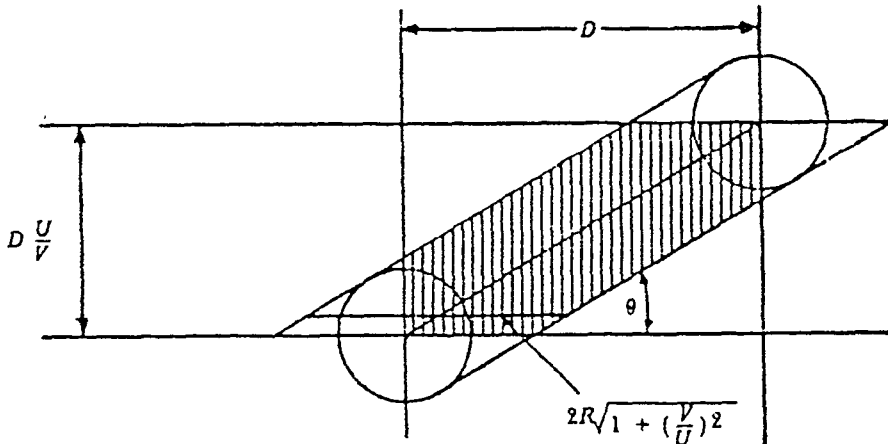


Figure 2-3. Relative Area Swept for One Leg.

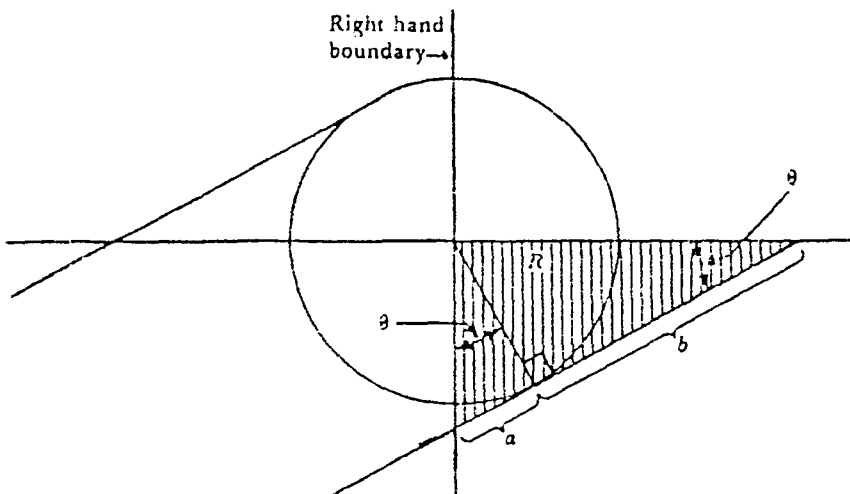


Figure 2-4. Area Within the Parallelogram.

The parallelogram height is shown in Figure 2-3 to be a function of the patrol front length D and the angle θ .

Specifically, $\tan \theta = (\text{height}/D)$, also $= (u/v)$, so $\text{height} = D(u/v)$. The area of the parallelogram, then, is base times height, or

$$2RD \frac{u}{v} \sqrt{1 + \left(\frac{v}{u}\right)^2}$$

The shaded area of Figure 2-4 shows the area to be subtracted from the parallelogram area one end of the patrol leg.

Dropping a perpendicular from the center of the search area circle to the hypotenuse of the triangle in question makes the solution simple. The length of that line is R, which can be thought of as the height of the triangle.

The base of the triangle. The base of the triangle is the sum of the lengths of a and b, which resulted from the drop of the perpendicular R. Since $\tan \theta = a/R$ in one subtriangle and R/b in the other, the base of the triangle becomes $R \tan \theta + R/\tan \theta$; The area of the triangle is:

$$R(\tan \theta + 1/\tan \theta)/2$$

which reduces by trigonometric identity to $R(u+v)/2uv$, and the total shaded area outside the patrol zone is $R(u+v)/uv$. The total relative area for one patroller search leg is seen to be the length of the patrol front D times the height of the parallelogram Du/v , or Du/v .

Finally, probability of detection for back-and-forth sweep where the patroller reverses course at the patrol area boundary is found:

$$P = 2(R/D) \sqrt{1 + (v/u)^2} - (R/D)^2 (1 + (v/u)^2) \quad (\text{eq 1}).$$

III. SIMULATION

A. ALGORITHM AND FLOWCHART

The variable definition that follows:

N = # of trials (Monte Carlo runs)

D = width of patrol front in yards

L = length of patrol area in yards

W = sweep width of patroller in yards

v = patroller speed in knots

u = transitor speed in knots

X1, Y1 = patroller position

X2, Y2 = transitor position

R = maximum detection range

C = the range between patroller and transitor

M = direction value

RND = uniform distribution random number

Referring to the flowchart that follows, the program logic has:

- * Used a 3-minute time step, i.e., moved patroller and transitor in jumps (distance) equivalent to 3 minutes; for example, a ship doing 5 knots would move $5 \times 100 = 500$ yards every 3 minutes.
- * Included a step to check the range (C) between patroller and transitor after each move; note that if $C < W/2$, then a detection would occur.
- * Started a new run either if a detection occurs or if the transitor gets through the patrol (a total length of L) without being detected.
- * Made provisions to have patroller reverse course each time he reaches the end of his patrol area. A direction value M is used to accomplish this; when $M=+1$, the patroller moves in the other. Note that multiplying M by -1 each time the patroller reaches a boundary alternately changes the sign of the direction vector M.

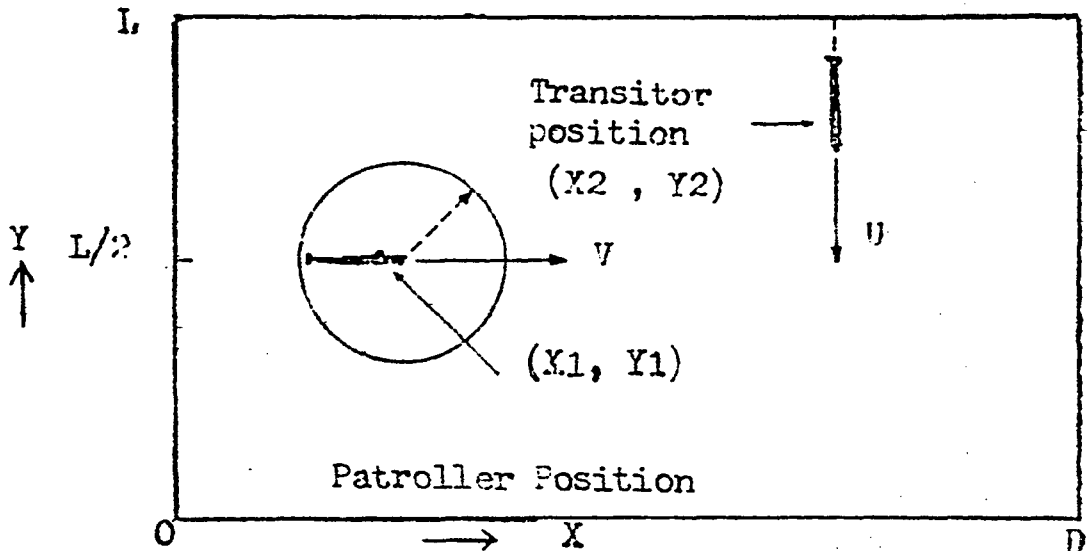


Figure 3-1. Sketch of Barrier Model.

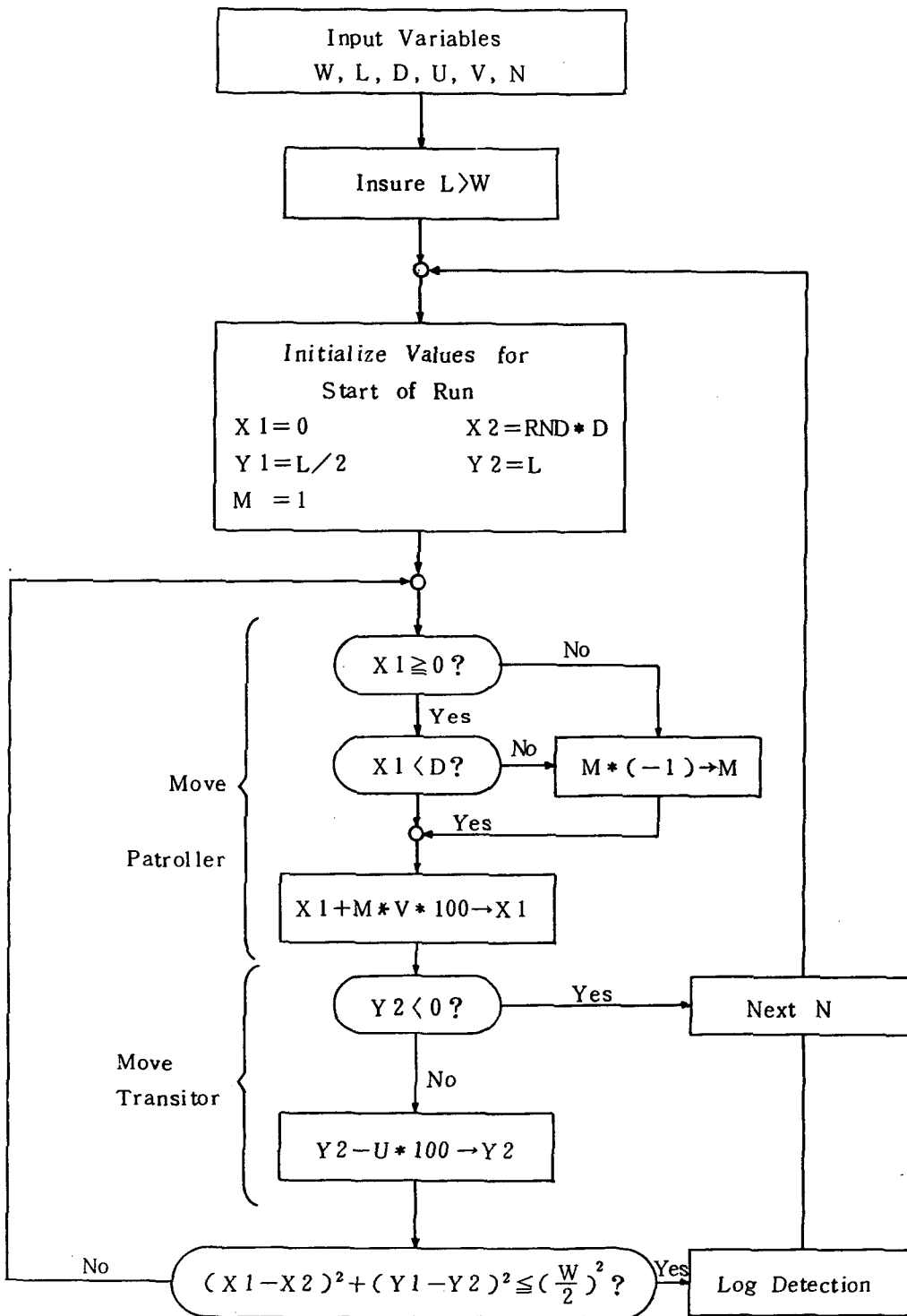


Figure 3-2. Flowchart.

B. RESULTS

The results table and graph that follows;

V \ D	8800	10560	12320	14080	15840	17600
4	.404	.344	.299	.265	.237	.215
8	.409	.248	.303	.268	.240	.218
12	.418	.356	.310	.274	.246	.223
16	.429	.366	.319	.282	.253	.229
20	.443	.378	.330	.292	.262	.237
24	.459	.392	.342	.303	.272	.247
28	.477	.208	.357	.316	.284	.258
32	.497	.426	.372	.330	.297	.269
36	.518	.444	.389	.345	.310	.282
40	.539	.463	.406	.361	.325	.295

Table 1. Patroller Speed VS Width of Patrol. (U=40 KTS, R=2000 yds, L=8800 yds)

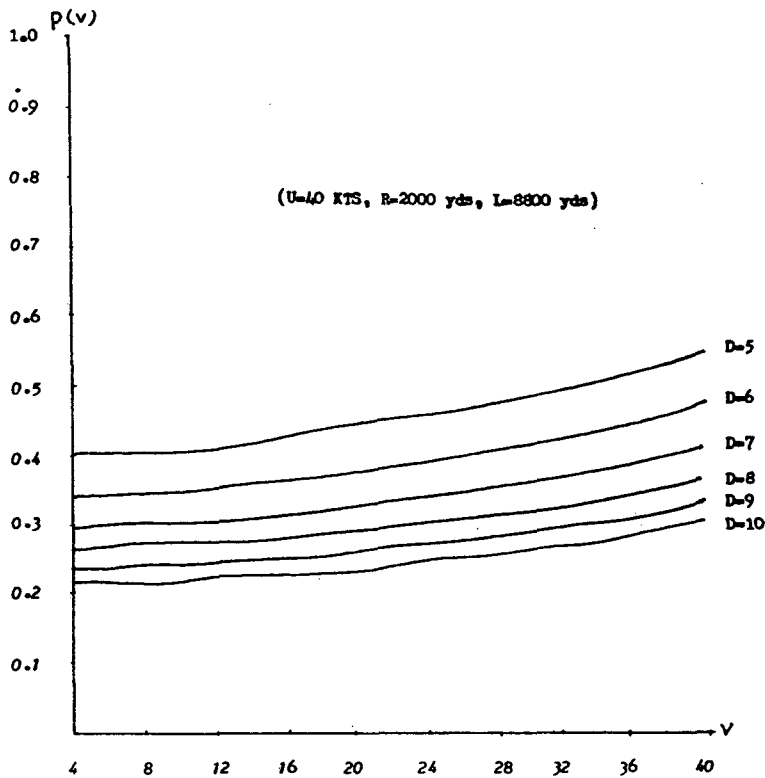


Figure 3-3. Patroller speed vs detection probability.
(D; Mile, V: KTS)

U \ D	8800	10560	12320	14080	15840	17600
4	.986	.929	.863	.798	.739	.687
8	.732	.642	.570	.512	.464	.424
12	.595	.514	.452	.403	.363	.330
16	.526	.451	.395	.351	.316	.287
20	.487	.417	.364	.323	.290	.263
24	.464	.396	.346	.306	.275	.250
28	.449	.383	.334	.296	.265	.241
32	.439	.374	.326	.289	.259	.235
36	.431	.368	.320	.284	.254	.231
40	.426	.363	.316	.280	.251	.227

Table 2. Transitor Speed VS Width Patrol.

(V=15 KTS, R=2000 yds, L=8800 yds)

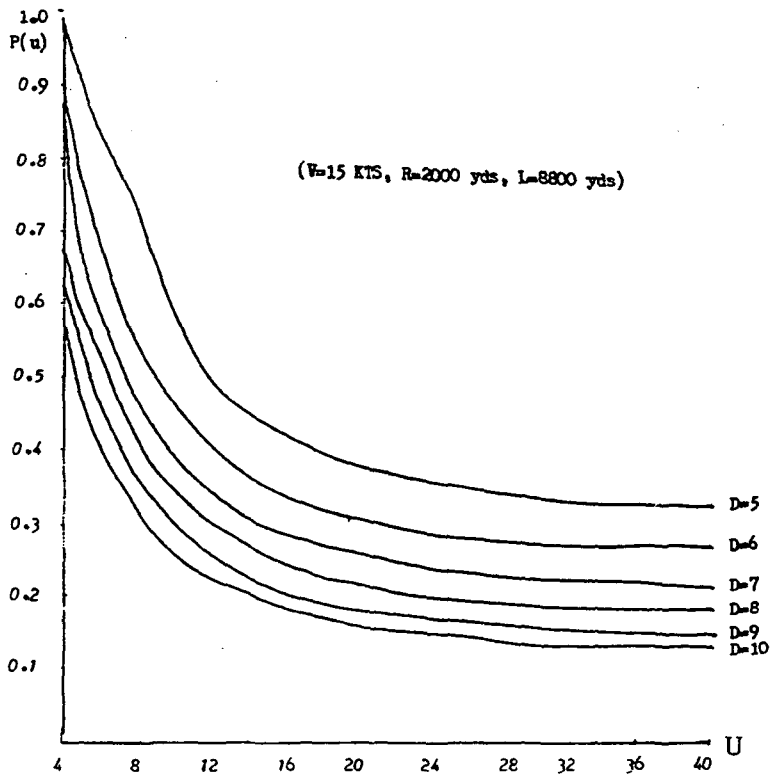


Figure 3-4. Transitor Speed VS Detection Probability.

(D: Mile, U: KTS)

IV. SUMMARY AND RECOMMENDATION

In this paper we have developed a simple method for the analysis design of operational tests and evaluations the selection of barrier patrol parameters so as to attain the model with high detection probability is difficult and important problem.

When user want to test a barrier patrol, this model should be useful. Also, it can be useful to assign for decision maker the detection area and estimate the naval fleet force size as each fleet area, we can develop the patrolling model which is likely the current situation.

An interactive program to compute detection probability for several parameters values have been developed.

References

1. Naval Institute Press, "Naval Operations Analysis" 3rd, 1984.
2. Byron J.T. Morgan, "Elements of simulation", 1984, The University press, Cambridge.
3. Averill M. Law and W. David Kelton, "Simulation modeling and Analysis", 1982, McGraw-Hill Company.