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論 文

유전체 공진기의 공진주파수 및 공진모드 해석

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An Analysis of the Resonant Frequencies and Modes in Dielectric Resonators

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요 약 유전체 공진기를 실제로 사용할 때 발생하는 여러가지 영향을 고려한 원통형 유전체공진기의 공진주파수 및 field계수를 결정하기 위한 보다 효율적인 수치해석법과 컴퓨터프로그램을 개발했다. 이 새로운 해석법에는 차폐금속면, 마이크로스트립substrate, 유전체지지대 및 유전체공진기의 제작시 발생되는 허용오차를 보상하기 위한 유전체post 등의 영향이 고려됐다.

ABSTRACT An efficient numerical technique and computer program has been developed for the determination of the resonant frequencies and field coefficients of cylindrical dielectric disc resonators in a configuration which models the various effects present in real technical applications. It accounts for the influences due to shielding, planar circuit substrate, dielectric support and a dielectric post used to tune out fabrication tolerances.

1. Introduction

With the development of low-loss, temperaturestable ceramic materials for microwave frequencies during the last ten years, dielectric resoators have become potentially suitable for high-Q applications which up to now were reserved for cavity and coaxial resonators. Although the properties of dielectric resonators have been known in principle since the times of Debye (1), with their recent application to the realisation of narrow-band filters and resonator-stabilised oscillators, a more detailed investigation of their electromagnetic behaviour has become necessary. In particular, the integration of dielectric resonators into planar microwave circuits—see, for example, Reference 2— requires a detailed analysis taking into account the effects on the resonator frequency and field of adjacent parts of the circuit, such as the planar circuit substrate, a metallic resonator shielding and a dielectric disc support introduced to avoid a reduction of the resonator Q-factor. In addition, it is useful to consider the influence of a dielectric cylindrical tuning post protruding from the top shielding in order to be able to compensate for a shift of the resonant frequency due to fabrication tolerances of the dielectric resonator. Numerous aspects concerning the design and the application of dielectric reson-

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ators have been treated individually in the technical literature. The resonant frequencies of shielded dielectric resonators have been computed analytically and numerically--see, for example, References 3 and 4. A rigorous and comprehensive analysis which considers the effects present in standard practical configurations has not been published previously, and is described here. Some representative results are also given.

2. Numerical method

The general resonator structure analysed is shown in Fig. 1 and contains all the elements mentioned. It is of radial symmetry and consists of five regions, I... V, each of which can be considered as a short length of waveguide either homogeneous (I, IV) or partially filled with dielectric (II, III, V). The different dielectric materials are assumed to be lossless and isotropic. Therefore, the electromagnetic field in each of the regions I...V can be expanded into a complete set of modal fields of the corresponding cylindrical waveguide, TE and TM modes.

Owing to the radial symmetry of the structure, the class of solutions which are possible decomposes into different angular dependences cos((m φ)) or sin (m φ), so that the angular index m can be predescribed. With m=0, the technically most important applications of dielectric resonators are covered. Although the method of analysis employed is also applicable to angular dependencies m ≠ O, the computer program written has been restricted to solutions of radial summetry. The reason for this is that, with m ≠ O complex waves- see, for example, Reference 5--would have to be encountered in the partially dielectric filled regions II, III and V of Fig. 1. This would introduce analytical and numerical complications, the treatment of which cannot be justified in view of the relative technical unimportance of these cases. So, finally, the problem is formulated in terms of the TE_{on} and TM_{on} modes of the waveguides corresponding to regions I...V for the associated transverse electric and magnetic modal fields. For region q,q=I...V, this means a transverse field representation of the form

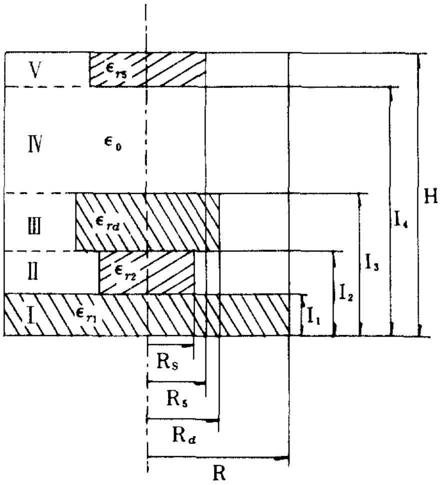

Fig. 1 General structure of shielded dielectric resonator including dielectric support and tuning post.

$$\bar{E}_t^q = \sum_{n=1}^{N} (\underline{A}_n \cos \beta_{on}^q z + \underline{B}_n \sin \beta_{on}^q z)\, \bar{E}_{ton}^q$$

$$\bar{H}_t^q = -j \sum_{n=1}^{N} (\underline{A}_n \sin \beta_{on}^q z - \underline{B}_n \cos \beta_{on}^q z)\, \bar{H}_{ton}^q$$

where β_{on}^q denotes the modal propagation constants, $\underline{A}_n$ and $\underline{B}_n$ the associated field coefficients and j is the imaginary unit. N is the upper summation limit of the truncated modal expansions in each region used in the analysis. The main advantage of the above formulation is that it allows the reduction of the existing three-dimensional dielectric resonator problem into successive one-dimensional steps. In addition, it allows the use of orthogonality relations which apply to the modal fields in the cross-sections of regions I... V (6), and thus a significant reduction in the complexity of the analysis. All the boundary conditions to be satisfied on the metallic enclosure are taken into account analytically in the above formulation.

In a first step of the numerical analysis, the modal propagation constants β^q_{on} and associated transvere expansion functions $\overline{E}^{-q}_{ton}$, $\overline{H}^{-q}_{ton}$ are determined for a given start frequency. The integral coupling coefficients between successive regions q=I...V are then computed, which is equivalent to satisfying the continuity conditions at the interfaces I, II...IV, V by mode matching. Then, the elements of the characteristic system of equations governing the dielectric resonator problem are set up. Owing to the analytical elimination of unknown field coefficients on the basis of orthogonality relations, the corresponding system matrix is of reduced order N′=4N. It is split up into two submatrices which contain the separated TE and TM modes of the system. By systematic variation of the start frequency, the resonator solutions are found as the zeros of the determinant of the system matrix containing the associated coefficients. Typically, with N = 4 or 5, accuracies of the resonant frequencies of a small fraction of a percent can be achieved. The order N′=4N of the system matrix can be kept low in most cases of practical interest.

3. Numerical Results

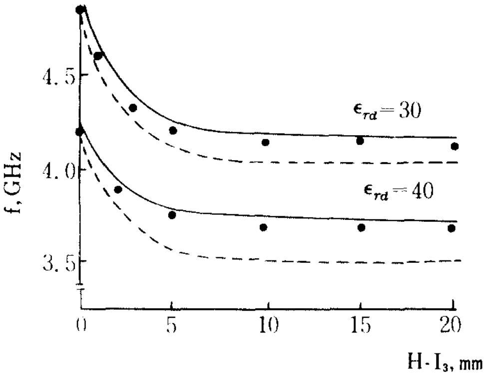

Fig.2 Variation of the first TE-resonant frequencies with the shielding height H-l_3
$l_1=l_2=3$ mm. $l_3=6$ mm. R=7.5mm, R=17.5mm, $\epsilon_{r1}=2.5$
—— from Reference 4
- - N = 1 } this method
...... N = 2 } this method

As a representative example, Fig. 2 shows the results of a computation of the technically important $TE_{01\sigma}$ resonant frequency in a situation where the height H of the shielding is varied. With $L_1=L_2$, i.e. without dielectric support, with $L_4=H$, i.e. without dielectric tuning post and the remaining parameters as specified in Fig. 2, a comparison can be made with Fig. 5 of Reference 4. Specifically, for the case of H = 19mm, the first-order solution N=1 of Fig. 2 differs by only about 3% from the converged solution of Reference 4. With N=2 the difference decreases to about 0.3%, and with further increase of N it cannot be resolved within the accuracy of the figures. Measured results from Reference 8 have been taken for comparison in cases where the $TE_{01\sigma}$ frequency is not affected by the presence of sidewalls. The absence of boundaries is simulated in the method described here by removing the metallic shielding far enough from the resonator. As is to be expected, it is found that the speed of

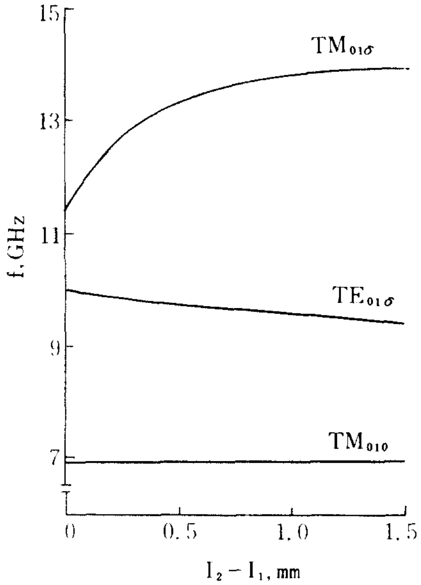

Fig3 Variation of resonance frequencies with the dielectric support height l_2-l_1
H = 8.896mm, R = 15mm, $l_3-l_2=2.14$mm, $l_1=0.70$mm, R_d 3.01mm, R_s = 1 mm, $\epsilon_{r1}=9.5$, $\epsilon_{r2}=5$, $\epsilon_{rd}=36.2$.

convergence decreases with an increase of the ratio of the shielding radius to the radius of the dielectric resonator pill. However, with N = 8 ...10, even the truly open case can be accurately modelled by the computer algorithm described. Fig. 3 shows the influence of a dielectric support introduced between the substrate and the dielectric resonator on the resonant frequencies of a specific resonator configuration in the frequency band 1-15 GHz. The lowest frequency visible in Fig. 3 corresponds to the quasi-TM_{010} resonant mode of the cylinder cavity which is partially loaded with dielectric material. With an increase of the support thickness $l_2 - l_1$ the first TE resonant frequency of the dielectric resonator decreases, while the first TM frequency increases. This corresponds to the results of experiments carried out in Reference 7. Finally, it is shown in Fig. 4 how dielectric resonators can be tuned by means of a cylindrical tuning post which consists of the same material but is not resonating itself because of its smaller dimensions [9].

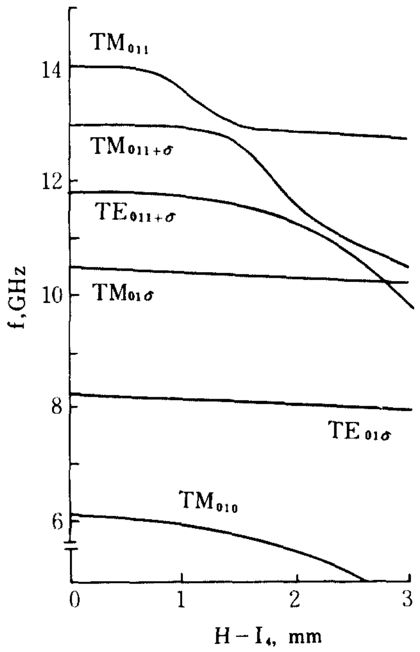

Fig 4 Resonant frequencies as a function of the distance of a tuning post from the resonator top
$H = 8.9mm, R = 15mm, H_d = l_3 - l_2 = 4.22mm, R_d = 3.03mm, l_1 = l_2 = 0.70mm, R_s = 3mm, \epsilon_{r1} = 9.5, \epsilon_{rd} = \epsilon_{rs} = 36.2.$

4. Conclusions

A post of radius r=3mm and height 3mm is utilised with a lowest-order TE resonance at 12.077 GHz. The $TE_{01\sigma}$ resonance of the dielectric resonator under consideration is at 8.1996 GHz if the tuning post is at a distance of 3mm or more from the resonator top. By reduction of this distance down to the point where it makes contact with the resonantor, a shift of the $TE_{01\sigma}$ resonance to a frequency of 7.988 GHz is achieved which is equivalent to a tuning range of 2.5%. If a post with a lower dielectric constant is used, the effect becomes smaller. For this reason, and in order to avoid a decrease of the resonater Q-factor, it should be advantageous to choose the same material, eventually with different temperature coefficients, for the tuning post and the dielectric resonator.

References

(1) D. Hondros and P. Debye, "Dieletric waveguide", Ann. Phys. vol. 32, pp. 465-476, 1910.

(2) 홍의석, "마이크로웨이브에서의 유전체공진기", 한국전기학회지, 제32권, 제3호 pp. 150-155, 1983.

(3) R. Bonetti and A. Atia, "Design of cylindrical dielectric resonators in inhomogeneous media", IEEE Trans. MTT-29, pp. 323-326, 1981.

(4) U. Crombach and R. Michelfeit, "Resonanzfrequenzen und feldstaerken in geschirmten dielektrischen scheiben-und ringresonator", Frequenz, vol. 35, pp. 324-328, 1981.

(5) P. J. B. Clarricoats and B. C. Taylor, "Evanescent and propagating modes of dielectric-loaded circular waveguide", Proc. IEE, vol. 111, pp. 1951-1956, 1964.

(6) U. S. Hong and R. H. Jansen, "Veraenderung der resonanzfrequenzen dielektrischer resonatoren in mikrostripschaltungen durch umgebungsparameter", AEU, vol. 38, pp. 106-112, 1984.

(7) W. R. Day, "Dielectric resonators as microstrip circuit elements", IEEE Trans., MTT-18, pp. 1175-1176, 1970.

(8) M. W. Pospieszalski, "Cylindrical dielectric resonators and their applications in TEM line microwave circuits", IEEE Trans., MTT-27, pp. 233－238, 1979.

(9) U. S. Hong, "Zur Berechnung geschirmter dielektrischer resonatoren", Ph. D. Dissertation, T. H. Aachen, W. Germany, 1982.

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