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Optimum Inventory Level and Optimal Selling Price to Realize a Pre-determined Level of Profit

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Abstract

In this paper, the one period multi-item inventory model is considered in which it is required to determine the production quantity and selling price of each item which maximize the probability of realizing predetermined level of profit.

The objective function of this model is the sum of weighted probabilities which represent the possibility of obtaining the predetermined level of profit for each item. Budget constraint, inventory site constraint and constraints of price are considered.

Finally this paper shows a numerical example in which random demand of each item has exponential distribution.

Introduction

Inventory models have been developed for many years on the assumption that in economic environment uncertainty decision maker's behavior is governed by the expected profit maximization (or expected cost minimization) decision making criteria. Recent research shows that decision makers tend to maximize the probability of attaining the budget objectives rather than to maximize the expected profit; They try to maximize the probability of reaching or exceeding a predetermined level of profit. If it is so, our inventory models should be adapted to this different point of view.

There have been a few efforts at analizing mathematical models of inventory which reflects the probability maximization decision criteria. Kabak and Schiff(1978) developed an one period single item stochastic inventory model. In their model, the objective function, the probability of reaching or exceeding a predetermined level of profit R, is dependent only on the order quantity. By differentiating this

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objective function, optimal order quantity is derived. E. Sankarasubramanian and S. Kumaraswamy (1983) tried to derive some analytic solutions of order quantity for some kinds of probability distributions of demand, such as exponential distribution, uniform distribution and pareto distribution. In both models the parameters like selling price, ordering cost, shortage cost, salvage value are assumed to be known. There are no considerations of holding cost and constraints. Only the order quantity is decision variable.

On the general point of view, the raise of selling price causes the demand to decrease and the lowering of selling price causes the demand to increase. This tendency of demand fluctuation affected by selling price can be transformed into a mathematical function. In this case, selling price must be considered as a decision variable. The objective of this study reported here is to build one period multi-item inventory which determines the production quantities and selling prices maximizing the probability of obtaining predetermined level of profit for each item.

Multi-Item Inventory Model with Constraints

Notations

The notations used in this paper are as follows:

- M : maximum available budget level
- N : number of items
- x_i : demand quantity of item i
- p_i : selling price of item i
- y_i : production quantity of item i
- $f_i(x_i, p_i)$: pdf of demand for item i
- c_i : unit production cost of item i
- s_i : unit shortage cost of item i
- q_i : unit salvage value of item i
- h_i : unit holding cost of item i
- Z_i : total profit of item i
- R_i : satisfying level of profit of item i
- P_{ui} : selling price upper bound of item i
- a_i : unit inventory area of item i
- A : total inventory site area

Assumptions

The model is developed under the following assumptions:

1) Demand for each item has a continuous probability density function.
2) Demand distribution is dependent on the selling price for each item.
3) Upper bound and lower bound of the selling price are given for each item.
4) The satisfying level of profit is given (predetermined) for each item.

5) Demand occurs at the beginning of period.
6) The remainder, if any, is kept during the period.
7) Available budget is limited.
8) Total storage site is limited.

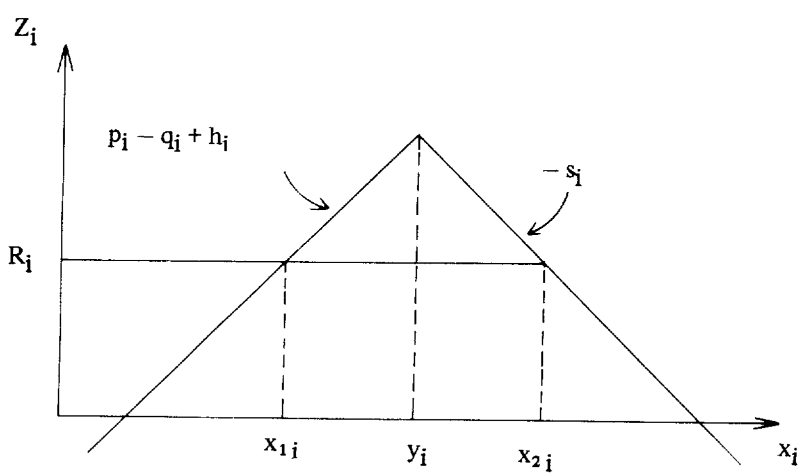
Deveropment of the model

The profit Z_i, being dependent on the random demand x_i and selling price p_i, is itself a random variable and is easily seen to be

$$Z_i = \begin{cases} -c_i y_i + p_i x_i + q_i(y_i - x_i) - h_i(y_i - x_i), & x_i \leq y_i \\ -c_i y_i + p_i y_i - s_i(x_i - y_i), & x_i > y_i \end{cases}$$

$$= \begin{cases} (q_i - c_i - h_i)y_i + (p_i - q_i + h_i)x_i & x_i \leq y_i \quad (1) \\ (p_i - c_i + s_i)y_i - s_i x_i & x_i > y_i \quad (2) \end{cases}$$

The graphical representation of (1), (2) is given below:



Z_i increase for $x_i \leq y_i$ and decrease for $x_i < y_i$.
A predetermined level of profit R_i will be realized when x_i is equal to either x_{1i} or x_{2i} where

$$x_{1i} = \frac{R_i - (q_i - c_i - h_i)y_i}{p_i - q_i + h_i} \quad (3)$$

$$x_{2i} = \frac{(p_i - c_i + s_i) - R_i}{s_i} \quad (4)$$

The profit Z_i will be greater than R_i whenever $x_{1i} < x_i < x_{2i}$.
Therefore the probability that Z_i is greater than or equal to R_i is given by

$$P[Z_i \geq R_i] = P[x_{1i} \leq x_i < x_{2i}]$$

$$= \int_{x_{1i}}^{x_{2i}} f_i(x_i, p_i)\, dx_i \qquad (5)$$

The objective function becomes

$$\text{Max } G_i(y_i, p_i; R_i) = \text{Max} \int_{x_{1i}}^{x_{2i}} f_i(x_i, p_i)\, dx_i \qquad (6)$$

Now, let's consider the constraints. Total production cost should not exceed the maximum available budget level.

Therefore,

$$\sum_{i=1}^{N} c_i y_i < M \qquad (7)$$

In the same way, the total area for the storage should not exceed the storage limit. Thus,

$$\sum_{i=1}^{N} a_i y_i < A \qquad (8)$$

(6) is the function of y_i and p_i.

One may derive the analytic optimal solutions of (6) in the same way that was used in the previous papers. (Kabak and Schiff 1978, Sankarasubramanian and Kumaraswamy 1983). But in constraints case, y_i^*, p_i^*, the optimal solution of y_i, p_i in (6), do not always satisfy (7) and (8).

In that case, we are obliged to reduce the production quantities of some items and at the same time the probability of obtaining predetermined level of profit will be decreased. For which items will the reduction of production quantity be imposed?

$G_i(y_i, p_i; R_i)$ is the probability of gaining predetermined level of profit. Thus $1 - G_i(y_i, p_i; R_i)$ is the probability of not obtaining predetermined level of profit R_i, that is risk. The risk for each item should not be treated with the same improtant. The higher R_i is ranked, the severer the risk of not obtaining R_i is perceived by the decision makers. By this reason, we must evaluate the risk $1 - G_i(y_i, p_i; R_i)$ by weighting with $R_i/\Sigma R_i$. Then the objective function of this model becomes to minimize the weighted average risk over all items, which is shown as

$$\text{Min} \quad \sum_{i=1}^{N} \frac{R_i}{\sum R_i} \{ 1 - G_i(y_i, p_i; R_i) \} \qquad (9)$$

By deleting constant term, we can rewrite (9) as

$$\text{Max} \quad \sum_{i=1}^{N} \frac{R_i}{\sum R_i} \quad G_i(y_i, p_i; R_i)$$

Therefore the multi item inventory model with constraints is the form of

$$\text{Max} \quad \sum_{i=1}^{N} \frac{R_i}{\sum R_i} \quad G_i(y_i, p_i; R_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} c_i y_i \leq M \qquad (10)$$

$$\sum_{i=1}^{N} a_i y_i < A$$

$$c_i \leq p_i \leq p_{ui}$$

$$y_i \geq 0$$

In general, the objective function in (10) has a nonlinear form. If there exists a pair of analytic optimal solutions for y_i and p_i, and if it satisfies the constraints in (10), then we are done. Unfortunately, however, we cannot generally certify the existence of analytic solutions for y_i, p_i, especially for p_i, for various demand probability distributions because of the complexity of the way p_i included in. If there does not exist a pair of analytic optimal solutions for y_i and p_i, then it must be found in another way. In both cases, a pair of optimal solutions for y_i and p_i can be found by the nonlinear search method. As this model has a nonlinear form objective function and linear inequality constraints, Gradient Projection Method is properly applicable.

Numerical Example

Suppose x_i has respectively the exponential distribution with mean $1/\lambda_i$. Also suppose λ_i is proportional to selling price p_k. Then

$$f_i(x_i; p_i) = \lambda_i \exp[-\lambda_i x_i] = \frac{p_i}{k_i} \exp\left[-\frac{p_i}{k_i} x_i\right], \qquad (11)$$

where k_i is a proper constant which can be determined through the investigation of relationship between a selling price and a demand distribution.

From (6), after some algebraic procedure, the objective function becomes

$$\text{Max} \quad G_i(y_i, p_i; R_i)$$

$$= \max \left\{ \text{ex}\left[-\frac{p_i}{k_i} \cdot \frac{R_i - (q_i - c_i - h_i) y_i}{p_i - q_i + h_i}\right] - \exp\left[-\frac{p_i}{k_i} \cdot \frac{(p_i - c_i + h_i) y_i - R_i}{s_i}\right] \right\} \qquad (12)$$

with the same constraints in (10).

The values of various parameters are assumed as follows;

	a_i	c_i	q_i	s_i	h_i	R_i	k_i	p_{ui}
item 1	2	6	3	5	1	70	1250	20
item 2	3	9	5	2	1	100	1000	23
item 3	1	5	2	2	1	50	1500	15

M = 800
A = 500

The result of this example is as follows;

y_1	y_2	y_3	p_1	p_2	p_3	opt. value
55.03	24.79	49.34	20	23	15	0.7400

These solutions make the first constraint in (10) active.
If M = 1500, then the solutions for y_i and p_i become as follows;

y_1	y_2	y_3	p_1	p_2	p_3	opt. value
64.59	26.57	58.11	20	23	15	0.7439

where M = 1500, the optimal solutions do not make first constraint in (10) active, which is the case where solutions for y_i and p_i can be obtained as a analytic form.

When M = 800, the optimal production quantity of each item diminishes by some degree. This result tells us that the higher R_i is ranked, the less production quantity diminishes.

Conclusion

An extension of a one period single item inventory model, maximizing the probability of obtaining a predetermined satisfiable level of profit, is made to include the situation in which the probability density function of a demand is dependent on the selling price for multi-item case.

Futher study may be performed to give explicit solutions for the optimal production quantity and optimal selling price of each item for various type of probability density function of demand.

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