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COMPACT CR-SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR IN A COMPLEX SPACE FORM

Dedicated to professor Y. Tashiro on his sixtieth birthday

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0. Introduction

A submanifold M of a Kaehlerian manifold $\bar{M}$ is said to be *complex* or *totally real* if each tangent space to M is mapped into itself or the normal space by the complex structure J of $\bar{M}$. These two classes of submanifolds are most typical examples among all submanifolds of $\bar{M}$. On the other hand, as a generalization to these submanifolds, a concept of CR-submanifolds is introduced by A. Bejancu [1] and others. A submanifold M of a Kaehlerian manifold $(\bar{M}, J)$ is called a *CR-submanifold* if there is a differentiable distribution such that it is invariant under J and the complementary orthogonal distribution is totally real. Many subjects for CR-submanifolds were investigated from various different points of view. In [2, 4, 5, 15], A. Bejancu, B. Y. Chen, M. Kon and K. Yano studied fundamental properties of CR-submanifolds M in a Kaehlerian manifold. In particular, under the assumption that the normal connection of M is flat or the second fundamental forms are all commutative, some characterizations and some classifications of CR-submanifolds with parallel mean curvature vector in a complex space form were obtained (see [3, 8, 10, 16, 17]), and, in previous paper [9] totally real submanifolds with parallel normal section in a complex space form were studied as a general case of [7].

The purpose of this paper is to investigate the manifold structure of CR-submanifolds in a complex space form in the case where the mean curvature vector is parallel in the normal bundle.

1. CR-submanifolds of a Kaehlerian manifold

Let $(\bar{M}, G)$ be a Kaehlerian manifold of real dimension $2m$ equipped

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with an almost complex structure J and with a Hermitian metric tensor G. Let $\bar{M}$ be covered by a system of coordinate neighborhoods $\{\bar{U}, y^A\}$ and denoted by G_{AB} components of G and by $J_B{}^A$ those of J. We then have

(1.1) $$J_A{}^B J_B{}^C = -\delta_A{}^C, \quad G_{CD} J_B{}^C J_A{}^D = G_{BA},$$

where here and in the sequel the following convention on the range of indices are used, unless otherwise stated:

$$A, B, C, \ldots = 1, \ldots, n, \quad n+1, \ldots, 2m;$$
$$h, i, j, \ldots = 1, \ldots, n;$$
$$x, y, z, \ldots = n+1, \ldots, 2m.$$

The summation convention will be used with respect to those system of indices. Since $\bar{M}$ is Kaehlerian, we get

(1.2) $$\nabla_B J_C{}^A = 0,$$

where ∇_B denotes the covariant derivative with respect to G_{BA}.

Let M be an n-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{U; x^h\}$ and immersed isometrically in $\bar{M}$ by the immersion $\phi : M \rightarrow \bar{M}$. When the argument is local, M need not be distinguished from $\phi(M)$, and then the immersion ϕ can be represented by $y^A = y^A(x^h)$ and $B_j = (B_j{}^A)$ are also n-linearly independent local tangent vectors of M, where $B_j{}^A = \partial_j y^A$ and $\partial_j = \partial/\partial x^j$. $2m-n$ mutually orthogonal unit normals $C_x = (C_x{}^A)$ may then be chosen. The induced Riemannian metric g_{ji} on the submanifold M is given by $g_{ji} = G_{BC} B_j{}^B B_i{}^C$ because the immersion ϕ is isometric. Therefore, by denoting by ∇_j the covariant derivative of van der Waerden–Bortolotti formed with g_{ji}, the equations of Gauss and Weingarten for M are respectively obtained:

(1.3) $$\nabla_j B_i{}^A = h_{ji}{}^x C_x{}^A, \quad \nabla_j C_x{}^A = -h_j{}^i{}_x B_i{}^A,$$

where $h_{ji}{}^x$ are the second fundamental forms in the direction of C_x and related by $h_j{}^h{}_x = h_{jix} g^{ih} = h_{ji}{}^y g^{ih} g_{yx}$, $g_{yx} = G_{BA} C_y{}^B C_x{}^A$ being the metric tensor of the normal bundle, and $(g^{ji}) = (g_{ji})^{-1}$.

DEFINITION 1.1. A submanifold M of a Kaehlerian manifold $\bar{M}$ of real $2m$ dimension is called a *CR-submanifold* if there is a differentiable distribution $T : p \rightarrow T_p \subset M_p$ on M satisfying the following conditions, where M_p denotes the tangent space at each point p to M:

(1) T is invariant under the complex structure J, i.e., $JT_p = T_p$ for each point p in M.

(2) The complementary orthogonal distribution $T^{\perp} : p \rightarrow T_p{}^{\perp} \subset M_p$ is

totally real, i.e., $JT_p^{\perp}\subset M_p^{\perp}$ for each p in M, where $M_p^{\perp}$ denotes the normal space to M at p.

If dim $T_p^{\perp}=0$ (resp. $\dim T_p=0$), then the CR-submanifold M is a Kaehlerian submanifold (resp. totally real submanifold) of $\bar{M}$. If dim $T_p^{\perp}=\dim M_p^{\perp}$, then M is said to be *generic*.

The transformation of $B_j{}^A$ and $C_x{}^A$ by the almost complex structure J are represented in each coordinate neighborhood as follows:

$$J_B{}^A B_j{}^B = f_j{}^i B_i{}^A - J_j{}^x C_x{}^A, \tag{1.4}$$

$$J_B{}^A C_x{}^B = J_x{}^i B_i{}^A + f_x{}^y C_y{}^A, \tag{1.5}$$

where $f_{ji}=G(JB_j, B_i)$, $J_{jx}=-G(JB_j, C_x)$, $J_{xj}=G(JC_x, B_j)$ and $f_{xy}=G(JC_x, C_y)$. From these definitions the following equations

$$f_{ji}+f_{ij}=0,\ J_{jx}=J_{xj},\ f_{xy}+f_{yx}=0$$

can be easily verified. By the properties of the almost complex structure, it follows from (1.4) and (1.5) that

$$f_j{}^t f_t{}^h = -\delta_j{}^h + J_j{}^x J_x{}^h,\quad f_x{}^y f_y{}^z = -\delta_x{}^z + J_x{}^t J_t{}^z, \tag{1.6}$$

$$f_j{}^t J_t{}^y + J_j{}^x f_x{}^y = 0,\quad J_x{}^t f_t{}^h + f_x{}^y J_y{}^h = 0, \tag{1.7}$$

where $f_j{}^h = f_{ji}g^{ih}$, $J_j{}^x = J_{jy}g^{yx}$ and $f_x{}^y = f_{xz}g^{zy}$, and g^{yz} is contravariant components of g_{yz}.

By the way, the CR-structures on the submanifold M of the Kaehlerian manifold $\bar{M}$ are characterized as follows ([15]):

LEMMA 1.2. *A necessary and sufficient condition for a submanifold M of $\bar{M}$ to be a CR-submanifold is that the tensors $f_j{}^i$ and $J_j{}^x$ satisfy*

$$f_j{}^t f_t{}^s f_s{}^i + f_j{}^i = 0,\quad J_j{}^x f_i{}^j = 0. \tag{1.8}$$

Accordingly it follows from (1.6), (1.7) and (1.8) that

$$J_j{}^x f_x{}^y = 0,\quad f_x{}^y f_y{}^z f_z{}^w + f_x{}^w = 0, \tag{1.9}$$

so the CR-submanifolds of a Kaehlerian manifold admit the f-structure both on M and its normal bundle.

If the covariant derivative ∇_j is applied to (1.4) and (1.5) and if the equation from (1.1) to (1.5) are taking account of, then the following relations are obtained respectively:

$$\nabla_j f_i{}^h = h_{ji}{}^x J_x{}^h - h_j{}^h{}_x J_i{}^x, \tag{1.10}$$

$$\nabla_j J_i{}^x = h_{jt}{}^x f_i{}^t - h_{ji}{}^y f_y{}^x, \tag{1.11}$$

$$\nabla_j f_x{}^y = h_j{}^t{}_x J_t{}^y - h_{jt}{}^y J_x{}^t, \tag{1.12}$$

where $h_{jix} = h_j{}^t{}_x g_{ti} = h_{ji}{}^y g_{yx}$.

In the sequel, the ambient Kaehlerian manifold $\bar{M}$ is assumed to be of constant holomorphic curvature $4c$ and of real dimension $2m$, which is called a complex space form and denoted by $M^{2m}(c)$. Then the curvature tensor $\bar{R}$ of $M^{2m}(c)$ is given by

$$\bar{R}_{DCBA}=c(G_{DA}G_{CB}-G_{CA}G_{DB}+J_{DA}J_{CB}-J_{CA}J_{DB}-2J_{DC}J_{BA}).$$

Thus, from (1.3), (1.4), (1.5) and (1.8) it follows that equations of Gauss, Codazzi and Ricci for M are respectively obtained:

$$(1.13)\quad R_{kjih}=c(g_{kh}g_{ji}-g_{jh}g_{ki}+f_{kh}f_{ji}-f_{jh}f_{ki}-2f_{kj}f_{ih}) +h_{kh}{}^{x}h_{jix}-h_{jh}{}^{x}h_{kix},$$

$$(1.14)\quad \nabla_k h_{ji}{}^{x}-\nabla_j h_{ki}{}^{x}=c(-f_{ji}J_k{}^{x}+f_{ki}J_j{}^{x}+2f_{kj}J_i{}^{x}),$$

$$(1.15)\quad R_{kjyx}=c(J_{kx}J_{jy}-J_{jx}J_{ky}-2f_{kj}f_{yx})+h_{ktx}h_j{}^{t}{}_{y}-h_{jtx}h_k{}^{t}{}_{y},$$

where R_{kjih} and R_{kjyx} are the Riemannian curvature tensor of M and that with respect to the connection induced in the normal bundle of M, respectively. The Ricci tensor of M can be expressed as

$$(1.16)\quad R_{ji}=c(n+2)g_{ji}-3cJ_j{}^{z}J_{zi}+h^{x}h_{jix}-h_{jrx}h_i{}^{r}{}_{x},$$

where $h^{x}=g^{ji}h_{ji}{}^{x}$.

2. Parallel tensor in the normal bundle

Let M be an n-dimensional CR-submanifold in a complex space form $M^{2m}(c)$ of constant holomorphic curvature $4c$. A normal vector field $\xi=(\xi^{x})$ is called a *parallel section* in the normal bundle if is satisfies $\nabla_j\xi^{x}=0$, and furthermore a tensor field F on M is said to be *parallel* in the normal bundle if it is in the normal bundle and $\nabla_j F$ vanishes identically.

In this section, the f-structure in the normal bundle is assumed to be parallel. In this case, (1.12) turns out to be

$$(2.1)\quad h_{jtx}J^{ty}-h_{jt}{}^{y}J_x{}^{t}=0.$$

Remark. Notice that $f_y{}^{x}$ vanishes identically if M is a generic submanifold of a Kaehlerian manifold $\bar{M}$. Thus, a generic submanifold of $\bar{M}$ has always a trivial f-structure in the normal bundle.

Let $\mathfrak{g}$ be a mean curvature vector field of the submanifold. Namely, it is defined by

$$\mathfrak{g}=g^{ji}h_{ji}{}^{x}C_x/n=h^{x}C_x/n,$$

which is independent of the choice of the local field of orthonormal frames $\{C_x\}$. Since the fact that the mean curvature vector is parallel in the normal bundle is assumed, we may choose a local field $\{e_x\}$ in such a way that $\mathfrak{H}= aC_{n+1}$, where $a=\|\mathfrak{H}\|$ is constant. Because of the choice of the local field, the parallelism of $\mathfrak{H}$ yields

$$(2.2)\qquad \begin{cases} h^x=0, & x\geqq n+2, \\ h^*=na, \end{cases}$$

where here and in the sequel the index $n+1$ is denoted by $*$. Since the f-structure in the normal bundle is parallel, it is easily seen from the first equation of (1.9) and (1.11) that $h^z f_z{}^y f_{yx}=0$. Because $f_y{}^x$ is the f-structure in the normal boundle, it follows that $h^z f_z{}^x=0$, which together with (2.2) gives

$$(2.3)\qquad f_*{}^x=0.$$

Therefore the second relationship of (1.6) gives rise to

$$(2.4)\qquad J_{jx}J^{j*}=\delta_x{}^*.$$

Since the mean curvature vector $\mathfrak{H}$ is normal, the curvature tensor R_{kjyx} of the connection in the normal bundle shows that R_{kj*x} vanishes identically for any index x. Thus the Ricci equation (1.15) yields

$$(2.5)\qquad h_{kt}{}^x h_j{}^{t*}-h_{jt}{}^x h_k{}^{t*}=c(J_{k*}J_j{}^x-J_{j*}J_k{}^x)$$

by means of (2.3).

On the other hand, because of (1.6), (1.8), (1.14) and (2.4), we have

$$(2.6)\qquad \|\nabla_k h_{ji}{}^*-c(f_{kj}J_i{}^*+f_{ki}J_j{}^*)\|^2=\|\nabla_k h_{ji}{}^*\|^2-2c^2(n-J_{jz}J^{jz}).$$

Now, the mean curvature vector being parallel in the normal bundle, the Laplacian $\Delta h_{ji}{}^*$ of $h_{ji}{}^*$ is given by

$$\Delta h_{ji}{}^*=R_{js}h_i{}^{s*}-R_{kjih}h^{kh*}+c\nabla_k(J_*{}^k f_{ij}-J_{j*}f_i{}^k-2J_{i*}f_j{}^k).$$

Accordingly, by the straightforward calculation, the last equation can be reduced to

$$(2.7)\qquad \begin{aligned} \Delta h_{ji}{}^* =& c(n+3)h_{ji}{}^*-ch^*g_{ji}+h^*h_{jt}h_i{}^{t*}-h_{kh}{}^x h^{kh*}h_{jix} \\ &+c(3h^*J_{j*}J_{i*}-6h_{st}{}^*f_j{}^s f_i{}^t-3h_{it}{}^*J_z{}^t J_j{}^z \\ &-h_{it}{}^x J_x{}^t J_j{}^*-2h_{jt}{}^x J_*{}^t J_{ix}+h_{it}{}^x J_*{}^t J_{jx}-h_{jt}{}^x J_x{}^t J_i{}^*). \end{aligned}$$

3. Normal f-structure on the submanfold

This section is devoted to investigating the manifold structure of the CR-submanifold with parallel f-structure in the normal bundle in $M^{2m}(c)$. The f-structure induced on M is said to be *normal* ([13], [17]), if the second fundamental forms $h_{ij}{}^x$ and the f-structure induced on the submanifold M are commutative each other, that is, $h_j{}^{tx}f_t{}^h - f_j{}^t h_t{}^{hx}=0$ for any indices, or equivalently

$$h_{jt}{}^x f_i{}^t + h_{it}{}^x f_j{}^t = 0. \tag{3.1}$$

Transforming (3.1) by $f_k{}^i$ and taking account of (1.6), we find

$$-h_{jk}{}^x + h_{jt}{}^x J_y{}^t J_k{}^y + h_{it}{}^x f_j{}^t f_k{}^i = 0. \tag{3.2}$$

By the properties of CR-structure on M, it follows from the last equation that

$$h_{jt}{}^x J_z{}^t = P_{yz}{}^x J_j{}^y, \tag{3.3}$$

where $P_{yz}{}^x$ is defined by $P_{yz}{}^x = h_{ji}{}^x J_y{}^j J_z{}^i$ and hence it satisfies

$$P_{yz}{}^x f_x{}^w = 0. \tag{3.4}$$

Denoting $P_{xyz} = g_{zw} P_{xy}{}^w$, we see that P_{xyz} is symmetric for all indices, because of a direct consequence of (2.1). When $x=n+1$ in (3.3), we have

$$h_{jt}{}^* J_z{}^t = P_{yz}{}^* J_j{}^y. \tag{3.5}$$

Differentiating (3.5) covariantly along M and substituting (1.11) and (1.12), we find

$$(\nabla_k h_{jt}{}^*) J_z{}^t + h_j{}^{t*}(h_{ksz} f_t{}^s - h_{kt}{}^y f_{yz})$$
$$= (\nabla_k P_{yz}{}^*) J_j{}^y + P_{yz}{}^*(h_{kt}{}^y f_j{}^t - h_{kj}{}^w f_w{}^y),$$

from which, together with (1.14), (2.4), (2.5) and (3.1) it follows

$$2(c f_{kj}\delta_z{}^* - h_{jt}{}^* h_s{}^t{}_z f_k{}^s + P_{yz}{}^* h_{jt}{}^y f_k{}^t)$$
$$= (\nabla_k P_{yz}{}^*) J_j{}^y - (\nabla_j P_{yz}{}^*) J_k{}^y.$$

By a consequence of the simple calculation, the equation above is reduced to

$$h_{jt}{}^* h_i{}^{tx} = P_z{}^{x*} h_{ji}{}^z + c\delta^{x*}(g_{ji} - J_j{}^z J_{iz}) + (P_{yw}{}^x P_z{}^{w*} - P_{yzw} P^{wx*}) J_j{}^z J_i{}^y. \tag{3.6}$$

This and (2.5) mean that

$$P_{zw}{}^x P_y{}^{w*} - P_{yw}{}^x P_z{}^{w*} = c(\delta_z{}^* J_y{}^i J_i{}^x - \delta_y{}^* J_z{}^i J_i{}^x).$$

Thus, P_{zyx} being symmetric for all indices, it follows that

$$P_{zyx} P^{yx*} = P^x P_{zx}{}^* + c\delta_z{}^*(J_{ix} J^{ix} - 1), \tag{3.7}$$

$$P_{zx}{}^* P_y{}^{z*} = P_{zyx} P^{z**} + c(J_y{}^i J_{ix} - \delta_y{}^* \delta_x{}^*), \tag{3.8}$$

where $P^x = P_z{}^{zx}$. Using above two relationships, (3.6) implies that

$$h_{jt}{}^* h_i{}^{t*} = P_{z*}{}^* h_{ji}{}^z + c(g_{ji} - J_i{}^* J_j{}^*). \tag{3.9}$$

By taking account of some equations obtained above, the equation (2.7) then turns out to be

$$\Delta h_{ji}{}^{*}=c(-2h_{ji}{}^{*}+2h^{*}J_{j*}J_{i*}+2P_{yz}{}^{*}J_{j}{}^{y}J_{i}{}^{z}-P_{z}J_{i}{}^{z}J_{j}{}^{*}-P_{z}J_{j}{}^{z}J_{i}{}^{*}),$$

which together with (3.6) and (3.9) implies

$$h^{ji*}\Delta h_{ji*}=2c^{2}(J_{jx}J^{jx}-n). \tag{3.10}$$

By combining (2.6) and (3.10), it follows that

$$\Delta(h_{ji}{}^{*}h^{ji}{}_{*})=2\|\nabla_{k}h_{ji}{}^{*}-c(f_{kj}J_{i}{}^{*}+f_{ki}J_{j}{}^{*})\|^{2}. \tag{3.11}$$

From (2.6) and (3.11) we conclude

PROPOSITION 3.1. *Let M be an n-dimensional compact CR-submanifold with parallel f-structure in a complex Euclidean space C^m. If the mean curvature vector is parallel and if the f-structure induced on M is normal, then the second fundamental tensor A^* is parallel.*

REMARK. Let M be a totally real submanifold with parallel f-structure in the normal bundle in $M^{2m}(c)$. It is shown that if the non-trivial mean curvature is parallel, then A^* is parallel. This fact is proved in [7] and [9].

REMARK. Let M be a generic submanifold with flat normal connection in C^m. It is known in [8] that if the mean curvature vector is parallel and if the f-structure induced on M is normal, then the second fundamental form of M is parallel.

We next prove the following:

THEOREM 3.2. *Let M be an n-dimensional compact CR-submanifold imbedded in a $2m$-dimensional complex Euclidean space C^m. Assume that an f-structure in the normal bundle is parallel and that the mean curvature vector of M is parallel. If the f-structure induced on M is normal, then M is a product submanifold $M_1\times\cdots\times M_\alpha$, where M_a $(a=1,\dots,\alpha)$ is a compact n_a-dimensional submanifold imbedded in C^{m_a} and M_a is contained in a hypersphere in C^{m_a}.*

Proof. Since the ambient space is complex Euclidean, it can not admit compact minimal submanifolds. So, the mean curvature vector $\mathfrak{g}$ is non-trivial. By the way, $\mathfrak{g}$ being parallel, Proposition 3.1 says that the second fundamental form $h_{ji}{}^{*}$ in the direction of $\mathfrak{g}$ is parallel, that is, $\nabla_k h_{ji}{}^{*}=0$ on M. When a function h_m for any integer $m\geqq 1$ is given by

$$h_m = h_{i_1}{}^{i_2 *} h_{i_2}{}^{i_3 *} \ldots h_{i_m}{}^{i_1 *},$$

it is easily seen that h_m is constant on M for any integer m, which means that each eigenvalue of the shape operator A^* is constant on M, because $h_{ji}{}^*$ is parallel. By $\mu_1, \ldots, \mu_\alpha$ mutually distinct eigenvalues of A^* are denoted. Let $n_1, \ldots, n_\alpha$ be their multiplicities. Thus, distinct eigenspaces $D_a (a=1, \ldots, \alpha)$ of A^* define parallel distributions on M; say $A^* X = \mu_a X$ for all X in D_a. Then, de Rham's decomposition theorem tells us that M can be written as a product of Riemannian manifolds $M_1 \times \ldots \times M_\alpha$ where the tangent bundle of M_a corresponds to D_a. Since the mean curvature vector $\mathfrak{g}$ is parallel in the normal bundle, each shape operator A_y satisfies $[A^*, A_y]=0$. This implies that $A_y D_a \subset D_a$ for any indices y and a. By means of Moore's theorem [12], $M = M_1 \times \cdots \times M_\alpha$ is a product submanifold imbedded in $\boldsymbol{C}^m = \boldsymbol{C}^{m_1} \times \cdots \times \boldsymbol{C}^{m_\alpha}$. Let $\pi_a(\mathfrak{g})$ be the component of $\mathfrak{g}$ in the subspace $\boldsymbol{C}^{m_a}$. Then $\pi_a(\mathfrak{g})$ is a parallel mean curvature vector of M_a in $\boldsymbol{C}^{m_a}$, and M_a is umbilical with respect to $\pi_a(\mathfrak{g})$. Therefore it follows that M_a lies in a small hypersphere in $\boldsymbol{C}^{m_a}$ which is orthogonal to $\pi_a(\mathfrak{g})$. For datails, see [4], for instance. Hence it is a compact minimal submanifold in the hypersphere. This completes the proof of the theorem.

COROLLARY 3.7. *Let M be a compact generic submanifold with parallel mean curvature vector in $\boldsymbol{C}^m$. If the f-structure induced on M is normal, then M is the same type as that of Theorem* 3.2.

REMARK. Let M be a compact totally real submanifold with parallel mean curvature vector in $\boldsymbol{C}^m$. It is shown in [7] and [9] that if the f-structure in the normal bundle is parallel, then M is the same type as that of Theorem 3.2.

REMARK. Let M be a complete generic submanifold of $\boldsymbol{C}^m$ with flat normal connection and with parallel mean curvature vector. It is already seen in [8] that if the f-structure induced on M is normal, then M is a product of spheres.

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