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측정 불가능한 미지외란을 포함한 계통에 대한 관측기 설계방법에 관한 연구

A Generalized Method to Design Observers for the Systems with Unknown Disturbances

李 起 常*
(Kee-Sang Lee)

요 약

측정 불가능한 미지외란이 투입된 계통에 대한 관측기의 설계를 위해 새로운 설계방법을 제안하고 미지외란의 존재에도 불구하고 매우 정확한 상태추정을 할 수 있는 최소차수관측기의 존재조건을 제시했다.

본 연구에서 제시된 관측기설계방법은 근래에 개별적으로 제안, 발전된 외란모델링방법 (John O'Reilly)과 대수적접근방법(Wang, Kudva)을 결합한 형태로서 관측기차수 및 존재조건의 완화라는 면에서 양자의 장점을 취하고 단점을 보완하는 특징을 갖고있다. 따라서 기존의 방법으로는 처리가 불가능하거나, 가능하더라도 관측기의 차수가 매우 커서 실현이 불가능했던 대부분의 경우에 실질적 적용이 가능하며 동시에 미지외란을 모델링함으로써 발생되는 추정오차도 감소시킬 수 있다.

마지막으로 제안된 기법의 유용성을 보이기위해 대수적 접근방법에 의해서는 처리가 불가능하고 외란모델링방법의 적용시 최소한 5차 이상의 관측기만이 설계가능한 계통에 이 방법을 도입함으로서 매우 정확한 상태추정을 하는 3차의 관측기를 설계했고 그 시간특성을 그림으로 보였다.

Abstract

The design of observers for the systems with unknown and unmeasurable disturbances is treated. A generalized observer design method is proposed and existence conditions are established by combining the existing two different approaches; disturbance modelling approach by O'Reilly and algebraic approach by Kudva et. al.. The proposed approach, therefore, takes the advantages and removes the shortcomings of the existing two approaches in view points of dimensionality and existence conditions of the observer. To show the usefulness of the approach, a numerical example is given.

1. Introduction

The design of observers for the systems with unknown inaccessible external disturbances has received much attention in recent days.

A promissible method to design such an observer is disturbance modelling approach motivated by the work of C.D. Johnson (1971) (1975) and developed by several researchers such as Meditch and Hostetter (1974), John O'Reilly (1978) (1979) etc.[1)-3)]

In this approach, unknown disturbances

*正 會 員：檀國大 工大 電氣工學科 助教授·工博
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are modelled by output of a dynamical system or polynomial type equations. The system equations are then augmented with this disturbance model and an observer is designed for the overall system. Since such an observer exists if and only if the augmented system is completely observable, the exitence condition can be easily met and an observer can be designed by usual manner (Luenberger 1971).[4] This approach, however, increases the dimension of the observer considerably and implementation may be impractical and/or difficult due to the high dimensionality.

Another method to design observers for the systems with unknown inaccessible disturbance is the direct rejection approach proposed by Wang et. al. (1975).[7]

In this approach, the observer gain matrix is obtained by algebraic method so as to reject the effects of disturbances on estimated vector and to assign poles of the observer to prescribed values. The order of the observer is, thus, low. However, the existence condition can be seldomly met due to the fact that disturbance rejection and poleassignment must be simultaneously achieved by constant gain matrix.

It is now apparent that one approach suffers from dimensionality but enjoy the weak existence conditions while the other suffers from strong existence conditions but enjoy the low dimensionality.

A question that may be raised at this point is whether or not the two approaches can be combined into one to remove the short-comings and to utilize the merits of them and, if possible, what it is. The main purpose of this paper is, therefore, to propose a generalized method to design observers for the systems with unknown and inaccessible disturbance vectors by modifying above methods. And basic idea behind the generalized approach is the fact that the disturbance vector can be partitioned into two vectors, where one can be directly rejected, the other is the disturbance vector that cannot be rejected by constant gain matrix and, therefore, should be modelled.

To finish the answer, existence conditions for the observer designed by using generalized approach are given in terms of original system parameters. And design procedures are described in detail.

2. Problem Description

Consider the linear time-invariant system with unmeasurable disturbances driven by following equations.

$$\dot{X}(t)=AX(t)+BU(t)+DW(t) \qquad (1)$$
$$Y(t)=CX(t)$$

where $X(t)\in R^n$ is state vector, $U(t)\in R^m$ is input vector, $Y(t)\in R^p$ is output vector and $W(t)\in R^q$ is disturbance vector. A,B,D,C are (n,n), (n,m), (n,q) and (p,n) matrices respectively. Assume that Rank (C)=P and (C,A) is completely observable pair. Now the problem is to design an observer that gives good estimates in the face of unmeasurable disturbances. And a potentially useful approach currently available is disturbance modelling approach (John O'Reilly 1978).[3]

The disturbance vector is assumed to be modelled by the differential equation, therein.

$$\dot{Z}(t)=EZ(t) \qquad (2)$$
$$W(t)=HZ(t)$$

where E, H are $(\delta q, \delta q)$, $(q, \delta q)$ matrices with following form.

$$E=\begin{pmatrix} 0 & I_{(\delta-1)q} \\ 0 & 0 \end{pmatrix} \text{ and } H=(I_q \,\vdots\, 0) \qquad (3)$$

and δ is obtained from polynomial type equation for single disturbance.

$$W(t)=\sum_{i=0}^{\delta-1} a_i t^i, \quad \delta \geq 1 \qquad (4)$$

By augmenting the original system of eq(1) with the disturbance model of eq(2), the overall system may be obtained by eq(5).

$$\tilde{X}(t)=\tilde{A}\tilde{X}(t)+\tilde{B}\tilde{U}(t) \text{ and}$$
$$Y(t)=\tilde{C}\tilde{X}(t) \quad (5)$$

, where $\tilde{X}(t)=(X^T(t),\ Z^T(t))^T$ and

$$\tilde{A}=\begin{pmatrix} A & DH \\ O & E \end{pmatrix},\ \tilde{B}=\begin{pmatrix} B \\ O \end{pmatrix},\ \tilde{C}=(C \quad O) \quad (6)$$

In eq(6), $\tilde{A}$,$\tilde{C}$ are (n+δq,n+δq), (p,n+δq) matrices, respectively. And an observer can be constructed if and only if ($\tilde{C}$,$\tilde{A}$) is completely observable pair.

Although the existence condition is quite simple and can be easily satisfied, the dimension of the resultant observer considerably increases and implementation of this observer may be impractical. Reduction of dimensionality is, therefore, essential for this observer to be practical one. And procedure to design such observers is proposed in following section.

3. A Generalized Approach to Design a Minimal Order Observer for the Systems with Unknown Disturbances.

In this section, a generalized method by which the dimension of observer can be reduced and estimation errors due to the disturbance modelling error may be decreased is proposed.

The basic idea behind the method is partitioning disturbance vector into two part as following:

$$W(t)=(W_1^T(t):W_2^T(t))^T$$

where $W_1(t)\in R^r$ is directly rejectable disturbance vector and $W_2(t)\in R^{q-r}$ is disturbance vector to be modelled. Then original equation(1) becomes

$$\dot{X}(t)=AX(t)+BU(t)+(D_1\ D_2)\begin{pmatrix} W_1(t) \\ W_2(t) \end{pmatrix} \quad (7)$$

where $D_1\in R^{n\times r}$ and $D_2\in R^{n\times(q-r)}$

Since $W_2(t)$ cannot be rejected, it should be modelled as follows.

$$\dot{Z}_2(t)=E_2Z_2(t) \quad (8)$$
$$W_2(t)=H_2Z_2(t)$$

$$\text{where}\quad E_2=\begin{pmatrix} 0 & I_{(\delta-1)(q-r)} \\ 0 & 0 \end{pmatrix}$$
$$H_2=(I_{q-r} \quad 0) \text{ and} \quad (9)$$

E_2,H_2 are (δ(q-r), δ(q-r)), (q-r, δ(q-r)) dimensional matrices. The augmented system, then, becomes

$$\dot{\hat{X}}(t)=\hat{A}\hat{X}(t)+\hat{B}\hat{U}(t)+\hat{D}W_1(t) \quad (10)$$
$$Y(t)=\hat{C}\hat{X}(t)$$
$$\hat{X}(t)=(X^T(t),\ W_2^T)^T \text{ and}$$
$$\hat{A}=\begin{pmatrix} A & D_2H_2 \\ O & E_2 \end{pmatrix},\ \hat{B}=\begin{pmatrix} B \\ O \end{pmatrix},\ \hat{C}=(C \quad O) \text{ and}$$
$$\hat{D}=\begin{pmatrix} D_1 \\ O \end{pmatrix} \quad (11)$$

In eq(10) and eq(11), $\hat{A}$,$\hat{B}$,$\hat{C}$ and $\hat{D}$ are (n+ δ(q-r), n+ δ(q-r)), (n+ δ(q-r), m), (p, n+ δ(q-r)) and (n+ δ(q-r), r) matrices, respectively. If ($\hat{C}$,$\hat{A}$) is completely observable pair, then, a minimal order observer with dimension of (n+ δ(q-r)-p) can be constructed.

From eq(6) and eq(10), we can see that the difference in order of two observers is δr where δ is the dimension of single disturbance model and r is the number of directly rejectable disturbances.

An other important difference between eq(6) and eq(11) is the existence of disturbance.

Since eq(10) contains the term $DW_1(t)$ and since $W_1(t)$ is not measurable, the observer gain matrix for the augmented system of eq(11) must be determined so as to zeroing the effects of disturbance $W_1(t)$ on estimated values. In order to design such an observer, eq(10) must be partitioned as following equation(12) with the assumption that C is (I_p: O).

$$\begin{pmatrix} \dot{\hat{X}}_1(t) \\ \dot{\hat{X}}_2(t) \end{pmatrix}=\begin{pmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{pmatrix}\begin{pmatrix} \hat{X}_1(t) \\ \hat{X}_2(t) \end{pmatrix}+\begin{pmatrix} \hat{B}_1 \\ \hat{B}_2 \end{pmatrix}U(t)+\begin{pmatrix} \hat{D}_1 \\ \hat{D}_2 \end{pmatrix}W_1(t) \quad (12)$$

where $\hat{X}_1(t)$ is measured vector and $\hat{X}_2(t)$

is a vector to be estimated. Since $(\hat{C},\hat{A})$ is completely observable pair and observability of the pair implies that of the pair $(\hat{A}_{22}, \hat{A}_{12})$, an $(n+\delta(q-r)-p)$ dimensional observer of eq (13) can be constructed.

$$\dot{Z}(t)=FZ(t)+GY(t)+HU(t)+JW_1(t) \text{ and} \quad (13)$$
$\hat{X}_{2es}(t)=Z(t)+LY(t)$, where $\hat{X}_{2es}(t)$ is estimated vector of $\hat{X}_2(t)$ and
$F=\hat{A}_{22}-L\hat{A}_{12}$, $G=FL+\hat{A}_{21}-L\hat{A}_{11}$,
$H=\hat{B}_2-L\hat{B}_1$ and $J=\hat{D}_2-L\hat{D}_1$

In this case, however, $W_1(t)$ is not available, so that the observer of eq(13) can be implemented only if the gain matrix L can be chosen such that

i) $J=\hat{D}_2-L\hat{D}_1=0$ (14)
ii) $F=\hat{A}_{22}-L\hat{A}_{12}$ has stable eigenvalues. (15)

And existence condition of the observer gain matrix L that satisfies eq(14)(15) is as follows.
THEOREM (1): if

$$\text{Rank}(\hat{D}_1)=\text{Rank}\begin{pmatrix}\hat{D}_1\\ \hat{D}_2\end{pmatrix}=r$$
or eqaivalently (16)
$$\text{Rank}(\hat{C}\hat{D})=\text{Rank}(\hat{D})=r \text{ and } p>r$$

then there exists L satisfying eq(14) (15).

Although the proof is described in ref(6) in detail; we'll present it because some steps of design procedure are contained in.

PROOF: if eq (16a) is satisfied, the general solution of eq(14) can be written as eq(17).

$$L=\hat{D}_2\hat{D}_1^{\#}+K(I_r-\hat{D}_1\hat{D}_1^{\#}) \quad (17)$$

where # represents psuedo inverse and K is arbitrary matrix of appropriate dimension. To complete the proof, it is necessary to show that the observer gain matrix of eq(17) satisfies condition(16b).

For this, an orthogonal transformation matrix T is introduced to divide L into two part; fixed part to reject the disturbance and arbitrary part to assign observer poles. Then D_1 is transformed into following form.

$$T\hat{D}_1=\begin{pmatrix}\bar{D}_1\\ O\end{pmatrix} \quad (18)$$

Since $\text{Rank}(D_1)=r$, the transform is always possible. Let

$$TA_{12}=\begin{pmatrix}\bar{A}_{12}^1\\ \bar{A}_{12}^2\end{pmatrix} \text{ and } KT^T=(\bar{K}_1\,\bar{K}_2) \quad (19)$$

where $\bar{A}_{12}^1$ has r rows and $\bar{K}_1$ has r columns. Then following relationships are successively obtained by using eq(18) and eq(19).

$$\hat{D}_1^{\#}=((\bar{D}_1)^{-1}:O)T \quad (20)$$
$$(I_p-\hat{D}_1\hat{D}_1^{\#})=T^T\begin{pmatrix}O & O\\ O & I_{p-r}\end{pmatrix}T \quad (21)$$
$$L=(\hat{D}_2(\bar{D}_1)^{-1}:\bar{K}_2)T \quad (22)$$
$$F_1=\hat{A}_{22}-\hat{D}_2(\bar{D}_1)^{-1}\bar{A}_{12}^1 \quad (23)$$
$$F=F_1-\bar{K}_2\bar{A}_{12}^2 \quad (24)$$

Since $\bar{K}_2$ is an arbitrary matrix, all the eigenvalues of F except the unobservable modes of the pair $(\bar{A}_{12}^2, F_1)$ are freely assignable. For more detailed explanations, see ref (6).

From the above descriptions, the existence conditions of the observer for the augmented system(10) with unmeasurable disturbance $W_1(t)$ can be established as Theorem(2).

THEOREM (2) if:

(i) $(\hat{C},\hat{A})$ is completely observable pair

(ii) $\text{Rank}(\hat{D})=\text{Rank}\begin{pmatrix}\hat{D}_1\\ \hat{D}_2\end{pmatrix}=r$ or equivalently

(iii) $\text{Rank}(\hat{C}\hat{D})=\text{Rank}(\hat{D})=r$ and $p>r$

then there exists an observer of eq(13) that satisfies eq(14) and eq(15) provided that the pair $(\bar{A}_{12}^2, F_1)$ is detectable where $\bar{A}_{12}^2$ and F_1 are represented by eq(19) and eq(23), respectively.

It should be noted that the conditions in Theorem(2) are represented not by parameters of the original system equation (1) but by parameters of the augmented system equation (10). Due to the fact above, Theorem(2) cannot be used to identify the existence of observer of eq(13) with eq(14) (15) for the original system of eq (1). Another drawback of these conditions is the fact that it cannot be used to select the directly rejectable disturbances and the disturbances to be modelled. The latter is very important in a view point that the partitioning of disturbance vector is the first step of design procedure of the observer. The conditions, therefore, must be replaced with the conditions with original system parameters as following Theorem.

THEOREM (3):

For the system equation (1), if

(i) (C,A) is completely observable pair

(ii) rank $\begin{pmatrix} A & D_2 \\ C & O \end{pmatrix} = n+q-r$

(iii) rank $\begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix} = \text{rank}(D_{11}) = r$

where $D_1 = \begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix}$ or equivalently

(iv) rank (CD_1) = rank (D_1) = r and $p > r$

(v) Disturbance model is completely observable, then there exists a reduced order observer of eq(13) with properties of eq(14) and eq(15) if $(\bar{A}^2_{12}, F_1)$ is detectable pair.

PROOF: The first part of proof is to show (i) (ii) (vi) implies that (C,A) is completely observable pair and the detailed proof of this part is described in ref(2) and thus omitted. Now, the remaining part can be easily proved in same manner that used in the proof of Theorem(1) by the use of following useful facts and therefore omitted.

FACT I: Rank $\begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix}$ = Rank (D_{11}) = r implies that

Rank $\begin{pmatrix} \hat{D}_1 \\ \hat{D}_2 \end{pmatrix}$ = Rank $(\hat{D}_1)$ = r

FACT II: Rank (CD_1) = Rank (D_1) = r implies that

Rank $(\hat{C}\hat{D})$ = Rank $(\hat{D})$ = r

These facts are followed by following relation; from

$\hat{D} = \begin{pmatrix} D_{11} \\ D_{21} \\ 0_{(\sigma q-r, r)} \end{pmatrix}$ $\hat{D}_1 = D_{11}$ and $\hat{D}_2 = \begin{pmatrix} D_{21} \\ 0 \end{pmatrix}$

1. Rank (D_{11}) = Rank $(\hat{D}_1)$

2. Rank $\begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix}$ = Rank $\begin{pmatrix} D_{11} \\ D_{21} \\ 0 \end{pmatrix}$ are obtained and it means FACT I.

And from Rank (D_1) = Rank $\begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix}$,

Rank $(\hat{D})$ = Rank $\begin{pmatrix} D_{11} \\ D_{21} \\ 0 \end{pmatrix}$

1. Rank (D_1) = Rank $(\hat{D})$

2. Rank $(\hat{C}\hat{D})$ = Rank (CD_1) are obtained and it means FACT II.

REMARK (1): In Theorem(3), Condition(ii) determines the existence of an observer for the augmented system and indicates that the existence of observer do not depend upon dimension of disturbance model, while dimension of observer depend on it. And condition (v) can always be meet by appropriate disturbance modelling.

REMARK (2): Condition(iii) or (iv) in Theorem (3) provides the partitioning method of disturbance vector and corresponding transmission matrix in the form of

$$DW(t) = (D_1 \ \ D_2)\begin{bmatrix} W_1(t) \\ W_2(t) \end{bmatrix}$$

such that $D_1 = \begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix}$ has following property;

There exists L_0 satisfying $D_{21} - L_0 D_{11} = 0$ where $W_1(t)$ is directly rejectable subvector and $W_2(t)$ is subvector to be modelled.

And such L_0 exists if and only if D_1 satisfies condition (iii) or (iv).

By applying these conditions, disturbance patitioning can be performed for original system equation (1) and eq(10) (12) can be obtained.

REMARK (3): As a special cases, (i) if we can partition the disturbance vector and transmission matrix as $W_1(t)=W(t)$ and $D_1=D$, then all the disturbances can be rejected by algebraic method and observer design procedure is therefore reduced to pure algebraic approach [6] and (n-p)th order (lowest) observer may be constructed. (ii) if $W_2(t)=W(t)$ and $D_2=D$, then all the disturbances must be modelled and the resultant observer with highest order of $(n+\delta q-p)$ may be constructed by the use of Disturbance Modelling Approach [3]. It is notable that case (ii) is the worst case in view points of dimensionality and estimation errors of the observer.

4. Design Procedure

Step 1) Test the observability of the original system of eq(1).

Step 2) By applying conditions (iii) and (iv) of Theorem (3), select the directly rejectable disturbance vector and the disturbance vector to be modelled. In this step, all column exchange of matrix D is possible.

Step 3) Test the observability of the augmented system of eq(10) by applying condition (ii).

Step 4) Construct the disturbance model so that the pair (E_2, H_2) is observable. When modelling the disturbance vector the choice of δ is dictated by a trade-off between performance and the dimension of the observer to be constructed.

Step 5) Construct the augmented system equation by using original system equation and disturbance model.

Step 6) Construct the minimal order observer that gives good estimates for the augmented system with unmeasurable disturbance vector $W_1(t)$ by applying algebraic procedure given in proof of Theorem (1).

5. A Numerical Example and Simulation Results.

Consider the linear time-invariant system with unknown, unmeasurable disturbance vector.

$$\dot{X}(t)=\begin{pmatrix}-1 & 0 & 1\\ 0 & -2 & 1\\ 0 & 1 & -3\end{pmatrix}X(t)+\begin{pmatrix}0 & 0\\ 1 & 1\\ 1 & 0\end{pmatrix}W(t) \quad (25)$$

and

$$Y(t)=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}X(t)$$

(step 1) (C,A) is completely observable pair.

(step 2) Since $col_1(D)$ satisfies $Rank\begin{pmatrix}0\\1\\1\end{pmatrix}= Rank\begin{pmatrix}0\\1\end{pmatrix}=1$

$W_1(t)$ can be selected as rejectable disturbance and $W_2(t)$ is a disturbance to be modelled. Thus, r = 1 and q = 2.

$$\text{(step 3)}\quad Rank\begin{pmatrix}A & D_2\\ C & O\end{pmatrix}=Rank\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & -2 & 1 & 1\\ 0 & 1 & -3 & 0\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}=4$$

So (C,A) is an observable pair.

(step 4) By choosing $\delta = 2$, $W_2(t)$ can be modelled as

$$\dot{Z}(t)=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}Z(t) \qquad (26)$$

$$W_2(t)=(1\quad 0)Z(t)$$

(step 5) From (21) and (22), augmented system obtained as

$$\dot{X}_a(t)=\left(\begin{array}{cc:ccc}-1 & 0 & 1 & 0 & 0\\ 0 & -2 & 1 & 1 & 0\\ \hdashline 0 & 1 & -3 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{array}\right)X_a(t)$$

$$+\left(\begin{array}{c}0\\ 1\\ \hdashline 1\\ 0\\ 0\end{array}\right)W_1(t) \qquad (27)$$

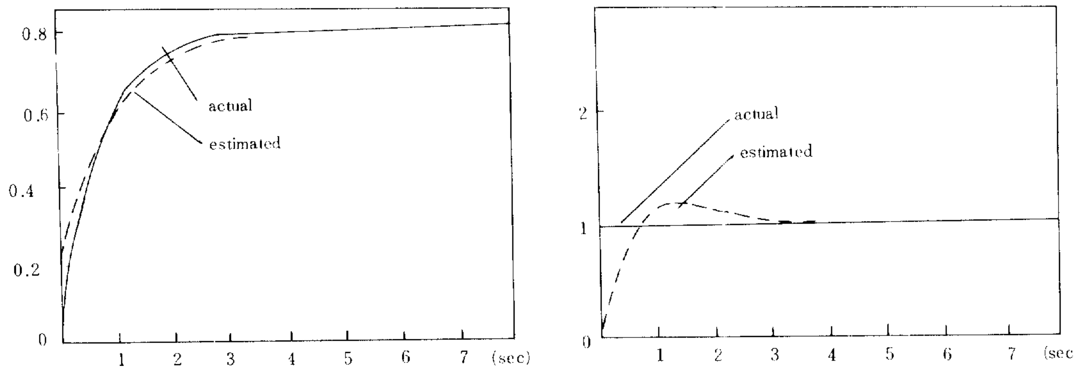
$$Y(t)=\left(\begin{array}{cc:ccc}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{array}\right)X_a(t)$$

(step 6) Since $\hat{D}_1=\begin{pmatrix}0\\1\end{pmatrix}$ and $\hat{D}_2=\begin{pmatrix}1\\0\\0\end{pmatrix}$.

T is selected as $T=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

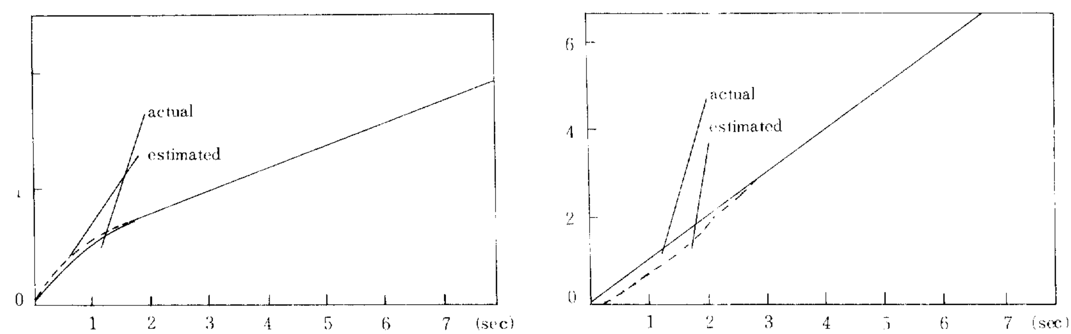
$T\hat{D}_1=\begin{pmatrix}1\\0\end{pmatrix}$ where $\bar{D}_1 = 1$ and Rank $\bar{D}_1 = 1 = r$.

By sequencially applying the equations (20), (21), (22), (23), we obtain



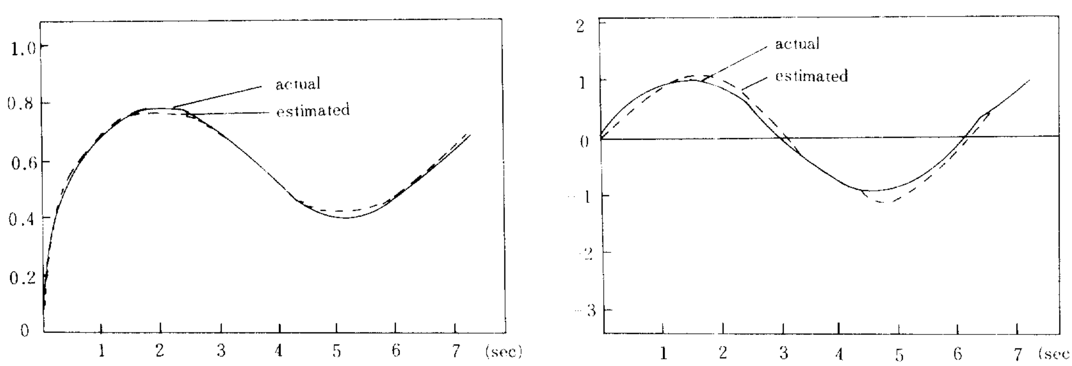
(a) x_3 and $\hat{x}_3$ (b) w_2 and $\hat{w}_2$

Fig.1. Estimated values for step disturbances where w_1 is directly rejected.



(a) x_3 and $\hat{x}_3$ (b) w_2 and $\hat{w}_2$

Fig.2. Estimated values for the disturbances, $w_1=1$ and $w_2=t$ where w_1 is directly rejected.



(a) x_3 and $\hat{x}_3$ (b) w_2 and $\hat{w}_2$

Fig.3. Estimated values for the disturbances, $w_1=1$ and $w_2=\sin(0.5t)$ where w_1 is directly rejected.

$$\bar{A}^1_{12}=(1 \quad 1 \quad 0) \text{ and } \bar{A}^2_{12}=(-1 \quad 0 \quad 0) \tag{28}$$

$$I_p-\hat{D}_1\hat{D}_1^{\#}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \tag{29}$$

$$F_1=\begin{pmatrix}-4 & -1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix} \tag{30}$$

From (24) and (26), we find the fact that (F_1, $\bar{A}^2_{12}$) is observable pair. Therefore, all eigenvalues of F in eq(24) are freely assignable. By selecting the observer poles as λ_i (F) = -1, -2, -5 the observer gain matrix L can be computed as

$$L=\begin{pmatrix}4 & 1\\ -17 & 0\\ -10 & 0\end{pmatrix} \tag{31}$$

And resultant minimal order observer for the original system with inaccessible disturbances is obtained as follows.

$$\dot{Z}(t)=\begin{pmatrix}-8 & -1 & 0\\ 17 & 0 & 1\\ 10 & 0 & 0\end{pmatrix} Z(t)+\begin{pmatrix}-11 & -5\\ 41 & 17\\ 30 & 10\end{pmatrix}Y(t)$$

$$\hat{X}_{2es}(t)=Z(t)+LY(t) \tag{32}$$

To show the effectiveness of the observer constructed by proposed procedure, a simulations are performed for the various kinds of disturbance vectors; say, constant, ramp, and sinusoidal form and the results are shown in Fig (1), (2), (3).

6. Conclusions

The design of an observer for the system with unknown unmeasurable disturbance vector is treated and a generalized design method is proposed by combining the direct rejection approach and disturbance modelling approach that are separately developed recently.

The proposed method is very powerful in view points that this method enjoys low dimensionality and existence conditions that can be easily met, while existing methods are suffering from high dimensionality and existence conditions that can hardly be met.

In addition, the existence conditions in Theorem (3) are essential because it can be directly used to partition the disturbance vector into two vectors where the partitioning contributes to reducing the dimension of the observer.

The numerical example and simulation results show that the observer designed by proposed method not only gives good estimates but has low dimensionality.

Finally, it should be noted that the two existing methods are special cases of this generalised method.

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