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ARMA 스펙트럼 추정을 위한 변형 기구변수법에 관한 연구

Modified Instrumental Variable Methods for ARMA Spectral Estimation

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요 약

고해상도를 갖는 ARMA 모델의 스펙트럼 추정을 위하여 AR 계수와 MA 계수를 동시에 추정할 수 있는 변형 기구 변수 알고리즘을 제시했다. 제시된 알고리즘은 계산량은 확장 최소자승법(ELS)와 같으나 ELS나 RML보다 빠른 수렴성을 보였다.

더 정확한 추정을 위하여 Overdetermined 알고리즘도 제시했고 이 알고리즘이 Narrow band 에서 높은 해상력을 가짐을 Computer Simulation을 통해 보였다.

Abstract

The signal can be modeled as a linear combination of its past values and present and past values of a hypothetical input to system whose output is given signal. Using this model spectral estimation problem can be reduced to estimate the ARMA parameters.

This paper presents recursive modified instrumental variable algorithm which can estimate AR and MA parameters. For more accurate estimation, overdetermined modified IV algorithm is also derived.

Computer simulations are presented to illustrate the above methods.

1. Introduction

High resolution spectral estimation has been the subject of system theory, geophysics, speech processing oil exploration and many other areas.[1,2]

Conventional methods based on fast Fourier transform(FFT) provide poor resolution when data record is short, and show Gibb's phenomenon[2]. Thus the maximum entrophy method[3] and the time series modeling techniques were developed in the last decade.[4,5]

A stationary time series $y(t)$ is modeled as the output of a time invariant linear system driven by a zero mean unit variance white noise input[6]. The linear system is assumed to have a rational transfer function.

$$H(z)=\frac{C(z)}{A(z)}$$

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The estimated power spectrum S(w) is computed by

$$S(w) = \frac{C(e^{jw})\ C(e^{-jw})}{A(e^{jw})\ A(e^{-jw})}$$

Using this model' spectral estimation problem can be reduced to the estimation of model parameters.

The most popular method for parameter estimation is least squares method(LSM), but LSM gives biased estimates for correlatel noise case. To overcome this shortage, instrumental variable method(IVM) is proposed[7),8)].

Since IVM cannot give estimates of MA parameters, extended least squares(ELS) and recursive maximum likelihood (RML) algorithms were presented[9),10)].

In this paper, recursive modified instrumental varirble algorithm for estimating ARMA parameters is presented. The computational burden of this algorithm is equal to that of ELS and less than that of RML, but this algorithm shows faster convergence than ELS and RML algorithms.

For more accurate estimation, the overdetermined MIV algorithm is also derived. And computer simulation is presented to efficiency show of these algorithms.

2. The Instrumental Variable Method

The observed data sequence y(t) can be modeled by ARMA process of order(p, q)

$$y(t) = -\sum_{i=1}^{p} A(i)y(t-i) + w(t) \tag{2.1a}$$

$$w(t) = \sum_{i=1}^{q} (i)e(t-i) + e(t) \tag{2.1b}$$

where e(t) is white process.

To estimate parameter vector θ_1, the eq.(2.1) can be written in matrix form as

$$y = X\theta_1 + W \tag{2.2a}$$

where

$$\theta_1 = [A(1),\ldots,A(p)]^T \tag{2.2b}$$

$$y = [y(o),\ldots,y(t)]^T \tag{2.2c}$$

$$X = \begin{bmatrix} 0 & ,\ldots, & 0 \\ -y(0) & \ldots, & 0 \\ \cdot & , & \cdot \\ -y(p-1),\ldots, & & -y(0) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ -y(t-1),\ldots, & & -y(t-p) \end{bmatrix} \tag{2.2d}$$

$$W = [w(o),\ldots,w(t)]^T \tag{2.2e}$$

Premultiplying by $(X^TX)^{-1}X^T$ we get

$$(X^TX)^{-1}X^T y = \theta_1 + (X^TX)^{-1} \times^T W \tag{2.3}$$

The left hand side is LS estimate and the second term on right is a nonzero bias vector when the w(t) is a correlate sequence. This is a severe drawback of LS method.

Next consider premultiplying(2.3) by the IV matrix Z^T, and rewriting it as

$$(Z^TX)^{-1}Z^Ty = \theta_1 + (Z^TX)^{-1}Z^TW \tag{2.4}$$

the left side is the IV parameter estimate vector. Choosing this matrix so that $E\{Z^TW\}=0$ will make the bias term vanish automatically.

The different type of instrumental variants are presented in[7)]. In the ARMA case, the instruments are usually delayed outputs.

3. The Modified Instrumental Variable Method

The IV method gives consistent estimate for ARMA process, but cannot estimate MA parameters. To estimate ARMA parameters simultaneously, the modified instrumental variable method is proposed.

If we assume, for the moment, that e(t) is measurable, then we can write the model of eq.(2.1). Matrix from as

$$y = Y\theta + E \tag{3.1a}$$

where

$$\theta = [A(1),\ldots,A(p),\ C(1),\ldots,C(q)]^T \tag{3.1b}$$

$$Y = \begin{bmatrix} 0 & ,\ldots, & 0\ ,\ 0\ ,\ldots,0 \\ -y(0) & ,\ldots, & 0\ ,\ e(o),\ldots,0 \\ \cdot & & \cdot\ ,\ \cdot \\ -y(p-1),\ldots, & & -y(0),\ e(p-1),\ldots,e(p-q) \\ \cdot & & \cdot\quad \cdot \end{bmatrix}$$

$$\begin{bmatrix} \cdot & & \cdot & \cdot \\ -y(t-1),\ldots, & -y(t-p), & e(t-1),\ldots, & e(t-q) \end{bmatrix} \quad (3.1c)$$

$$E=[e(0),\ldots,e(t)]^T \quad (3.1d)$$

Of course the sequence e(t) is not known. However the natural way to proceed is to replace e(t) by e(t) which is residual sequence given by

$$\hat{e}(t)=y(t)-\phi(t)^T\hat{\theta} \quad (3.2a)$$

where

$$\phi(t)=[-y(t-1),\ldots,-y(t-p),\ \hat{e}(t-1),\ldots,\hat{e}(t-q)]^T \quad (3.2b)$$

$$\hat{\theta}=[\hat{A}(1),\ldots,\hat{A}(p),\ \hat{C}(1),\ldots,\hat{C}(q)]^T \quad (3.2c)$$

Now consider premultiplying (3.1) by the IV matrix Z^T, and rewritten it as

$$(Z^TY)^{-1}Z^Ty=\theta+(Z^TY)^{-1}Z^TE \quad (3.3)$$

We choose the matrix Z so that $E(Z^TE)=0$, and $E(Z^TY)=R$ is nonsingular matrix. By choosing Z as (3.4), the above conditions are automatically satisfied from consistency condition of IVM[8] and whiteness property of residual process.

$$Z=\begin{bmatrix} 0 & ,\ldots, & 0, & 0 & ,\ldots, & 0 \\ \cdot & & \cdot & \hat{e}(0) & ,\ldots, & 0 \\ 0 & ,\ldots, & 0, & \cdot & & \cdot \\ -y(0),0,\ldots, & & 0, & \cdot & & \cdot \\ \cdot & & \cdot, & \cdot & & \cdot \\ -y(p-1),\ldots, & & -y(0), & \hat{e}(2p-1),\ldots, & & \hat{e}(2p-q) \\ \cdot & & \cdot, & \cdot & & \cdot \\ \cdot & & \cdot, & \cdot & & \cdot \\ -y(t-1-p),\ldots, & & -y(t-2p), & \hat{e}(t-1),\ldots, & & \hat{e}(t-q) \end{bmatrix} \quad (3.4)$$

The eq. (3.3) can be solved least squares sense, and this leads to the following recursive algorthms.

$$\phi(t)=[-y(t-1),\ldots,-y(t-p),\hat{e}(t-1),\ldots,\hat{e}(t-q)]^T \quad (3.5a)$$

$$z(t)=[-y(t-1-p),\ldots,-y(t-2p),\hat{e}(t-1),\ldots,\hat{e}(t-q)]^T \quad (3.5b)$$

$$\varepsilon(t)=y(t)-\phi(t)^T\hat{\theta}(t-1) : \text{prediction error} \quad (3.5c)$$

$$P(t)=(I-P(t-1)z(t)\phi(t)^T/(\lambda(t)^T+\phi(t)^TP(t-1)z(t)))\ P(t-1)/\lambda(t) \quad (3.5d)$$

$$K(t)=P(t)z(t)/(\lambda(t)+\phi(t)^TP(t)z(t)) \quad (3.5e)$$

$$\hat{\theta}(t)=\hat{\theta}(t-1)+K(t)\varepsilon(t) : \text{parameter update} \quad (3.5f)$$

$$\hat{e}(t)=y(t)-\phi(t)^T\hat{\theta}(t) : \text{residual error} \quad (3.5g)$$

The parameter $\lambda(t)$ is an exponential weight on the data (a forgetting factor). Typically $\lambda(t)$ is a constant close to unity or time varing,

$$\lambda(t+1)=\lambda\lambda(t)+(1-\lambda)\ (\lambda=0.99,\ \lambda(0)=0.95) \quad (3.6)$$

The initial conditions of algorithm are:

$$P(0)=\alpha I,\ \alpha=\text{sufficiently large number} \quad (3.7a)$$

$$\theta(0)=0 \quad (3.7b)$$

$$\phi(0)=0 \quad (3.7c)$$

The proposed algorithm is equal to ELS algorithm when z(t) is chosen by $\phi(t)$, and it is equal to RML algorithm when z(t) is chosen by prefiltered value of $\phi(t)$.

4. The Overdetermined MIV Method

The MIV method described above involves the solution of (p+q) equations. It was noticed by several researchers[11),12)] that improved estimates can be obtained by using a large number of equations. Increasing the number of equations is more effective in the narrow band case than in the broad case[13)].

The use of additional instruments improves estimation accuracy. Increasing the dimension of instrument vector lead to an overdetermined set of linear equation for the parameters, which can be solved in a least squares sense.

We choose an overdetermined modified instrumental variable matrix as

$$z=\begin{bmatrix} 0 & ,\ldots, & 0, & 0 & ,\ldots, & 0 \\ \cdot & & \cdot, & \hat{e}(0) & ,\ldots, & 0 \\ 0 & ,\ldots, & 0, & \cdot & & \cdot \\ -y(0) & ,\ldots, & 0, & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ -y(kp-1),\ldots, & & -y(0), & \hat{e}(kp-1),\ldots, & & \hat{e}(kp-kq) \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ -y(t-1-p),\ldots, & & -y(t-kp-p), & \hat{e}(t-1),\ldots, & & \hat{e}(t-kq) \end{bmatrix} \quad (4.1)$$

If $k=1$, above problem is MIV case. In the case of $k>1$, the least squares solution of eq.(3. 1) is

$$\hat{\theta}=(Y^T ZZ^T Y)^{-1} Y^T ZZ^T y \quad (4.2)$$

This estimate is known to be asymptotically consistent, inceasing k will improve the accuracy of estimates.

From the eq.(4. 2) we can derive recursive algorithms using matrix inversion lemma (see appendix).

$$\phi(t)=[-y(t-1),...,-y(t-kp), \hat{e}(t-1),...,\hat{e}(t-kq)]^T \quad (4.3a)$$

$$z(t)=[-y(t-1-p),...,-y(t-kp-p), \hat{e}(t-1),..., \hat{e}(t-kq)]^T \quad (4.3b)$$

$$U(t)=S(t)z(t) \quad (4.3c)$$

$$S(t+1)=\lambda(t)S(t)+\phi(t)z(t)^T \quad (4.3d)$$

$$R(t+1)=[U(t+1) : \phi(t)] \quad (4.3e)$$

$$\lambda^2(t)G(t+1)=\begin{bmatrix} -z(t+1)^T z(t+1) & \lambda(t) \\ \lambda(t) & 0 \end{bmatrix} \quad (4.3f)$$

$$K(t+1)=P(t)R(t+1)\lambda^2(t)G(t+1) + R(t+1)^T P(t) R(t+1)) \quad (4.3g)$$

$$P(t+1)=(P(t)-K(t+1)R(t+1)^T P(t))/\lambda(t^2) \quad (4.3h)$$

$$V(t+1)=\begin{bmatrix} z(t)^T L(t) \\ y(t) \end{bmatrix} \quad (4.3i)$$

$$L(t+1)=\lambda(t)L(t)+z(t)y(t) \quad (4.3j)$$

$$\hat{\theta}(t+1)=\hat{\theta}(t)+K(t+1)(V(t+1) - R(t+1)^T \hat{\theta}(t)) \quad (4.3k)$$

The parameter $\lambda(t)$ is a forgetting factor and the initial conditions are;

$$S(0)=\mu[I : O],\ P(0)=\frac{1}{\mu^2} I,\ L(0)=0,\ \hat{\theta}(0)=0 \quad (4.4)$$

where μ is a small scalar parameter.

Computational burden of overdetermined MIV is about k times that of MIV. But this algorithm gives more accurate estimates and fast convergence.

5. Simulation Results

Computer simulations are presented to illustrate the behavior of the proposed algorithms compared to ELS and RML algorithms.

Example 1

The ARMA(4, 2) process for this example is

$$y(t)=-2.7607y(t-1)+3.8106y(t-2)-2.6535y(t-3)+0.9238y(t-4)+w(t)$$

$$w(t)=e(t)-0.102e(t-1)+0.173e(t-2)$$

where e(t) is a white noise with zero mean and unit variance.

This model was estimated by ELS, RML, MIV and OMIV algorithms respectively. The estimated parameters in each case are presented in table 1.

Table 1. Estimated parameters by ELS, RML, MIV and OMIV (for $k=3$)

True Para. / Esti. Para. / Data		A(1)	A(2)	A(3)	A(4)	C(1)	C(2)
		−2.7607	3.8106	−2.2535	0.9238	−0.1020	0.1730
128	ELS	−1.9386	2.0985	−1.1469	0.4837	0.1581	0.7394
	RML	−2.2798	2.5709	−1.4983 0	0.4282	−0.0695	−0.0660
	MIV	−2.1209	4.4789	−3.1953	0.0514	−0.1156	0.2276
	OMIV	−3.7897	3.7753	−2.5696	0.8558	−0.1045	0.1167
256	ELS	−2.2904	2.8896	−1.8718	0.7152	−0.1484	0.5264
	RML	−2.5308	3.1994	−2.0259	0.6603	−0.1260	0.0011
	MIV	−2.8286	3.9146	−2.7264	0.9312	0.0091	0.1240
	OMIV	−2.7487	3.7526	−2.5845	0.8883	−0.1438	0.1126
512	ELS	−2.6263	3.5614	−2.4500	0.4500	−0.1037	0.3760
	RML	−2.6932	3.6414	−2.4874	0.8582	−0.2250	0.1478
	MIV	−2.7918	3.8657	−2.7016	0.9388	−0.1432	0.0993
	OMIV	−2.7669	3.8186	−2.6623	0.9279	−0.1536	0.1638

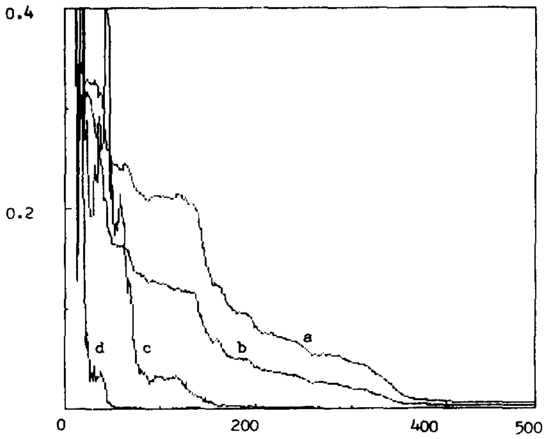

Fig. 1. Normalized errer for recursive algorithms;a;ELS b;RML c;MIV d;OMIV(k=3).

In order to facilitate quick comparision, the normalized error(NE) in each case are in Fig.1. It is defined as

$$NE=\frac{\|\hat{\theta}-\theta\|^2}{\|\theta\|^2}$$

where $\|\theta\|^2=\theta^T\theta$

Above results show that the proposed algorithms gives better convergence.

Example 2

The wave form chosen for this example is

$$y(t)=\sqrt{6.3}\sin(0.4\pi t)+\sqrt{20}\sin(0.42\pi t)+w(t)$$
$$w(t)=e(t)-0.102e(t-1)+0.173e(t-2)$$

where e(t) is white process with zero mean and

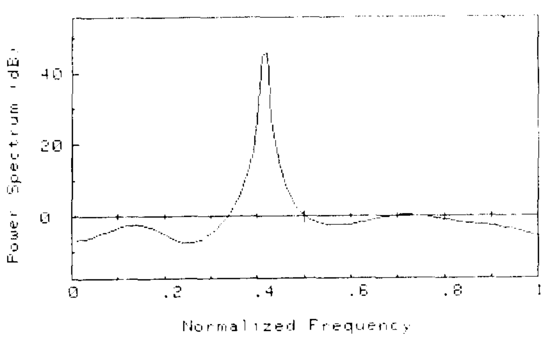

Fig. 2. Estimated spectrum by ELS.

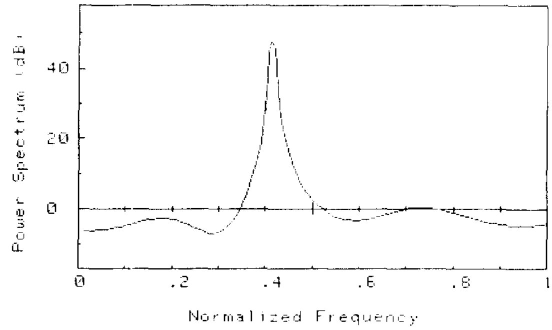

Fig. 3. Estimated spectrum by RML.

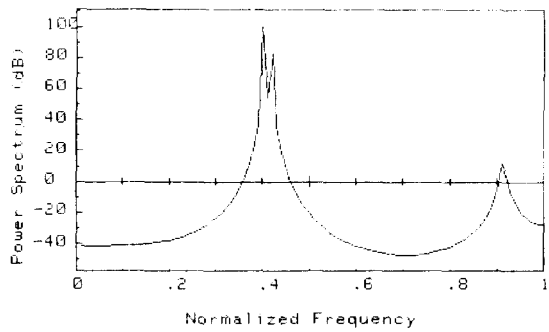

Fig. 4. Estimated spectrum by MIV.

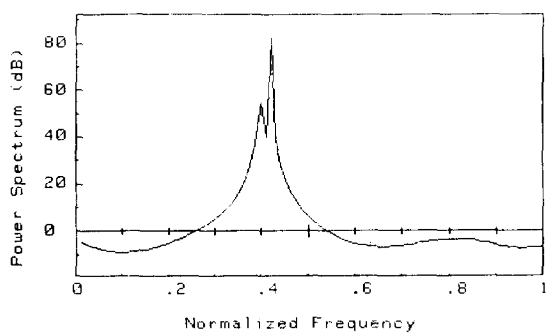

Fig. 5. Estimated spectrum by OMIV(K=3).

unit variance. Figs. 2-5 are estimated spectrums. The number of data points was taken as T=512 and ARMA(8, 8) model was used. The corresponding signal to noise ratio of two sinusoids are 5 dB and 10 dB. From the results, it was shown, that proposed algorithms give good resolution.

6. Conclusions

For high resolution spectral estimation, the recursive modified instrumental variable algorithm for ARMA process is presented. This algorithm can estimate AR and MA parameter simultaneously. A computational burden of this algorithm is equal to the extended least suares algorithm. But parameter estimates of this algorithm show faster convergence than RML as well as ELS.

For more accurate estimation, the recursive overdetermined modified IV algorithm is also derived. It provides good resolution for the narrow band case. The computational complexity is increased, but it improves accuracy. So, in some application, it is well worth to use.

Finally, the proposed algorithms can be implemented in lattice structure.

Appendix : Derivation of the OMIV Algorithm

The recursive OMIV algorithm can be derived by similar way in.

Let Y(t) be Y of the eq..(3. 1c), Z(t) be Z of the eq.(4. 1) and y(t) be y of the eq.(2. 2c). The update variables can be written as

$$Y(t+1)=\begin{bmatrix} Y(t) \\ \phi(t+1)^T \end{bmatrix},\quad Z(t+1)=\begin{bmatrix} \lambda(t)Z(t) \\ z(t+1)^T \end{bmatrix},$$
$$y(t+1)=\begin{bmatrix} y(t) \\ y(t+1) \end{bmatrix} \qquad (A1)$$

where $\phi(t+1)$ and $z(t+1)$ are given in eqs.(4.1a) and (4.1b).

Let $P(t)^{-1} \triangleq (Y(t)^T Z(t) Z(t)^T Y(t))$ (A 2)

The inverse of this matrix can be updated as follows

$$P(t+1)^{-1}=\lambda(t^2(P(t)^{-1}+\lambda(t)U(t+1)\phi(t+\ )^T+ \lambda(t)\phi(t+1)U(t+1)^T+\phi(t+1)z(t+1)^T z(t+1)\phi(t+1)^T \qquad (A3)$$

where

$$U(t+1)=Y(t)^T Z(t{:}z(t+1) \qquad (A4)$$

This can be written more compactly as

$$P(t+1)^{-1}=\lambda(t)^2P(t)^{-1}+R(t+1)G(t+1)^{-1}R(t+1)^T \qquad (A5)$$

where

$$R(t+1)=[U(t+1) \vdots \phi(t+1)]$$
$$G(t+1)^{-1}=\begin{bmatrix} \lambda(t) & \dfrac{\lambda(t)}{z(t+1)^T z(t+1)} \end{bmatrix}, \text{ or}$$
$$\lambda(t)^2G(t+1)=\begin{bmatrix} -z(t+1)^T z(t+1) & \lambda(t) \\ \lambda(t) & 0 \end{bmatrix} \qquad (A7)$$

Inversion of (A5) yields

$$P(t+1)=(P(t)-P(t)R(t+1)(\lambda(t^2)G(t+1)+R(t+1)^T P(t)R(t+1))^{-1} R(t+1)^T P(t))/\lambda(t)^2 \qquad (A8)$$

To derive the update for $\theta(t)=Y(t)^T Z(t)$ (A9)

which cao be recursively updated by

$$(L(t+1)=\lambda(t)L(t)+z(t+1)y(t+1),\ S(t+1)=\lambda(t) S(t)+\phi(t+1)z(t+1)^T \qquad (A10)$$

From eq. (4. 2), we get

$$\theta(t+1)=P(t+1)S(t+1)L(t+1) \qquad (A11)$$

From (A3) and (A10), we get

$$\theta(t+1)=\theta(t)+P(t+1)R(t+1)G(t+1)^{-1}(V(t+1) R(t+1)^T\theta(t)) \qquad (A12)$$

where

$$V(t+1)=\begin{bmatrix} z(t+1)^T L(t) \\ y(t+1) \end{bmatrix} \qquad (A13)$$

Eqs. (4), (A6), (A7), (A8), (A10), (A12) and (A13) provide a complete set of recursions.

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