韓國 軍事運營分析 學會誌
第13券, 第1號, 1987, 6.

Multiple Objective Linear Programming with Minimum Levels and Trade Offs through the Interactive Methods

Chun, Man Sul *
Kim, Man Sik *

Abstract

This paper studies to develop the procedure which is combined by the progressive goals and progressive weights generation method. This procedure minimizes the number of questions the decision maker has to make, and also satisfies the generated minimum goal of each objective function. With the procedure developed, we are able to improve the previous multiple objective linear programming techniques in two points.

1. Introduction

In many problems a comparison between actions must be made on the basis of multiple consequences of the heterogeneous nature. For example, an investment problem should consider such considerations as cashflow, market share, future investment possiblities, qualities, etc. The decision maker (DM) wants to attain more than one objective or goal in selecting the desired course of action while satisfying the constraints dictated by environment, process, resources, etc. However, these problems become complicated when the objectives are apparently conflict each other. Mathematically, the multiple objective problem can be represented as;

Find the most desirable K-tuple $(f_1(\underline{X}), f_2(\underline{X}), ..., f_k(\underline{X}))$

* Dept. of Industrial Engineering, Han Yang University.

such that $\underline{X} \in \bar{X} = \{\underline{X} \mid g_i(\underline{X}) \leq 0 \; i = 1, ..., m\}$

where $\underline{X}$ is an n-dimensional decision variable vector

$f_j(\underline{X})$ is an objective function j=1,...,k

$g_i(\underline{X})$ is one of the constraints i=1,...,m.

The problem consist of n-decision variables, m-constraints, and k-objectives.

The distinctive difficulty is the absence of a precise definition of "the most desirable."

Traditionally, there are two approach for solving the above problem. One approach is to optimize one of the objectives while appending the other objectives to the constraint set, so that the optimal solution would satisfy these objectives at least up to a predetermined level:

$$\text{Max } f_i(\underline{X})$$
$$\text{s.t. } g_j(\underline{X}) \leq 0 \; j=1, ..., m$$
$$f_I(\underline{X}) \geq a_I \; I=1, ..., k \text{ and } I \neq i$$

where a_I is any acceptable predetermined level for objective I.

The other approach is to optimize a "super-objective" function created by multiplying each objective with a suitable weight and then adding them together.

$$\text{Max } \sum_{i=1}^{k} w_i \; f_i(\underline{X})$$
$$\text{s.t. } \underline{X} \in \bar{\underline{X}} = \{\underline{X} \mid g_j(\underline{X}) \leq 0 \quad j=1, \cdots, m\}$$

where w_i's are predetermined weights.

The difficulty with the first approach is to choose the acceptable level(a_I's) for the objectives which have conflicting nature. For the second approach, the major problem lies in determing the proper weights (w_i's). It is also unclear that any linear combination of the objective function will represent the true preferences.

2. The Developed Interactive Methods

Hwang (1979) surveys a considerable amount of efforts directed toward multiple objective decision making over the last decade. Among the several types of the method categorized by Hwang, we will be studying method of the type described as "progressive articulation methods of preference information." These methods are based on asking the decision maker to state comparisions of preferences among several feasible alternatives. The decision maker's responses are used to generate new feasible alternatives, which are then used to ask further questions about his preferences.

This method has been widely used because of the following advantages;

(1) There is no need for a prior preference information about the objectives.

(2) It is possible for the decision maker to develop and clearify his preference on the basis of the feasible solutions generated.

(3) It reduces the decision space where a decision maker can choose the better solution.

This approach has been applied to the problems having objectives because when the f_i's are convex combination of the f_i's, there will be an optimal solution at an extreme point of $\bar{X}$. Once the weights on the f_i are known, the problem becomes an ordinary convex optimization problem (linear program if f_i's are linear). One method like this approach is developed by Ziont – Wallenius and White.

Through the procedure the preferences the decision maker supplies are used to obtain successively more accurate estimates of the weights, and if $\bar{X}$ is a polytope, the procedure to find the most desirable solution must terminate because there are only finite extreme points.

Another method used in this approach is to reduce the feasible space $\bar{X}$ by the level of each objective f_i the decision maker supplies successively. Because $\bar{X}$ is polytope and the objective is linear, the reduced $\bar{X}$ is also polytope and has finite number of extreme points.

However this interactive goal or level method might ask too many questions to the decision maker for finding the level of objective f_i.

The Ziont – Wallenius method (1976) is the one of the first methods. This method assumes that the decision maker's utilities function is additive and linear. In most cases the objective functions themselves are linear (but it can be extended to the convex function by linearization) and the constraint forms a convex set.

This assumes that the decision maker's preferences are determined by a utility function that is linear function of in the f_j's. This method has been modified technically in three objective function case by White (1980).

Nijkamp and Spronk (1981) developed interactive goal programming method by progressive goal level and progressive goal priorities, which belongs to second method.

In this paper, we are interested in devising the procedure of this type which is combined by the progressive goals and progressive weights generation method. So that this minimize the number of questions the decision maker has to make, and which also satisfy the generated minimum goal of each objective function.

3. Method Development

In most cases of multiple objective linear programming case, the decision maker has to consider several objectives at same time and to find the most desirable solution among the feasible alternatives.

As described before, the most desirable solution is located at the extreme point and/or boundary of the polytope $\underline{\bar{X}}$.

To find the most desirable solution the successive questions are asked to clarify the decision maker's preference weights so that the new feasible region $\underline{\bar{X}}$' is reduced indirectly, or asked to reduce the feasible region to $\underline{\bar{X}}$' directly by the levels of each objective. Both cases the successive numbers of extreme points to consider will be reduced.

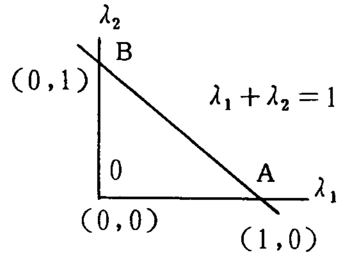
First the progressive articulation of preference weights of the decision maker has come from the parameter space theory (Zeleny, 1974).

If we have three objective function case as an example, the model can be explained as follow.

$$\text{Max} \sum_{i=1}^{3} \lambda_i f_i$$

$$\text{s.t.} \sum_j a_{ij} X_j \leq b_i \qquad i \in M, \; j \in N$$

$$\text{where} \sum_{i=1}^{3} \lambda i = 1$$



Because $\lambda_1 + \lambda_2 + \lambda_3 = 1$, we can draw the space of $\lambda_i \in \Lambda$ in two dimension like above figure.

If the true preference weight is at A, then it is single objective function problem with first objective f_1; at B, f_2; at 0, f_3 respectively.

If it is at line segment OA, then f_2 is ignored; at AB, f_3; at BO, f_1 respectively. If it is inside of ΔOAB, then all three objectives are considered.

As described by Zeleny (1974), preference space Λ is devided into several subregion Λ_j's. All the subregion Λ_j's are separated by the line which indicates the possible trade-off among the extreme points. More over each Λ_i indicates unique extreme points, and the trade – offs are originated from non basic efficient trades in the simplex tableau.

In other words, at certain feasible extreme point, if the decision maker would prefer a certain trade – off among the nonbasic variable then, this nonbasic variable will enter the basis and the preference subregion Λ_j will move to next to the current region Λ_k.

In this method, it is very important to estimate the closest λ_j's to the true λ_j^*s with the set of preference responses. Zionts (1976) uses a simple LP procedure to find λ_j's.

Using another set of LP or using center points of λ_j to find the estimates of λ_j's, takes more times and more steps. Also this method considers only the weight of objective and does not consider each objective function's minimum level.

Therefore, in this research, we will ask decision maker the minimum level of objective after finding ideal values of objectives, and will find the proper preference weights of objectives later on.

The mathematical form is shown below.

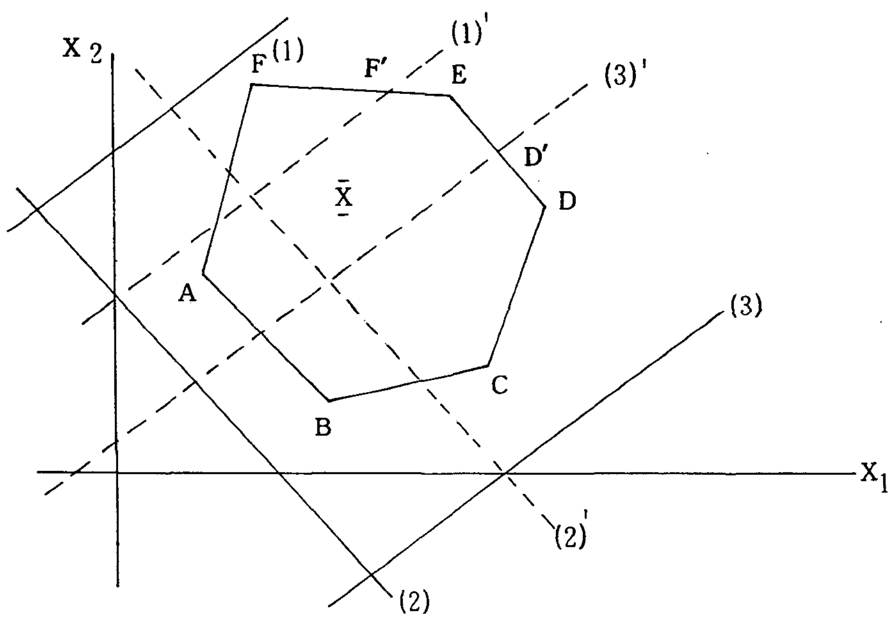
$$\text{Max} \sum_{i=1}^{k} w_i f_i$$

$$\text{st } X \in \bar{X}$$

$$f_i \geqslant g_i \; \forall \, i$$

$$\sum_i W_i = 1$$

If feasible region $\bar{X}$ is shown below and possible extreme points are A. B. C. D. E. F.



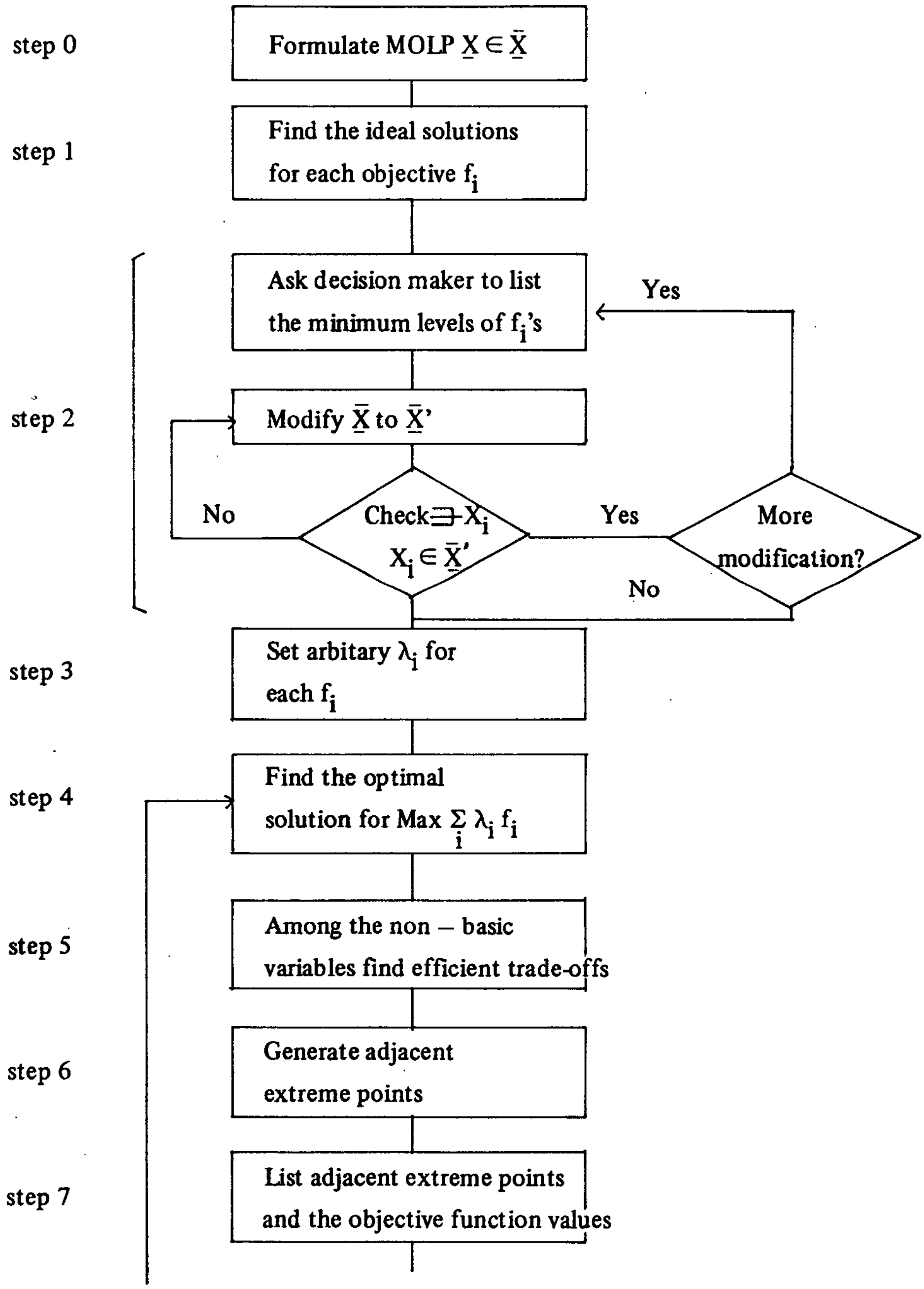
Objective function (1), (2), (3) are shown above at certain levels of objective values. If we could draw the (1)', (2)', (3)' by trading value of (1) with (2) and (3) then efficient extreme points are reduced to D', E, F' from B, C, D, E, F.

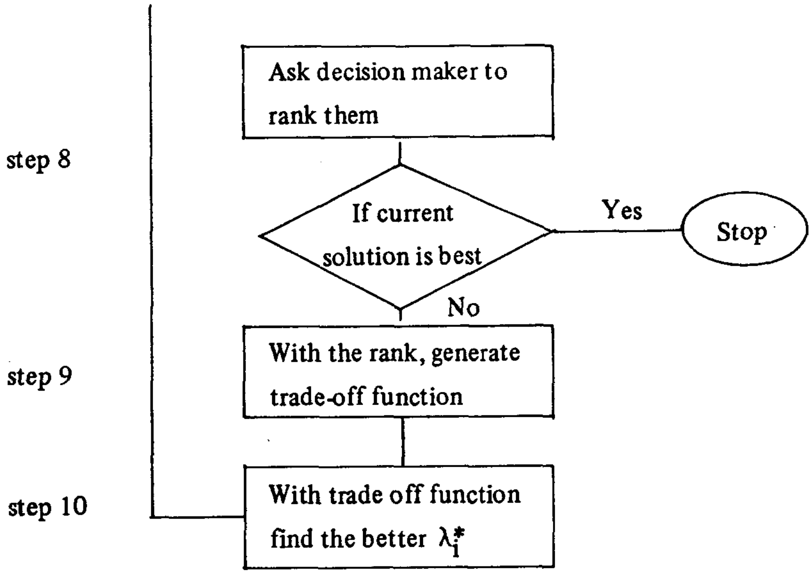
Because we have finite number of extreme points in the polytope, if the minimum levels of all objective functions have been achieved, then the trade off questions could be asked to decision maker to find the most desirable solution.

However, the minimum levels of objectives given by the decision maker might not be achieved in the feasible region. In this case, the modification of minimum level should be done by decision maker through the questions.

4. Solution Procedure

The procedures for this method are shown in the following chart.





The detailed explanations are as follows;

step 0; Formulate the ordinary multiple objective linear programming problem.

step 1; For each objective function, find the optimal solution among feasible set of $\bar{X}$

step 2;
1) After writing the ideal points for objectives, ask decision maker to list the minimum levels of f_i's by quantities or by percentage.
2) Check if there exists feasible solution in $\bar{X}$ so that the minimum goals are satisfied.
3) If there is a feasible solution, modify minimum goals,
4) If we like to increase minimum goals, modify them.
5) Otherwise go to step 3.

step 3;
1) Set arbitary λ'_i so that we can generate utility function.
Generally $\lambda_i = \frac{1}{k}$ or if there is any preferable objective give more weight for that objective,
2) Generate utility function

$$\text{Max} \sum_{i=1}^{k} \lambda'_i f_i$$

step 4;
1) Solve regular linear program and find the optimal solution.
2) At optimal point, find the basic variable and non-basic variables.

step 5; Among the non-basic variables, find the efficient non-basic variable which provide the efficient trade-offs (by Zionts method, 1980).

step 6; By trading off the objectives so that the efficient non-basic variable becomes basis, generate adjacent extreme point for every non-basic efficient trade-offs.

step 7; List the objective values at every adjacent point and current value.

step 8; 1) Ask the decision maker to rank them.

2) If current solution is the best, then we have done the procedure, and current solution is the most desirable solution.

step 9; If not, generate modified preference space Λ and trade – offs functions.

step 10; 1) Given trade – offs functions, find the better estimates of λ^*_i

2) And go to step 4.

Computer program of this method has been made in FORTRAN V 4.3 and tested through the VAX / VMS.

Most of cases, to find the feasible minimum ideal of objective, less than about 5 questions and mostly 2 or 3 questions has been asked to decision maker.

To find the most desirable solution, less than about 10 questions are asked for the examples which have 5 objectives. Most of cases, less than 5 questions are asked which are much less than questions with Zionts-Wallenius methods.

5. Conclusion

With this method developed, we have improved the previous multiple objective linear programming techniques in two point.

First, with Zionts-Wallenius method, we could not have any minimum levels of objectives. This is the weakest point for Zionts-Wallenius method. However this has been modified and the results of giving minimal level of each objective reduce the efficient extreme points to consider significantly.

Second, in Zionts-Wallenius methos, the marginal trade-offs are asked to decision maker but here we list the objective function value at every adjacent extreme point.

This gives more clear idea to decide the preference and less number of questions to be asked.

However we have not mentioned the nonlinear utility function but has been studied through the linearization by Zionts (1976).

References

1. Hwang, Ching-Lai, Multiple objective decision making method and application, Springer - Verlag. Berlin, 1979.

2. Spronk, Japp, Interactive multiple goal programming; Application to financial planning, Martin-Nijhoff, 1981.

3. White, DJ, Multiple – objectives interactive programming, J. of Operation Research (No. 3, 1980) pp. 517-523.
4. Zeleny, M., Linear multi objective programming, Springer-Verlag, Berlin, 1974.
5. Zionts, S. and J. Wallenius, An Interactive programming method for solving the multiple criteria problem, Management Science (No. 6, 1976) pp. 652-663.
6. Zionts, S. and J. Wallenius, Identify efficient vectors; Some theory and computational results, Operation Research (No. 3, 1980) p. 785-793.