國數學敎育學會誌〈數學敎育〉
36. 6, Vol. XXIV, No. 3

Journal of the Korea Society of Mathematical Education
Jun. 1986, Vol. XXIV, No. 3

On θ-Continuous Functions into Urysohn Spaces

by Sin Min Kang

Gyeongsang National University, Jinju, Korea

1. Introduction

S. V. Fomin [1] has introduced the concept of θ-continuity. The present note is to vestigate closedness under θ-continuous functions into Urysohn spaces.

Throughout this note, spaces always mean topological spaces. Let S be a subset of a ace X. The closure of S and the interior of S are denoted by $Cl(S)$ and $Int(S)$, reectively. A point $x \in X$ is said to be *θ-cluster* point of S [7] if $S \cap Cl(U) \neq \phi$ for each ıen neighborhood U of x. The set of all *θ-cluster* point of S is called the *θ-closure* of and is denoted by $Cl_\theta(S)$. If $Cl_\theta(S)=S$, then S is called *θ-closed*. The complement of ı *θ-closed* set is called *θ-open*.

Definition 1. 1. A function $f: X \longrightarrow Y$ is said to be *θ-continuous* [1] and *weakly-ɔntinuous* [2] if for each $x \in X$ and each open neighborhood V of $f(x)$, there exists an ıen neighborhood U of x such that $f(Cl(U)) \subset Cl(V)$ and $f(U) \subset Cl(V)$, respectively.

It is known that θ-continuous functions are always weakly-continuous, but the conerse is not true [2].

For the function $f: X \longrightarrow Y$, the subset $\{(x, f(x)) \mid x \in X\}$ of the product space $X \times Y$ is ılled the *graph* of f and is denoted by $G(f)$.

Definition 1. 2. The graph $G(f)$ is said to be *θ-closed with respect to* $X \times Y$ [4] (resp. *-closed with respect to* X [4], *δ-closed* [6] and *strongly-closed* [3]) for each $, y) \notin G(f)$, there exist open neighborhoods U and W of x and y, respectively, such that $Cl(U) \times Cl(W)] \cap G(f) = \phi$ (resp. $[Cl(U) \times W] \cap G(f) = \phi$. $[Int(Cl(U)) \times Int(Cl(W))] \cap G(f) = \phi$ nd $[U \times Cl(W)] \cap G(f) = \phi$).

2. Main Theorems

Lemma 2. 1. *If $f: X \longrightarrow Y$ is θ-continuous functions, then $f^{-1}(Cl(V))$ is θ-open in* ˹ *for every open set V of Y.*

Proof. Let V be any open sets of Y and $x \in f^{-1}(V)$. Then there exists an open set ˀ in Y such that $f(x) \in G \subset V$. Therefore, there exists an open neighborhood U of x uch that $f(Cl(U)) \subset Cl(G)$. Hence, we obtain $x \in U \subset Cl(U) \subset f^{-1}(Cl(V))$. This shows hat $f^{-1}(Cl(V))$ is θ-open in X.

Theorem 2. 2. *If $f: X \longrightarrow Y$ is θ-continuous and Y is Urysohn spaces, then* $(x_1, x_2) \mid f(x_1) = f(x_2)\}$ *is θ-closed in $X \times X$.*

Proof. Let $\Phi=\{(x_1, x_2) | f(x_1)=f(x_2)\}$. *If* $(x_1, x_2)\notin\Phi$, then $f(x_1)\neq f(x_2)$. Hence there exist open neighborhoods V_1 and V_2 of $f(x_1)$ and $f(x_2)$, respectively, such that $Cl(V_1)\cap Cl(V_2)=\phi$. Since f is θ-continuous, $f^{-1}(Cl(V_1))$ and $f^{-1}(Cl(V_2))$ are θ-open neighborhoods of x_1 and x_2, respectively, by Lemma 2.1. Hence $f^{-1}(Cl(V_1))\times f^{-1}(Cl(V_2))$ is an θ-open neighborhood of (x_1, x_2) by [5]. By [8, p. 88], this neighborhood can not meet Φ. Thus Φ is θ-closed in $X\times X$.

Theorem 2. 3. *If $f: X\longrightarrow Y$ is θ-continuous and Y is Urysohn spaces, then $G(f)$ is θ-closed with respect to $X\times Y$.*

Proof. Let $(x, y)\notin G(f)$. Then $y\neq f(x)$ and there exist open neighborhoods V and W of $f(x)$ and y, respectively, such that $Cl(V)\cap Cl(W)=\phi$. By θ-continuity of f, there exists an open neighborhood U of x such that $f(Cl(U))\cap Cl(W)=\phi$. Hence, we obtain $[Cl(U)\times Cl(W)]\cap G(f)=\phi$.

The following Corollary 2.4 and 2.5 follow immediately from Theorem 2.3.

Corollary 2.4. *If $f: X\longrightarrow Y$ is θ-continuous and Y is Urysohn spaces, then*

i) *$G(f)$ is θ-closed with respect to X.*

ii) *$G(f)$ is δ-closed in $X\times Y$.*

Corollary 2.5 ([3]). *If $f: X\longrightarrow Y$ is weakly-continuous and Y is Urysohn spaces, then $G(f)$ is strongly-closed in $X\times Y$.*

References

1. S. V. Fomin, *Extensions of topological spaces,* Dokl. Akud. Nauk SSSR, 32(1941), 114~116=Ann. of Math., 44(1943), 471~480.
2. N. Levine, *A decomposition of continuity in topological spaces,* Amer. Math. Monthly, 68(1961), 44~46.
3. P. E. Long and L. L. Herrington, *Functions with strongly-closed graphs,* Bollettino Un. Math. Ital., 12(1975), 381~384.
4. P. E. Long and L. L. Herrington, *Strongly θ-continuous functions,* J. Korean Math Soc., 18(1981), 22~28.
5. P. E. Long ang L. L. Herrington, *The T_θ-topology and faintly continuous function,* Kyungpook Math. J., 22(1982), 7~14.
6. T. Noiri, *On δ-continuous functions,* J. Korean Math. Soc., 16(1980), 161~166.
7. N. V. Veličko, *H-closed topological spaces,* Amer. Math. Transl., 78(1968), 103~10
8. S. Willard, *General Topology,* Addison-Wesley Pub. Co. 1970.