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A Procedure for Statistical Thermal Margin Analysis Using Response Surface Method and Monte Carlo Technique

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반응 표면 및 Monte Carlo 방법을 이용한 통계적 열여유도 분석 방법

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Abstracts

A statistical procedure, which uses response surface method and Monte Carlo simulation technique, is proposed for analyzing the thermal margin of light water reactor core. The statistical thermal margin analysis method performs the best-estimate thermal margin evaluation by the probabilistic treatment of uncertainties of input parameters. This methodology is applied to KNU-1 core thermal margin analysis under the steady state nominal operating condition. Also discussed are the comparisons with conventional deterministic method and Improved Thermal Design Procedure of Westinghouse.

It is deduced from this study that the response surface method is useful for performing the statistical thermal margin analysis and that thermal margin improvement is assured through this procedure.

요 약

경수로심의 열 여유도를 분석하기 위하여 반응표면 및 Monte Carlo 방법을 이용하는 통계적 분석 방법이 제시되었다. 통계적인 열 여유도 분석 방법은 입력변수들의 불확실도를 확률론적으로 처리함으로써 열 여유도의 최적 평가를 수행한다. 이 방법은 원자력 1호기 정상상태의 원자로심 분석에 응용되었으며 또한 종래의 결정론적 방법 및 웨스팅하우스의 개선된 열설계 방법과도 비교되었다.

본 연구를 통하여 반응표면 분석 방법은 통계적인 열 여유도 분석에 유용함을 알 수 있었으며, 이 방법을 통한 열 여유도의 증가도 확인되었다.

I. Introduction

The thermal hydraulic design of light water reactor should be accomplished with an appropriate thermal marin to prevent the core from the unexpected transient power overshooting, hence to ensure safe operation. On the other hand, it is also important to provide the core with the operational flexibility and to reduce the possibility of spurious reactor trips at the point of view of utility's cost loss.

Conventional thermal hydraulic design approach has been performed in the conservative way. Since the operation and design parameters' uncertainties are considered simultanelousy, there can be no more margins if design changes occur.[1),2)]

Some probabilistic approaches have been developed to quantify more realistic thermal margin by statistically combining the parametric uncertainties while satisfying the DNB design basis. Some problems, however, occur in the adoption of probabilistic design approach in the following aspects.

(1) the functional relationships between the outputs and input parameters may not be explicitly known.

(2) computer codes are long running.

(3) the number of input variables is large.

Therefore, the evaluation of the uncertainty distribution of output variable, which is a major concern, is much complicated and time consuming task.

In this regard, a statistical thermal margin analysis method is proposed in this paper, which uses experimental design tchnique[3),4)] and response surface method[4),5),6)] to consider the parametric uncertainties in a reasonable way. The procedure taken in this method is as follows;

(1) Input parameters important to DNBR are slected.

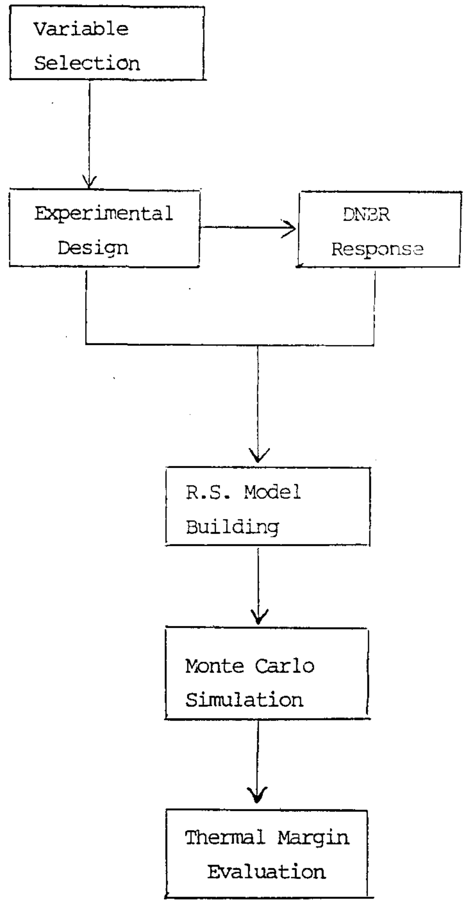


Fig. 1. Schematic Diagram of Statistical Thermal Margin Analysis Procedure

(2) Experimental design technique is used to generate design points more systematically.

(3) DNBR response is calculated by COBRA-IV computer code[7)] for each design point selected according to the above steps.

(4) Response surface which approximates the functional relationship between DNBR and input parameters is obtained by a regression technique.

(5) Uncertainty distribution of DNBR is estimated through the direct application of Monte Carlo method to the response surface obtained.

Figure 1 shows this procedure schematically. The purpose of this study is to use this statistical procedure for the analysis of thermal margin gain of KNU-1 reactor core against the conventional deterministic design approach and to discuss the differences and merits of this procedure against the Improved Thermal Design Procedure (ITDP) of Westinghouse.[8]

II. Description of Response Surface Method

II. 1. Basic Concept

When the output variable, y is a complex function of a number of input parameters x_i $(i=1, 2, \ldots, k)$, say,

$$y=f(x_1, x_2, \ldots, x_k) \quad (1)$$

the input-output relationship is approximated in the form of polynomial regression as

$$y=b_0+\sum_{i=1}^{k}b_ix_i \text{ (1st order regression)} \quad (2)$$

or

$$y=b_0+\sum_{i=1}^{k}b_ix_i+\sum_{i=1}^{k}\sum_{j\geq i}^{k}b_{ij}x_ix_j$$

$$\text{(2nd order regression)} \quad (3)$$

where,

$$x_i=\frac{z_i-z_i^0}{\sigma_i} \quad (4)$$

=coded value or level,

z_i=real value,

z_i^0=nominal value,

σ_i=standard deviation.

The term response surface refers to the geometric interpretation of a function of several independent variables. If the functional relationship is not highly nonlinear in the region of x's, then first order regression equation can be used. Otherwise second or higher order regression equation will fit the relationship reasonably. Sometimes in case that nonlinearity is obvious in the wide range of dependent variables, the first order model can be used in the narrow interested region of independent variables, too.

II. 2. Experimental Design[3),4)]

There are several methods in designing experiments to establish a set of sampling points in the space of the x's, at which y will be observed. The most commons of these are two level factorial design, three level factorial design, and central composite design, which are discussed in this paper.

Two level factorial design utilizes two levels of x's (coded values of ± 1). When the number of factors (here, number of input parameters) are k, the required number of code runs are 2^k for the full (complete) factorial design and 2^{k-p} for fractional factorial design, where 2^p is the fraction needed to reduce the required run numbers. In the fractional factorial design, $k-p$ parameters are combined completely but remaining p parameters are composed of product of already defined $k-p$ parameter levels. Obviously, the higher the degree of fraction is, the less is the degree of resolution in analyzing the lower order effects (main or cross-term effects) compared to the higher order interaction effects.

Three level full factorial and fractional factorial design is the same as the two level design except that they use three level of x's $(-1, 0, +1)$. But in these designs the number of runs is increasing more rapidly than two level design.

In central composite design, however, three distinct portions are included; (1) two level factorial points, (2) two axial points for each parameter, (3) one center point. Fig. 2 illustrates

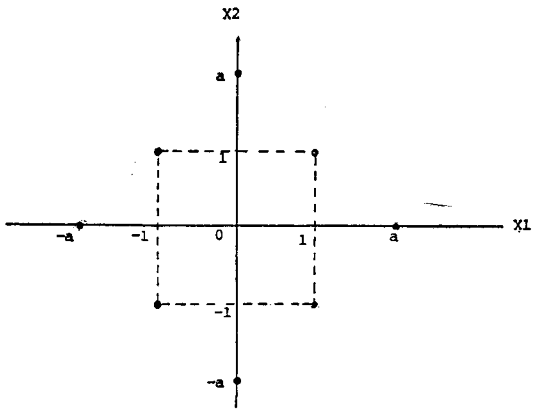


Fig. 2. Central Composite Design Points for Two Factors.

Table 1. Comparison of the Number of Unknowns and Code Runs between the Experimental Designs

No. of Parameters	1st Order Regression			2nd Order Regression			
	No. of Unknown	No. of Runs		*No. of Unknown	No. of Runs		
k		2^k	2^{k-p}		3^k	3^{k-p}	Central Composite**
3	4	8	$2^{3-1}=4$	10	27		8+6+1=15
4	5	16	$2^{4-1}=8$	15	81	$3^{4-1}=27$	16+8+1=25
5	6	32	$2^{5-1}=16$	21	243	$3^{5-1}=81$	32+10+1=43
7	8	128	$2^{7-1}=64$	36	2187	$3^{7-1}=729$	128+14+1=143
			$2^{7-2}=32$			$3^{7-2}=243$	
			$2^{7-3}=16$			$3^{7-3}=81$	
			$2^{7-4}=8$				

* $2k+kC_2+1$
** 2^k+2k+1

these design points for two factors, where 'a' is an arbitrary coded value.

Among the above mentioned types of experimental design, the two level factorial and central composite designs are commonly used for response surface work. The reason is that the former can be used to fit first order polynomial with minimum number of code runs, while the latter is suitable for fitting the second order polynomial. The major advantage of central composite design is utilizing the information of two level design to consider the nonlinearity with a few additional data points in case the linear model is not appropriate.

Table 1 shows the comparison of the number of unknowns and code runs required for these experimental design methods. It is obvious that the central composite design needs relatively few number of runs compared to the three level factorial design.

II. 3. Estimation of Response Surface Coefficients

The method of least square is used to estimate the coefficients of the response surface equation.

Eq. 2 or 3 can be represented by matrix form as

$$y=\overline{X}\vec{b}+\vec{e} \quad (5)$$

where,

$y=m\times 1$ observed data vector,

$\overline{X}=m\times n$ experimental design matrix,

$\vec{b}=n\times 1$ regression coefficient vector,

$\vec{e}=m\times 1$ error vecctor, i.e., the difference between observed data and estimated value

Least square method estimates $\vec{b}$ such that the sum of squares of the error becomes minimum.

$$L=\vec{e}^T\vec{e}=(y-\overline{X}\vec{b})^T(y-\overline{X}\vec{b}) \quad (6)$$

Setting the partial derivative of L with respect to $\vec{b}$ be zero, then

$$\vec{b}=(\overline{X}^T\overline{X})^{-1}\overline{X}^Ty. \quad (7)$$

In Eq. 5, the design matrix $\overline{X}$ contains the first column whose elements are only 1s. If first order regression model is used, then the order of design matrix $\overline{X}$ is $n=k+1$ and $(\overline{X}^T\overline{X})^{-1}$ becomes $1/n$, where k=number of parameters. Thus, the computation is very simple. In case of second order regression model, $\overline{X}$ must be augmented to include the quadratic and cross terms x_ix_j. In this case the use of computer is inevitable to perform the matrix inversion.

III. Implementation to Thermal Margin Analysis

III. 1. Variable Selection and Uncertainty Distribution

Sensitivity studies of DNBR to the core thermal hydraulics design and operating para-

Table 2. Mean, Standard Deviation, and Uncertainty Distribution Type of Parameters

Variable No.	Parameter	Mean	Standard Deviation	Uncertainty Distribution
$x1$	Core Inlet Temp. (°F)	541.2	2.31	Uniform
$x2$	Core Power (% of RTP)	100	1.15	Uniform
$x3$	$F_{\Delta H}$ (—)	1.435	4.28×10^{-2}	Normal
$x4$	Core Inlet Flow$\left(10^6\frac{lb_m}{ft^2\cdot hr}\right)$	2.45	0.1144	Normal
$x5$	System Pressure (psia)	2280	17.32	Uniform

meters have been performed for the KNU-1 reactor core to identify the importance of the input variables.[1),2)]

Five parameters selected important to DNB are; core inlet temperature, core power, enthalpy rise hot channel factor, core inlet flow, and the system pressure, whose uncertainties are to be statistically combined. The values of mean, standard deviation and their uncertainty distribution types are shown in Table 2. In this Table, the enthalpy rise hot channel factor incorporates both the nuclear($F^N_{\Delta H}$) and the engineering factor($F^E_{\Delta H}$). The core inlet flow is determined by subtracting the core bypass flow fraction from the best-estimated primary coolant flow. The nominal bypass fraction of 2.7% is assumed for the core inlet flow. Therefore, the standard deviations for these variables are added for each component's standard deviation. The other values are extracted from reference 8.

III. 2. Building Response Surface Model

The core subchannel analysis computer code, COBRA-IV, is used to generate DNBR values according to the experimental design plans for the prescribed five statistical parameters. Two types of experimental design methods are employed for a comparative purpose of obtaining the response surface coefficients. These are the two level fractional factorial design (2^{5-1}) and the central composite design as discussed in section II.

The 2^{5-1} design requires sixteen design points, the levels of which are listed in Table 3 together with their DNBR responses calculated by COBRA-IV. The central composite design consists of sixteen factorial points, two axial points

Table 3. Factorial Design Points (2^{5-1}) with DNBR Responses

$x1$	$x2$	$x3$	$x4$	$x5$	DNBR
1	1	1	1	1	2.734
−1	1	1	1	−1	2.627
1	−1	1	1	−1	2.625
−1	−1	1	1	1	2.582
1	1	−1	1	−1	2.506
−1	1	−1	1	1	2.472
1	−1	−1	1	1	2.463
−1	−1	−1	1	−1	2.366
1	1	1	−1	−1	2.900
−1	1	1	−1	1	2.855
1	−1	1	−1	1	2.854
−1	−1	1	−1	−1	2.739
1	1	−1	−1	1	2.728
−1	1	−1	−1	−1	2.617
1	−1	−1	−1	−1	2.617
−1	−1	−1	−1	1	2.575

Table 4. Axial and Central Design Points with DNBR Responses

$x1$	$x2$	$x3$	$x4$	$x5$	DNBR
2	0	0	0	0	2.563
−2	0	0	0	0	2.714
0	2	0	0	0	2.564
0	−2	0	0	0	2.717
0	0	2	0	0	2.450
0	0	−2	0	0	2.849
0	0	0	2	0	2.829
0	0	0	−2	0	2.455
0	0	0	0	2	2.672
0	0	0	0	−2	2.606
0	0	0	0	0	2.639

Table 5. Response Surface Model for KNU-1 Steady State Nominal DNBR

b	Variale	Regression Coefficients	
		1st Order	2nd Order
b0	1	2.6413	2.6402
b1	x1	−3.7125-2	−3.7333-2
b2	x2	−3.8625-2	−3.8500-2
b3	x3	−9.8250-2	−9.8750-2
b4	x4	9.4375-2	9.4083-2
b5	x5	1.6625-2	1.6583-2
b11	$(x1)^2$		−5.8333-4
b22	$(x2)^2$		−8.3333-5
b33	$(x3)^2$		2.1667-3
b44	$(x4)^2$		2.9167-4
b55	$(x5)^2$		−4.5833-4
b12	x1 x2		0.0
b13	x1 x3		1.6250-3
b14	x1 x4		−2.0000-3
b15	x1 x5		2.5000-4
b23	x2 x3		8.7500-4
b24	x2 x4		−7.5000-4
b25	x2 x5		−7.5000-4
b34	x3 x4		−3.1250-3
b35	x3 x5		−1.2500-4
b45	x4 x5		7.5000-4

per each variable, and one center point. The axial and center points along with their DNBR responses are shown in Table 4. The central composite design thus requires data points of both Table 3 and Table 4 (i.e. 27 points) where ±2 means two standard deviation.

The number of unknowns of response surface coefficients, $\vec{b}$, for 2^{5-1} design and central composite design are six and twenty-one respectively as shown in Table 1. The values of $\vec{b}$ for these two design methods are determined by the Eq. 7 and the results are compared in Table 5. The first order response surface coefficients show good agreement with the main effect terms of the second order regression coefficients. Also indicated from Table 5 is that the effects of the quadratic and the interaction terms in the second order polynomial are relatively small compared to the main effect terms. Hence, it is reasonable, for the sake of calculational simplicity in the Monte Carlo simulation, to use the first order regression polynomial as the DNBR response surface model for this study. The response surface model can be represented by the following analytical approximation:

$$y=2.6413-3.7125\times10^{-2}x_1-3.8625\times10^{-2}x_2-9.8250\times10^{-2}x_3+9.4375\times10^{-2}x_4+1.6625\times10^{-2}x_5 \quad (8)$$

Replacing Eq. 8 with Eq. 4 gives an alternative expression for DNBR response as

$$y=13.7824-1.6071\times10^{-2}z_1-3.3587\times10^{-2}z_2-2.2956z_3+0.8250z_4+9.5987\times10^{-4}z_5 \quad (9)$$

The above equation is the final form of fast running approximation to be used in the Monte Carlo calculation, which is discussed in the following section.

Eq. 9 can also be used for sensitivity study of input variables. Investigated are DNBR sensitivity factors, S_i, which are defined as the percent variation of DNBR due to percent variation in input variables, i.e.

$$S_i=\frac{\Delta y_i/y^0}{\Delta z_i/z_i^0}=a_i\frac{z_i^0}{y^0} \quad (10)$$

where,

a_i=coefficients in Eq. 9

y^0=2.6413 (nominal DNBR) from Eq. 8

z_i^0=nominal value of parameter.

DNBR sensitivity factors as calculated by this response surface model are listed in Table 6 together with the values in reference 1, which

Table 6. DNBR Sensitivity Factors Determined by First Order Response Surface Model

Parameter	Sensitivity Factor (%/%)	
	1st Order RSM	Values in Ref. 1
Core Inlet Temp.	−3.29	−4.03
Core Power	−1.25	−1.69
$F_{\Delta H}$	−1.25	−1.65
Core Inlet Flow	0.76	0.79
System Pressure	0.83	0.87

was done by one-at-a-time calculation. The importance ranking is in good agreement between these two methods. Note that the a_i values here are obtained as the averaged concept in the uncertainty range of the input parameters, because these parameters are varied simultaneously on the base of experimental design and response surface method.

III. 3. Monte Carlo Analysis

Monte Carlo analysis is a method used for propagating the uncertainty distributions through a function. The objective of the Monte Carlo simulation in this study is to predict DNBR uncertainty distribution caused by input variable uncertainties using response surface model established in the previous section.

Obtaining the statistical distribution of DNBR leads to the determination of limit value for nominal DNBR which satisfies the DNB design criteria. In this study, the Monte Carlo statistical analysis program, MOCUP,[9] is used.

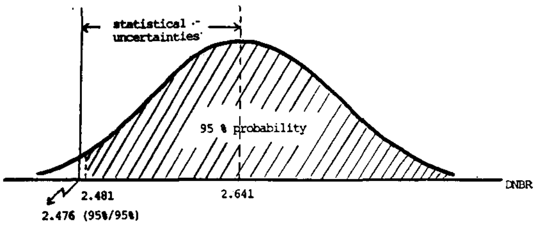


Fig. 3. DNBR Uncertainty Distribution (KNU-1 Core)

The calculational process of MOCUP is as follows;

(1) Generation of random values of input variables according to the given uncertainty distribution in Table 2.

(2) Use of response surface model to calculate DNBR values with the random input data.

(3) Estimation of DNBR uncertainty distribution from the sample DNBR data.

1200 Monte Carlo trials are conducted to produce a sample DNBRs. From this sample, the median and the standard deviation values of

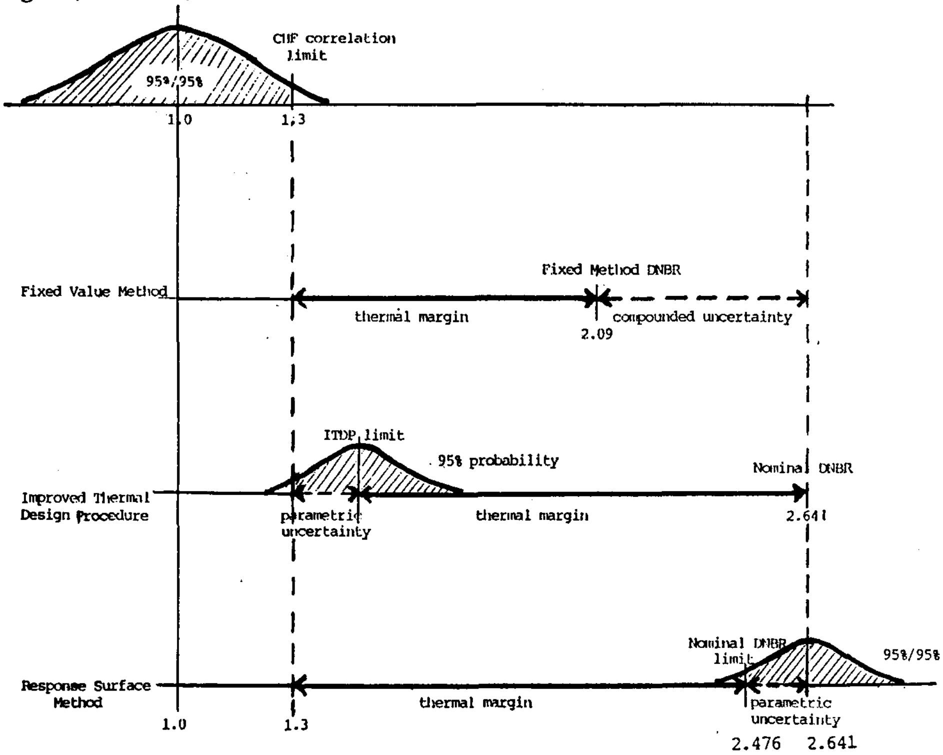


Fig. 4. Comparison of Thermal Margins for Three Different Thermal Design Methods.

DNBR are estimated for the KNU-1 reactor core under steady state nominal conditions. The median value of DNBR is 2.641 and its variance is 9.6595×10^{-3}. The one-side lower limit value at the 95% probablity is 2.481. The limit value of DNBR is then selected by considering the tolerance of MDNBR itself additional on the 95% confidence basis. This gives limit DNBR value of 2.476 as Eq. 11.

$$2.481-1.645\sigma/\sqrt{n}=2.476 \qquad (11)$$

where, σ=population standard deviation
$\simeq$square root of sample variance
n=number of sampling=1200
1.645=one-side 95% point of standard normal distribution.

Figure 3 shows the schematic of DNBR distribution curve for this case.

III.4. Results and Discussions

The statistical analysis of thermal margin is performed for the KNU-1 reactor core under steady state nominal condition using response surface methodology and Monte Carlo simulation.

In this study, five parameters are treated statistically and the remaining variables are held at their fixed values. COBRA-IV computer program calculates DNBR values according to

Table 7. ITDP Limit DNBR and Parametric Uncertainty Calculation

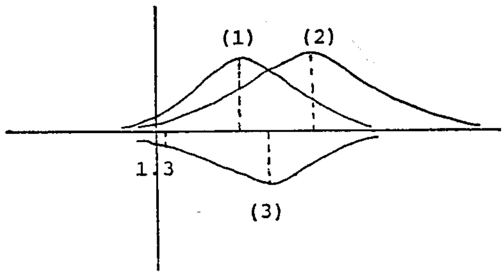
X_i	μ_i	σ_i	$\left(\frac{\sigma_i}{\mu_i}\right)$	$\left(\frac{\sigma_i}{\mu_i}\right)^2$	S_i	S_i^2	$S_i^2\left(\frac{\sigma_i}{\mu_i}\right)^2$
T_{in}	541.2	2.31	0.00427	1.822-5	−4.03(1)	16.2409	2.9591-4
					−5.68(2)	32.2624	5.8782-4
					−5.52(3)	30.4704	5.5552-4
Power	100	1.15	0.0115	1.3225-4	−1.69	2.8561	3.7772-4
					−2.27	5.1529	6.8147-4
					−1.87	3.4969	4.6246-4
G_{in}	2.45	0.1144	0.04669	2.1803-3	0.79	0.6241	1.3607-3
					0.87	0.7569	1.6503-3
					0.75	0.5625	1.2264-3
$F_{\Delta H}$	1.435	4.28-2	0.029825	8.8958-4	−1.65	2.7225	2.4219-3
					−2.16	4.6656	4.1504-3
					−1.40	1.9600	1.7436-3
Press	2,280	17.32	0.007596	5.7707-5	0.87	0.7569	4.3678-5
					0.85	0.7225	4.1693-5
					1.44	2.0736	1.1966-4

$\left(\frac{\sigma_y}{\mu_y}\right)^{2}$ (4)	$\left(\frac{\sigma_y}{\mu_y}\right)$	$\mu_y-1.645\sigma_y$ $\mu_y=0.995$ (5)	ITDP Limit DNBR $\left(\frac{1.3}{\mu_y-1.645\sigma_y}\right)$ (*)	ITDP Parametric (6) Uncertainty (*)−1.3
4.4999-3	0.06708	0.8852	1.467	0.167
7.1117-3	0.08433	0.8570	1.517	0.217
4.1076-3	0.06409	0.8901	1.461	0.161

Note (1) ref. 1: p. 246
(2) ref. 2: p. 156
(3) WCAP-8567 (ITDP): p. 4-21
(4) $\left(\frac{\sigma_y}{\mu_y}\right)^2=\sum_i S_i^2\left(\frac{\sigma_i}{\mu_i}\right)^2$
(5) WCAP-8567: p. 4-23
(6) cf. RSM parametric uncertainty:
2.641−2.476=0.165

the experimental design plans. The response surface coefficients are obtained using the least square method. Calculated results for DNBR uncertainty distribution indicate the limit value of nominal DNBR of 2.476. on the 95% probability/95% confidence basis.

Now that the minimum DNBR employing deterministic approach (fixed value method DNBR) is determined to be 2.09, the gain in DNBR is 0.39. Figure 4 shows the implications of thermal margin gain in this method against the conventional method. DNBR margin gain through statistical design is the quantity of uncertainty reduction compared to the fixed value method. Also shown in figure 4 is the relationship between thermal margin and uncertainties for the three different thermal design methods: Fixed value method, Improved thermal design procedure (ITDP),[8] Response surface method (RSM). Of these methods, both ITDP and RSM consider the parametric uncertainties in a statistical manner. But the fundamental difference between the two methods is that ITDP adopts the Central Limit Theorem and establishes the DNBR design limit based on the CHF (critical heat flux) correlation limit, while RSM constructs response surface based on the experimental design technique and utilizes Monte Carlo simulation to define the limit value of nominal DNBR.



(1) ITDP Parametric Uncertainty for small S_i case
(2) ITDP Parametric Uncertainty for large S_i case
(3) RSM Parametric Uncertainty (best-estimate)

Fig. 5. Comparison of ITDP and RSM Parametric Uncertainty Distributions

2^{k-p} factorial design is regarded as a good tool to contruct the response surface model because it needs comparatively lower runs but it treats the parametric uncertainty systematically. And the sensitivity coefficients obtained from this are more realistic because it is obtained as an averaged concept in the uncertainty range of the input parameters. Another merit of this procedure is that it considers 95% confidence level additionally while ITDP considers 95% probability only under the assumption of normal distribution of resulting DNBR through the Central Limit Theorem.

Table 7 shows ITDP limit DNBR and parametric uncertainties for three different sensitivity coefficients extracted from references 1, 2, and 8, respectively. Resulting parametric uncertainties are 0.167, 0.217, and 0.161, respecitvely compared to 0.165 of RSM. The important one to be pointed in this calculation is that the ITDP limit DNBR or parametric uncertainty is highly dependent on the sensitivity coefficients generated, but RSM is robust in this point of view. Figure 5 illustrates this point well. When small S_i's are used in the ITDP the resulting thermal margin is not conservative while not realistic for large S_i case. The robustness of RSM compensates for the additional computational effort (Monte Carlo simulation).

IV. Conclusion and Recommendation

Some conclusions are developed from this study:

(1) RSM is a useful method in performing statistical thermal margin analysis.

(2) Two level factorial design is a valuable method for investigating the sensitivities of input parameters, while the central composite design is good enough for fitting the second order polynomial.

(3) KNU-1 Reactor core thermal margin is increased using RSM and the obtaind DNBR gain over Fixed Value Method is 0.39 for steady state nominal operating condition.

(4) Future application of RSM can be extended to the statistical reactor safety analysis during transients and accidents.

(5) When ITDP is used to analyze the statistical thermal margin, much attention is needed due to the highly-dependence on sensitivity coefficients.

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