Simultaneous Addition and Subtraction of Optical Images by Using the Extended Incoherent Source

(인코히런트 광원을 이용한 영상의 동시 가감)

朴 炯 來*, 全 石 熙*, 朴 漢 奎*

(Hyung Rae Park, Seok Hee Jeon and Han Kyu Park)

要 約

본 논문에서는 인코히런트 광원을 이용하여 광 영상 합성을 실현하였으며, 0차 회절광과 2차 회절광의 효율이 같은 위상형 회절격자를 제작, 사용함으로써 영상의 동시 가감을 실현하였다. 광원의 공간적 코히런스와 스펙트럼 코히런스 조건은 각각 광원 부호화 마스크와 Interference Filter를 사용하여 만족하도록 하였으며, 코히런트 방식에 의한 실험도 병행함으로써 두 방식의 장단점을 비교, 분석하였다. 실험결과, 인코히런트 방식에서 Coherent artifact noise를 제거함으로써 보다 좋은 영상 합성을 실현할 수 있었다.

Abstract

A technique of optical image synthesis with an extended incoherent source is presented and compared with the coherent method. A holographic diffraction grating is fabricated by using Michelson interferometer, and by equalizing the 8th-order to the 2nd-order diffraction efficiency, complex amplitude addition and subtraction of optical images are simultaneously realized. The experiment shows that the quality of synthesized optical images in the incoherent method is improved in comparison with that of the coherent method by suppressing the coherent artifact noise.

I. Introduction

Optical image synthesis (subtraction and addition) is one of the most important method for information processing, and must be a powerful application to the recognition of similiar patterns or characters, fault detection of electronic circuits, urban development, and automatic tracking, etc. In 1965, Gabor et al. first introduced the optical image synthesis by complex amplitude addition and subtraction and in 1969, Bromley et al, proposed a holographic Fourier subtraction technique, for which a real-time image and a previously recorded hologram can be subtracted. In 1978, the technique of optical image synthesis using diffraction grating was proposed by Lee et al. However, coherent image processing is succeptible to coherent artifact noise, which frequently limits their processing capability, and the technique using incoherent source has been studied.

As a result, in 1979, Yu and Tai proposed a technique using incoherent point source, but to obtain the spatial coherence requirement for image processing, a very small point source is needed, and it is almost impossible to obtain a incoherent point source which has the power

*正會員, 延世大學校 電子工學科
(Dept. of Elec. Eng., Yonsei Univ.)
接受日字：1986年 2月 15日

efficiency applicable to image processing. Thus, to satisfy the coherence requirement source encoding method which samples the extended incoherent source according to the application of image processing has been studied. In 1981, Yu et al. introduced the technique which sampled the extended incoherent source and verified that the image quality of incoherent method was improved in comparison with that of coherent by suppressing the coherent artifact noise experimentally.

In 1984, Chartwright proposed the conception of the incoherent image processing using the cross-spectral density function which was first introduced by Mandel and Wolf.

In this paper, we adopt the method of Yu and Wu, and by equalizing the 8th order to the 2nd order diffraction efficiency of phase grating, we show that simultaneous addition and subtraction of optical images can be realized, and the number of the channel can be increased more than 3 times.

II. Propagation of Correlation Functions

Consider the case of which the light disturbances $V(x_1, t)$ and $V(x_1', t)$ at points x_1, x_1' respectively, of the plane A propagate to the plane B. If $V(x_2, t)$ and $V(x_2', t)$ are the complex disturbances at points x_2, x_2' of the plane B, by the Huygens-Fresnel principle, we have

$$V(x_2, t) = \int V\left(x_1, t-\frac{R_1}{C}\right) K(x_1, x_2, \bar{\nu})\, dx_1$$
$$V(x_2', t) = \int V\left(x_1', t-\frac{R_2}{C}\right) K(x_1', x_2', \bar{\nu})\, dx_1' \quad (1)$$

where $K=\frac{1}{j\lambda}\frac{\exp(jkR)}{R}\cos(\vec{n}, \vec{R})$ is the transmittance function of the system, and R_1, R_2 are distances between the two planes. The mutual intensity between two points x_2 and x_2' is given by

$$J(x_2, x_2') = \langle V(x_2, t)\ V^*(x_2', t)\rangle \quad (2)$$

By substituting eq. (1) into eq. (2), we have

$$J(x_2, x_2') \iint \left\langle V\left(x_1, t-\frac{R_1}{C}\right) V^*\left(x_1', t-\frac{R_2}{C}\right)\right\rangle K(x_1, x_2, \bar{\nu}) \cdot K(x_1', x_2', \bar{\nu})\, dx_1\, dx_1'$$
$$= \iint [V(x_1, t)\, V^*(x_1, t+\tau)]\, K(x_1, x_2, \bar{\nu})\, K^*(x_1', x_2', \bar{\nu}) \cdot dx_1 dx_1' \quad (3)$$

where $\tau=\frac{R_1-R_2}{C}$ represents time delay between R_1 and R_2.

If $|\tau|$ is so small that for all frequencies, ie, if $|\tau|$ is small compared to the coherence time of light, we may write as follows:

$$[V(x_1, t)\ V^*(x_1', t+\tau)] \approx [V(x_1, t)\ V^*(x_1', t)] = J(x_1, x_1') \quad (4)$$

Therefore, in the case of quasi-monochromatic source, the correlation functions are

$$J(x_2, x_2') = \iint J(x_1, x_1')\ K(x_1, x_2, \bar{\nu})\, K^*(x_1', x_2', \bar{\nu})\, dx_1\, dx_1'$$
$$\mu(x_2, x_2') = \frac{1}{\sqrt{I(x_2)}\sqrt{I(x_2')}} J(x_2, x_2') = C_1 \iint \mu(x_1, x_1')\ K(x_1, x_2, \bar{\nu})\, K^*(x_1', x_2', \bar{\nu})\, dx_1\, dx_1' \quad (5)$$

where $C_1 = \frac{\sqrt{I(x_1)}\sqrt{I(x_1')}}{\sqrt{I(x_2)}\sqrt{I(x_2')}}$

In practice, consider the case of which the light disturbances propagate along the z-axis in the region of fraunhofer diffraction.

The complex disturbances at points x_2, x_2', may be written

$$V(x_2, t) = \frac{1}{j\lambda z}\exp(jKz)\ \exp\left(j\frac{\pi}{\lambda z}x_2^2\right) \int_{-\infty}^{\infty} V\left(x_1, t-\frac{z}{C}\right)\ \exp\left(-j\frac{2\pi}{\lambda z}x_1 x_2\right) dx_1$$
$$V(x_2', t) = \frac{1}{j\lambda z}\exp(jKz)\ \exp\left(j\frac{\pi}{\lambda z}x_2'^2\right) \int_{-\infty}^{\infty} V\left(x_1', t-\frac{z}{C}\right)\ \exp\left(-j\frac{2\pi}{\lambda z}x_1' x_2'\right) dx_1' \quad (6)$$

where K is the propagation constant and z is the distance between two planes. Therefore, the correlation functions between two points x_2, x_2' are

$$J(x_2, x_2') = \langle V(x_2, t)\ V^*(x_2', t)\rangle$$

$$= \frac{\exp\left\{j\frac{2\pi}{\lambda z}(x_2^2 - x_2'^2)\right\}}{\lambda^2 z^2} \iint_{-\infty}^{\infty}\left[V\left(x_1, t-\frac{z}{C}\right) V^*\left(x_1', t-\frac{z}{C}\right)\right] \cdot \exp\left\{-j\frac{2\pi}{\lambda z}(x_1x_2 - x_1'x_2')\right\} dx_1dx_1'$$

$$= \frac{\exp\left\{j\frac{2\pi}{\lambda z}(x_2^2 - x_2'^2)\right\}}{\lambda^2 z^2} \iint_{-\infty}^{\infty} J(x_1, x_1') \exp\left\{-j\frac{2\pi}{\lambda z}(x_1x_2 - x_1'x_2')\right\} dx_1\, dx_1'$$

$$\mu(x_2, x_2') = \frac{1}{\sqrt{I(x_2)}\sqrt{I(x_2')}} J(x_2, x_2')$$

$$= C_2 \iint_{-\infty}^{\infty} \mu(x_1, x_1') \exp\left\{-j\frac{2\pi}{\lambda z}(x_1x_2 - x_1'x_2')\right\} dx_1dx_1' \quad (7)$$

where $C_2 = \frac{\sqrt{I(x_1)}\sqrt{I(x_1')}}{\sqrt{I(x_2)}\sqrt{I(x_2')}} \frac{\exp\left\{j\frac{2\pi}{\lambda z}(x_2^2 - x_2'^2)\right\}}{\lambda^2 z^2}$

III. System Analysis

Fig. 1 represents the partially coherent optical processor with an encoded extended incoherent source. According in eq. (2-7), the complex degree of coherence at the plane P_0 may be written as

$$\mu_3(x_3, x_3') = C_3 \iint \mu_2(x_2, x_2')\ t(x_2)\ t^*(x_2') \exp\left\{-j\frac{2\pi}{\lambda f}(x_2x_3 - x_2'x_3')\right\} dx_2dx_2' \quad (8)$$

where $\mu_2(x_2, x_2')$ is the complex degree of coherence at the input plane P_2 and C_3 is constant, while $t(x_2) = f(x_2 - h_0) + f(x_2 + h_0)$ is the input transparency function.

The amplitude transmittance function of the phase grating is given by,

$$t(x_3) = \exp\left\{j\frac{m}{2}\sin\ p_0(x_3 - \alpha)\right\}$$

$$= \sum_{n=-\infty}^{\infty} Jn\left(\frac{m}{2}\right) \exp\left\{jn\ p_0(x_3 - \alpha)\right\} \quad (9)$$

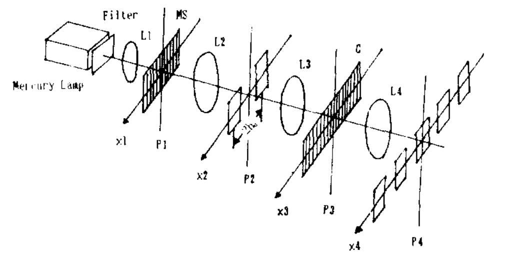

Fig. 1. Partially coherent optical image processor with an encoded extended incoherent source ; S, mercury arc lamp; MS, multislit mask; L_1, condensing lens, f_1 and f_2, object transparencies; G, phase grating; L_2, L_3, and L_4, Fourier transform lenses.

where $p_0 = 2\pi h_0/\lambda f$ and J_n is a Bessel function of the first kind, order n.

By the way, the complex degree of coherence immediately behind the grating is

$$\mu_3'(x_3, x_3') = t(x_3)\ t^*(x_3')\ \mu_3(x_3, x_3') \quad (10)$$

and if we consider the first order of the grating $\mu_3'(x_3, x_3')$ may be written,

$$\mu_3'(x_3, x_3') = \left[J_1\left(\frac{m}{2}\right)\exp\{jp_0(x_3 - \alpha)\} - J_1\left(\frac{m}{2}\right)\exp\{-jp_0(x_3 - \alpha)\}\right] \cdot \left[J_1\left(\frac{m}{2}\right)\exp\{-jp_0(x_3' - \alpha)\} - J_1\left(\frac{m}{2}\right)\exp\{jp_0(x_3' - \alpha)\}\right] \mu_3(x_3, x_3') \quad (11)$$

The intensity distribution at the output plane P_4 may be written as the form of Fourier transform of $\mu_3'(x_3, x_3')$, i. e.

$$I(x_4) = C_4 \iint \mu_3'(x_3, x_3') \exp\left\{j\frac{2\pi}{\lambda f}(x_3 - x_3')x_4\right\} dx_3dx_3' \quad (12)$$

where C_4 is a constant.

1. Image addition around the origin of the output plane P_4 ($n' = 0, \pm1, \pm2\ldots$)

when $p_0\alpha = \frac{(2n'+1)\pi}{2}$

$\exp(jp_0\alpha) = j(-1)^{n'}$

$\exp(-jp_0\alpha) = -j(-1)^{n'}$,

and by substituting eqs. (8), (9), and (11) into eq. (12), we have

$$I(x_4) = C_5 \iint [\mu_2(x_2, x_2')\{f_1(x_2-h_0)+f_2(x_2+h_0)\} \{f_1^*(x_2'-h_0)+f_2^*(x_2'+h_0)\} \cdot \{\delta(x_2-x_4-h_0)\ \delta(-x_2'+x_4+h_0) + \delta(x_2-x_4-h_0)\ \delta(-x_2'+x_4-h_0) + \delta(x_2-x_4+h_0)\ \delta(-x_2'+x_4+h_0) + \delta(x_2-x_4+h_0)\ \delta(-x_2'+x_4-h_0)\}] dx_2 dx_2', \tag{13}$$

where $C_5 = -C_4 J_1\left(\frac{m}{2}\right)$ and $\delta(x)$ is the dirac delta function.

Because $\mu_2(x_2, x_2')$ takes the form of $\mu_2(x_2 - x_2')$ and $\mu(x)$ can be written as $\mu^*(-x)$, intensity distribution may be re-written as follows:

$$I(x_4) = \mu_2(\theta)\{|f_1(x_4)|^2 + |f_2(x_4)|^2 + |f_1(x_4-2h_0)|^2 + |f_2(x_4+2h_0)|^2\} + \mu_2(2h_0)\ f_1(x_4)\ f_2^*(x_4) + \mu_2^*(2h_0)\ f_1^*(x_4)\ f_2(x_4) \tag{14}$$

Hence, the intensity distribution around the origin of the output plane P_4 is

$$I_0(x_4) = |\mu_2(2h_0)|\ |f_1(x_4)+f_2(x_4)|^2 + \{1-|\mu_2(2h_0)|\}\ \{|f_1(x_4)|^2 + |f_2(x_4)|^2\} \tag{15}$$

From the above equation we see that if the degree of coherence $|\mu_2(2h_0)|$ is high, i.e. $|\mu_2(2h_0)| \simeq 1$, eq. (15) reduces to

$$I_0(x_4) \simeq |f_1(x_4)+f_2(x_4)|^2 \tag{16}$$

Eq. (16) shows that image addition takes place around the origin of the output plane P_4 in the case of $p_0\alpha = (2n'+1)\pi/2$.

2. Image subtraction around the origin of the output plane P_4

when $p_0\alpha = n'\pi$ ($n' = \theta, \pm 1, \pm 2, \cdots$)

$\exp(jp_0\alpha) = \exp(-jp_0\alpha) = (-1)^{n'}$

In a similiar fashion, the intensity distribution is given by

$$I_0(x_4) = |\mu_2(2h_0)|\ |f_1(x_4-f_2(x_4)|^2 + \{1-|\mu_2(2h_0)|\}\ \{|f_1(x_4)|^2 + |f_2(x_4)|^2\} \tag{17}$$

Also, we see that in the case of $|\mu_2(2h_0)| \simeq 1$, output pattern around the axis of P_4 has the form of image subtraction, i.e.

$$I_0(x_4) \simeq |f_1(x_4)-f_2(x_4)|^2 \tag{18}$$

3. Coherence requirements

1) Spatial coherence requirement

From eqs. (16) and (18) we see that in the case of $|\mu_2(2h_0)| \simeq 1$, image subtraction or addition takes place around the origin of the output plane. In practice, the spatial coherence requirement for image synthesis can be obtained by sampling the extended incoherent source spatially, that is, the complex degree of coherence, $\mu_2(x_2, x_2')$ over the input plane P_2 can be written as,

$$\mu_2(x_2, x_2') = C_5 \int I(x_1)\ \exp\left\{-j\frac{2\pi}{\lambda f}(x_2-x_2')x_1\right\} dx_1 \tag{19}$$

where $I(x_1)$ is the intensity transmittance function of the sampling mask and C_5 is a constant. Suppose that we sample the extended incoherent source with N numbers of narrow slits, then we have

$$I(x_1) = \sum_{n=1}^{N} \mathrm{rect}\left(\frac{x_1 - nd'}{S}\right) \tag{20}$$

where S is the slit width and d is the spacing between slits. By substituting eq. (20) into eq. (19), the complex degree of coherence over the input plane P_2 may be written

$$\mu_2(x_2-x_2')=\frac{\sin\left\{N\pi\frac{d'}{\lambda f}(x_2-x_2')\right\}}{N\sin\left\{\pi\frac{d'}{\lambda f}(x_2-x_2')\right\}}\operatorname{sinc}\left\{\frac{\pi S}{\lambda f}(x_2-x_2')\right\} \quad (21)$$

and if d' is equal to d, the spacing of the phase grating (i.e. d'=d), the spatial coherence function becomes

$$\mu_2(x_2-x_2')=\frac{\sin\left(N\pi\frac{x}{h_0}\right)}{N\sin\left(\pi\frac{x}{h_0}\right)}\operatorname{sinc}\left(\pi\frac{sx}{dh_0}\right) \quad (22)$$

where $d=d'=f\lambda/h_0$. From eq. (22), we see that the complex degree of coherence exists at every $x_2-x_2'=\lambda f/d'$ in the form of narrow pulses and the width of the pulses is inversely proportional to the number of slits, N.

2) Spectral coherence requirement

In order to obtain the accurate image synthesis in the incoherent optical image processing, the spread of Fourier spectrum with wavelengthes must be within a small fraction of the grating space, d, i.e,

$$\frac{p_m f}{2\pi}\Delta\lambda \ll d \quad (23)$$

where p_m is the highest angular spatial frequency of the input objects, and $\Delta\lambda$ is the spectral bandwidth of the source.

By substituting $d=f\lambda/h_0$ into eq. (23), we have

$$\frac{\Delta\lambda}{\lambda}\ll\frac{2\pi}{h_0 p_m} \quad (24)$$

Eq. (24) shows that the higher the spatial frequency of the input objects, the narrower the spectral bandwidth required.

IV. Experiment and Discussion

1. Fabrication of the diffraction grating and the encoding mask

The system for fabrication of the holographic diffraction grating is shown in Fig. 2.

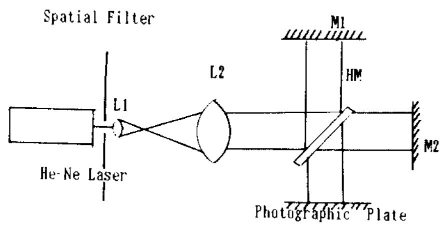

Fig. 2. Michelson interferometer used for the fabrication of the amplitude grating: L_1 and L_2, collimating lenses; M_1 and M_2, mirrors; HM, half mirror.

If the mirror M is tilted by θ, the angle on the photographic plate between the two collimated beams which pass through the half mirror is 2θ. Two collimated beam cause the interference on the photographic plate and the grating space is $d=\lambda/2\sin\theta$. Therefore, within the limit of resolution of the plate the desired fringe space is obtained by controlling θ.

In this experiment He-Ne laser with maximum power 5mW is used and the spatial noise is suppressed by using the pin-hole of the diameter being 100μm. We fabricate the phase grating by bleaching the prefabricated amplitude grating with chromium intensifier.

Also, for the satisfaction of spatial coherence requirement for image synthesis, the multi-slit mask-its width and space are 1 μm, 24.7 μm, respectively— is fabricated on the basis of Young's interference experiment by CAD.

2. Image synthesis and result discussion

In this experiment Mercury arc lamp with maximum power 300W as an incoherent source is used. The temporal coherence requirement for incoherent optical processor is satisfied by using a Yellow-Green interference filter; its mean-wave length and effective width are 5461Å, 100Å respectively.

1) Delicate adjustment of focus

In image synthesis the diffraction grating must be placed exactly on the back focal plane of Fourier transform lens for exact addition and subtraction of two input images.

In Fig. 1, when the diffraction grating is not placed exactly on the back focal plane of lens L_3 , the fringes are shown in the output pattern by effective optical path difference of the input image f_1 and f_2. Namely, the bright fringes are shown at the point where the effective optical path difference of two input images is integral multiple of mean-wavelength. Therefore, when the fringes on the output pattern are completely disappeared, we see that the diffraction grating must be placed exactly on the back focal plane.

The output pattern in which the diffraction grating is out of focus is shown in photo 1.

Photo 1. Fringe pattern in which diffraction grating is out of focus.

2) Addition around the origin of the output plane P_4

After the diffraction grating is completely aligned to the back focal plane of lens L_3 such as experiment 2.1., we observe the output pattern at the position to satisfy $p_0\alpha=(2n'+1)\pi/2$.

As the spacing of diffraction grating is minute in actual case, the diffraction grating is adjusted delicately by using the micrometer to the normal direction of optical axis and the output pattern which is added most clearly around the origin is observed.

Photo 2. represents the output pattern added around the axis which is the synthesis of two circles and two characters, being "YONSEI" and "YO SE".

3) Subtraction around the origin of the output plane P_4

In the case of $p_0\alpha=n'\pi$, the image subtraction occurs on the axis. Like image addition, first of all, the diffraction grating is placed on the back focal plane of lens L_3.

And by adjusting delicately we obtain the output pattern which is subtracted around the origin most clearly.

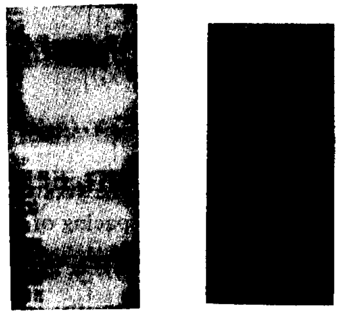

Photo 2. Output pattern added around the axis.

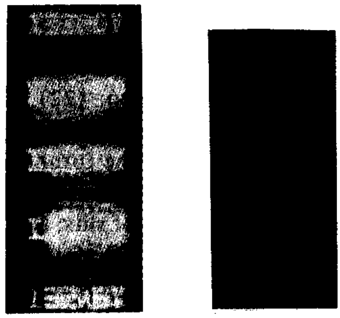

Photo 3. Output pattern subtracted around the axis.

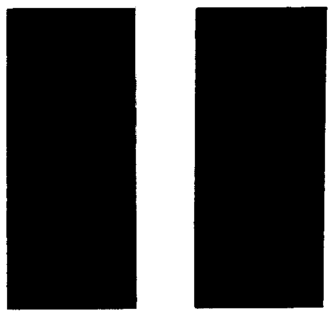

Photo 4. Output pattern synthesized by coherent method.

Photo 3. represents the output pattern subtracted around the origin.

When we examine photo 2 and 3, we see that the reversal synthesis for the pattern around the origin is occurred at the points $\pm nh_0$ apart from the origin.

But at the points, away from the origin more than $2h_0$, the difference of the efficiency of diffracted light used for image synthesis is very large. So we cannot obtain the result of right synthesis and because the diameter of Fourier transform lens which is used in the experiment is finite, there is a limit in the spacing and the size of the input image by the spherical aberration. The output pattern synthesized by coherent method is shown in photo 4.

By comparing photo 2 and 3 with 4, as expected, we see that the quality of synthesized image in incoherent method is improved in comparison with that of coherent method by suppressing the coherent artifact noise.

V. Conclusion

In this paper, the phase grating with the higher efficiency of the 1st order diffraction is fabricated and the technique of optical image synthesis with an extended incoherent source is presented and compared with a coherent method.

The phase grating is fabricated by bleaching the amplitude grating with chromium intensifier.

As the results the 1st order diffraction efficiency is increased from 5% to 21% (about 4 times), on the contrary the 8th order is from 22% to 7% (about 3 times decrement), and by equalizing the 8th order to the 2nd order diffraction efficiency, complex amplitude addition and subtraction of optical images are simultaneously realized.

In the case of incoherent method, for the satisfaction of spatial coherence requirement, source encoding mask is designed and fabricated.

In this paper, as the realization of simultaneous addition and subtraction, information processing time is reduced less than half and the quality of synthesized optical image in incoherent method is improved in comparison with that of coherent method by suppressing the coherent artifact noise.

References

[1] D. Gabor and G.W. Stroke, etc. "Optical image synthesis by holographic Fourier transformation", *Physics Letters*, vol. 18, no. 2, 1965.

[2] K. Bromley and M.A. Monahan, etc. "Complex spatial filtering by holographic fourier subtraction", *Applied physics letters*, vol. 14, no. 2, 1969.

[3] S.H. Lee, S.K. Yao, and A.G. Milnes "Optical image synthesis (complex amplitude addition and subtraction) in real time by a diffraction-grating interferometric method", *J. Opt. Soc. Am.*, vol. 60, no. 8, 1970.

[4] F.T.S. Yu and A. Tai "Incoherent image addition and subtraction: a technique", *Applied Optics*, vol. 18, no. 15, 1979.

[5] S.T. Wu and F.T.S. Yu "Image subtraction with an encoded extended incoherent sources", *Applied Optics*, vol. 20, no. 23, 1981.

[6] S.L. Zhaung and F.T.S. Yu "Coherence requirements for partially coherent optical processing", *Applied Optics*, vol. 21, no. 1982.

[7] Hwan Joo Jeong, *Simultaneous Optical Image Addition and Subtraction by Using a Phase Grating on Fourier Transform Plane*. KAIST 1982.

[8] S. Cartwright "Incoherent optical processing: a coherence theory approach", *Applied Optics*, vol. 23, no. 2, 1984.

[9] M. Born and E. Wolf, *Principles of Optics.*, 1975.

[10] J.W. Goodman, *Introduction to Fourier Optics*, 1968.