THE JOURNAL OF THE KOREAN ASTRONOMICAL SOCIETY
19: 1~9, 1986

AN OPTIMAL PHOTOMETRIC SYSTEM FOR HIGH RESOLUBILITY OF PHYSICAL QUANTITIES

PARK, NAM-KYU AND LEE, SEE-WOO
Department of Astronomy, Seoul National University

(Received June 2, 1986)

ABSTRACT

We present a method for deriving most optimal filter system by which the accuracy of derived physical quantities can be maximized. Using Kurucz(1978)'s model atmospheres, an optimal four filter system for F and G dwarfs is suggested, for which mean wavelengths are located at 3400Å, 3850Å, 4190Å, and 4600Å with half-bandwidth of 200Å. It is found that 35, 38 and 42 filters of the DDO system and the Strömgren u and v filters are close to those of the most optimal system.

I. INTRODUCTION

The best way to obtain physical parameters of a star is to examine its high resolution spectrum along with model atmospheres. However, this approach is limited only to bright stars. For a study of faint stars or variable stars (which require relatively rapid response of the detector), a photometric method is rather useful, although the detailed information of stellar spectra is missing. A multicolor photometric system is established by a set of filters which are characterized by mean wavelength and bandwidth. Their mean wavelengths, however, should be chosen by considering the characteristics of energy distributions of stars under consideration. The bandwidths of filters divide photometric system into wide, intermediate, and narrow band systems, and each system serves as a tool for investigating its own field of application (Steinlin and Buser 1974).

When a bandwidth is specified together with spectral type of stars, how can we determine an optimal set of mean wavelengths? A photometric system may be said to be superior to others if the system is capable of deriving the physical quantities more accurately than the other system can do. In this case, the resolubility of physical quantities is said to be high. For a given photometric system, its resolubility can be deduced by computing increments of colors with respect to a unit change of each physical quantity. In designing a new photometric system, it is customary to examine spectral features of stars qualitatively. In this paper, however, we make quantitative analysis to determine the optimal photometric system by examining the resolubility of physical quantities which are to be derived by this photometric system.

The following sections will be devoted first to define the differential volume and differential length in the color space as resolubility parameters. And then we introduce a most optimal photometric system suited for F, G type main sequence stars which gives the maximum resolubility. The resolubility parameter of other photometric systems widely used now is also derived and compared with that of our optimal system.

II. THE RESOLUBILITY PARAMETERS

A color is a difference of two heterochromatic magnitudes which is defined by

$$C_i = -2.5 \log \left(\frac{F_i}{F_{i+1}}\right) = -2.5 \log \left(\frac{\int R_i(\lambda)E(\lambda)d\lambda}{\int R_{i+1}(\lambda)E(\lambda)d\lambda}\right) \qquad (i = 1, 2, 3) \qquad (1)$$

where F_i is the flux measured with i-th filter with a total response function of $R_i(\lambda)$ and $E(\lambda)$ is a monochromatic stellar flux distribution. If we take three colors (to derive three physical parameters such as effective temperature $T_{\rm eff}$, surface gravity $\log g$ and metallicity [M/H]), a mathematical three dimensional vector space can be defined by taking photometric colors as three coordinate axes. The concept of the color vector space was utilized by Golay (1974) and Massa and Lillie (1978).

Let us see an example of two-dimensional color space. Figure 1 shows a grid of physical parameters on the $(b\text{-}y)$ *vs.* c_1 diagram of the *uvby* system. Since the grid provides iso-physical-variable lines, the longer the grid line element is, the more accurate values of T_{eff} and $\log g$ can be derived from the colors. In general, the grid line elements should be as long as possible and at the

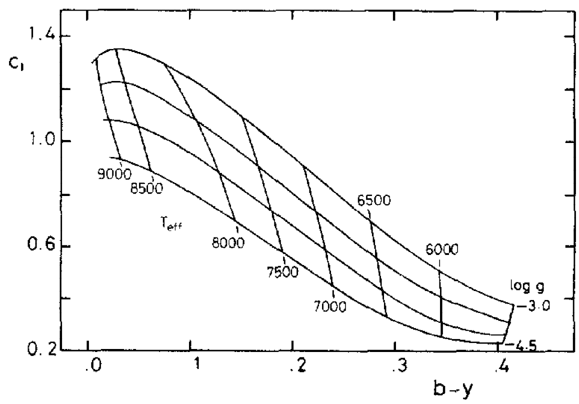


Fig. 1. The grids of $T_{\rm eff}$ and $\log g$ are shown on the $(b\text{-}y)$ *vs.* c_1 diagram. The resolubility of the physical quantities is proportional to the area made by grid elements.

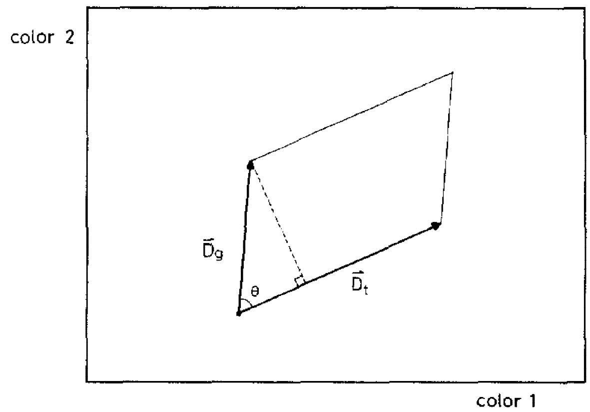


Fig. 2. $\boldsymbol{D}_g$ and $\boldsymbol{D}_t$ are differential vectors of colors due to unit variations of $T_{\rm eff}$ and $\log g$. $\boldsymbol{D}_g$ vector has an effective length of $|\boldsymbol{D}_g| \cdot \sin\theta$ orthogonol to $\boldsymbol{D}_t$.

same time as close to 90° with each other as possible in order to attain the maximum accuracy of the derived physical quatities (Steinlin and Buser 1974).

A given spectral type of stars characterized by a set of the three physical quantities can be mapped into a point vector in the color space. Let **C** be a vector in color space which has three components of C_1, C_2, C_3 and P_j is j-th physical quantity where $j=t$, g, m are abbreviations for T_{eff}, log g, [M/H], respectively. Since a color vector is a function of three physical quantities, it varies with the variation of these physical quantities. We define a differential vector $\boldsymbol{D}_j$ as an increment of the color vector by a unit change of j-th physical quantity,

$$\boldsymbol{D}_j=\left(\frac{\partial \boldsymbol{C}}{\partial P_j}\right)\cdot \Delta P_j \qquad (j=t,\ g,\ m) \tag{2}$$

This differential vector represents an amount of variation in colors with respect to j-th physical quantity. Thus the length of this vetor describes the degree of sensitivity of the photometric parameters to a specific physical parameter. In multi-dimensional color space the accuracy of the derived physical quantities increases with the length of differential vectors and the closeness of the perpendicularity between the two vectors under consideration; thus the accuracy is proportional to the area of parallelogram made by two differential vectors $\boldsymbol{D}_g$ and $\boldsymbol{D}_t$ (see in Figure 2). Accordingly, we have taken the quantity $|\boldsymbol{D}_g\times\boldsymbol{D}_t|$ as a resolubility parameter (Golay 1974). This argument can be extended to the three dimensional space. In this case the differential volume (DV) is defined by three differential vectors as

$$\begin{aligned}
\mathrm{DV}&=|\boldsymbol{D}_t\cdot(\boldsymbol{D}_g\times\boldsymbol{D}_m)|\\
&=\left|\Delta t\Delta g\Delta m\left|\frac{\partial \boldsymbol{C}}{\partial t}\cdot\left(\frac{\partial \boldsymbol{C}}{\partial g}\times\frac{\partial \boldsymbol{C}}{\partial m}\right)\right|\right|\\
&=\left|\Delta t\Delta g\Delta m\left\{\frac{\partial C_1}{\partial t}\left(\frac{\partial C_2}{\partial g}\frac{\partial C_3}{\partial m}-\frac{\partial C_3}{\partial g}\frac{\partial C_2}{\partial m}\right)+\frac{\partial C_2}{\partial t}\left(\frac{\partial C_2}{\partial g}\frac{\partial C_1}{\partial m}-\frac{\partial C_1}{\partial g}\frac{\partial C_3}{\partial m}\right)\right.\right.\\
&\quad\left.\left.+\frac{\partial C_3}{\partial t}\left(\frac{\partial C_1}{\partial g}\frac{\partial C_2}{\partial m}-\frac{\partial C_2}{\partial g}\frac{\partial C_1}{\partial m}\right)\right\}\right|
\end{aligned} \tag{3}$$

which yields a resolubility parameter of the three physical quantities. In equation (3), colors can be synthesized by equation (1) for a given set of response functions of selected filters, and their partial differentiations may be approximated by taking the differences of colors by varying each physical quantities.

The average sensitivity of colors to physical quantities increases with the differential volume. The resolubility of each physical quantity, however, cannot be estimated by the differential volume itself. In this sense, we define a differential length, say DL_g for the lenth of the gravity differential vector as

$$\mathrm{DL}_g=\left|\boldsymbol{D}_g\cdot\frac{(\boldsymbol{D}_t\times\boldsymbol{D}_m)}{|\boldsymbol{D}_t\times\boldsymbol{D}_m|}\right| \tag{4}$$

The quantity DL_g denotes the perpenticular component of $\boldsymbol{D}_g$ to the plane made by the other two differential vectors.

Now we are ready to examine the DV and DL_j with varying mean wavelengths to derive the optimal system. In order to do so, at first we have to take into account the observing accuracy of stellar energy flux at a given mean wavelength. Since the observing precision is proportional to the square-root of integrated energy $E=\dot{n}\tau$ (where $\dot{n}$ is photon flux per unit time and τ is an integration time), the integration time should be increased proportionally to square of $\sigma(\lambda_i)$ in order to maintain a given observing precision. The quantity $\sigma(\lambda_i)$ is the error of a heterochromatic magnitude at λ_i is given by

$$\begin{aligned}\sigma^2(\lambda_i) &= (\frac{\partial C_i}{\partial F_i})^2 \delta F_i^2 + (\frac{\partial C_i}{\partial F_{i+1}})^2 \ \delta F_{i+1}^2 \\ &= A' \ [\ (\frac{\delta F_i}{F_i})^2 + (\frac{\delta F_{i+1}}{F_{i+1}})^2 \ ] \\ &= A \ [\ (\frac{\sqrt{F_i}}{F_i})^2 + (\frac{\sqrt{F_{i+1}}}{F_{i+1}})^2 \ ] \\ &= A \ [\ \frac{1}{F_i} + \frac{1}{F_{i+1}} \ ] \end{aligned} \qquad (6)$$

In equation (6), A′ and A are constants and the error of flux F_i has been set to be proportional to the square-root of F_i. In our anaysis, we have taken the inverse of integration time for a given precision as a weight to i-th component of differential vectors. The differential vectors in equation (2) can be computed by using the known stellar energy distributions, each of which is then converted to the weighted quantity $\boldsymbol{D}_j$ defined as

$$\boldsymbol{DW}_j = \left[\hat{C}_1 \frac{\partial C_1}{\partial P_j} \Delta P_j \cdot \frac{1}{\sigma^2(\lambda)} + \hat{C}_2 \frac{\partial C_2}{\partial P_j} \cdot \frac{1}{\sigma^2(\lambda_2)} + \hat{C}_3 \frac{C_3}{\partial P_j} \Delta P_j \cdot \frac{1}{\sigma^2(\lambda_3)} \right] \qquad (5)$$

where $\hat{C}_1$ $\hat{C}_2$, $\hat{C}_3$ are unit vectors along axes C_1, C_2, C_3,. Substituting equation (5) into equation (3), the weighted differential volume DVW is computed by

$$DVW = \frac{DV}{\sigma^2(\lambda_1) \cdot \sigma^2(\lambda_2) \cdot \sigma^2(\lambda_3)} \qquad (7)$$

Similarly, the weighted differential lengths are also computed. For example, the weighted differential length DLW_g is given by

$$\begin{aligned} DLW_g = [& \sigma^4(\lambda_1) \ (\frac{\partial C_2}{\partial t} \frac{\partial C_3}{\partial m} - \frac{\partial C_3}{\partial t} \frac{\partial C_2}{\partial m})^2 \\ & + \sigma^4(\lambda_2) \ (\frac{\partial C_3}{\partial t} \frac{\partial C_1}{\partial m} - \frac{\partial C_1}{\partial t} \frac{\partial C_3}{\partial m}) \\ & + \sigma^4(\lambda_3) \ (\frac{\partial C_1}{\partial t} \frac{\partial C_2}{\partial m} - \frac{\partial C_2}{\partial t} \frac{\partial C_1}{\partial m})^2]^{-½} \end{aligned} \qquad (8)$$

The quantity DVW or DLW then represents a measure of resolubility which is proprtional to inverse of the integration time required for a given precision.

III. MOST OPTIMAL FILTER SYSTEM

We consider an optimal photometric system which consists of a V filter (as the reference) and three other filters with the same bandwidth of 200Å (FWHM). The total response curves of the filters are assumed to be Gaussian truncated at both limits of twice the halfwidth. Model spectra of Kurucz (1978) are chosen for $5500 < T_{\rm eff} < 7500$°K, $\log g = 4.5$ and $[M/H] = 0.0$. We varied physical parameters by $\Delta T_{\rm eff} = 500$°K, $\Delta \log g = -1.5$ and $\Delta [M/H] = -1.0$ to compute the color variations. The optimal sets of the mean wavelengths are then obtained by analyzing the resolubility parameters defined in the previous section.

Since DVW is a function of mean wavelengths of three filters, our problem is reduced to find the maximum value of a multivariable function. We utilized the steepest descent algorithm (Conte and Boor 1980) in which gradients of the multivariable functions are used to find the maximum value. Since this algorithm gives only the nearest local maximum to the starting point, we are forced to search for the maximum by supplying starting points at every 200Å interval from 1200Å to 6000Å.

The computed mean wavelengths of the optimal filter set (which satisfies the condition of the maximum DVW) are given in Table 1, where differential lengths are also listed for each filter set considered. The cubic-roots of DV are also given since they have dimension of color in magnitude and they also represent the maximum change in one photometric parameter with respect to the variations of each of $\Delta T_{\rm eff}$, $\Delta \log g$ and $\Delta [M/H]$. As can be seen from the table, the wavelengths for the optimal filter set for F and G dwarfs resides in the UV region of $\lambda < 4200$Å. It should be pointed out that the results in Table 1 were obtained without any consideration of the atmospheric extinction. When the atmospheric transmission (Lamla 1982, adopted the case of 3mm ozone

Table 1. The Optimal Sets for the Maximum DVW without Atmospheric Extinction.

$T_{\rm eff}$(°K)	(DVW)$^{1/3}$	(DV)$^{1/3}$	λ_1(Å)	λ_2(Å)	λ_3(Å)	DL$_g$	DL$_m$	DL$_t$
5500	3.5	.25	2850	3280	4190	2.7	2.3	3.0
6000	6.2	.23	2800	3280	4150	5.5	3.7	4.4
6500	10	.27	2490	3300	4180	10	5.7	5.5
7000	15	.24	2450	3290	4190	15	7.3	6.4
7500	20	.22	2440	3100	4190	24	9.1	8.0

Table 2. The Optimal Sets for the Maximum DVW with Atmospheric Extinction.

$T_{\rm eff}$(°K)	(DVW)$^{1/3}$	(DV)$^{1/3}$	λ_1(Å)	λ_2(Å)	λ_2(Å)	DL$_g$	DL$_m$	DL$_t$
5500	2.1	.18	3330	3830	4650	1.5	1.5	2.0
6000	3.4	.16	3340	3860	4580	3.4	2.7	2.8
6500	5.4	.16	3390	3860	4190	6.7	3.3	4.3
7000	7.5	.14	3440	3860	4190	11	4.2	5.6
7500	9.3	.13	3460	3870	4190	17	4.5	7.1

layer) is taken into account, the optimal filter wavelengths should be defined in the visual range (see Table 2). In this case, obviously the weighted differential volume is smaller than that of the case of free-atmosphere (see Table 1), and the resulting resolubility of physical quantities is degraded. We may also note that as T_{eff} increases, the mean wavelenghts λ_1 and λ_2 in Table 2 approaches those of 35, 38 filters of the DDO system (McClure 1976) and λ_3 to that of v filter of the *uvby* system.

So far, we examined the maximum resolubility for the three physical quantites as a whole rather than individual physical quantity. However, we may also find an optimal set of filters which maximizes the resolubility parameter for specific physical quantity. For this purpose, we have computed the mean wavelengths of the optimal filter set by searching for the maximum DLW_j in equation (7) for each physical quantity. The results are listed in Table 3, which reveals that the values of DLW are not far from those of the maximized DVW case in Table 2. The mean wavelengths listed in Table 3 are plotted against effective temperatures in Figure 3. In this figure, the solid, dot-dashed, dashed and dotted lines represent the cases of DVW, DLW*j* for gravity, metallicity and effective temperature and the crosses, filled and open circles refer to mean wavelengths, λ_1, λ_2 and λ_3 listed in Table 3, respectively. One of the important findings

Table 3-a. The Optimal Sets for the Maximum DLW_g with Atmospheric Exinction.

T_{eff}(°K)	DLW_g	DL_g	λ_1(Å)	λ_2(Å)	λ_2(Å)	DLW_m	DLW_t	$(DVW)^{1/3}$
5500	1.7	.26	3330	3810	3980	.36	.49	1.4
6000	3.5	.27	3330	3830	4020	1.1	1.6	2.9
6500	6.8	.22	3370	3850	4500	3.8	3.9	5.3
7000	11	.25	3400	3840	4520	4.9	5.2	7.3
7500	18	.29	3400	3840	4520	5.0	6.7	9.1

Table 3-b. The Optimal Sets for the Maximum DLW_m with Atmospheric Extinction.

T_{eff}(°K)	DLW_m	DL_m	λ_1(Å)	λ_2(Å)	λ_2(Å)	DLW_g	DLW_t	$(DVW)^{1/3}$
5500	1.9	.084	3280	3490	4650	.60	1.6	1.4
6000	3.0	.089	3270	3710	4640	2.5	2.7	2.9
6500	4.2	.10	3300	3790	4630	6.1	3.5	4.8
7000	5.3	.096	3280	3850	4620	7.6	4.2	6.3
7500	5.6	.072	3220	3890	4620	15	5.8	8.6

Table 3-c. The Optimal Sets for the Maximum DLW_t with Atmospheric Extinction.

T_{eff}(°K)	DLW_t	DL_t	λ_1(Å)	λ_2(Å)	λ_2(Å)	DL_g	DL_m	$(DVW)^{1/3}$
5500	2.1	.076	3480	4130	4590	.85	1.5	1.5
6000	3.2	.096	3280	3760	4170	3.0	2.2	3.2
6500	4.6	.089	3310	3780	4180	6.0	3.4	5.2
7000	6.2	.085	3320	3780	4190	10	4.2	7.2
7500	7.7	.076	3330	3790	4200	16	4.3	9.0

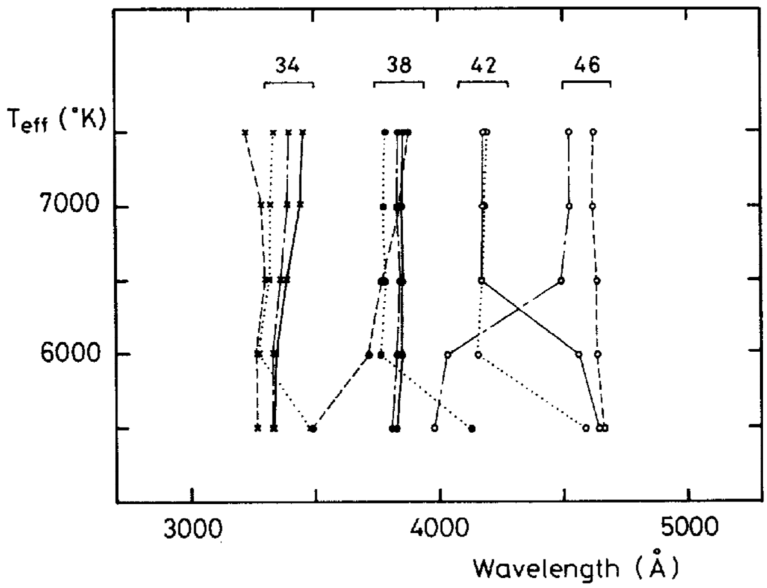


Fig. 3. The most optimal sets of mean wavelengths in Table 3 are plotted against effective temperatures. Solid, dot-dashed, dashed and dotted lines represent the most optimal filter sets in Table 2, Table 3-a, Talbe 3-b and Table 3-c, respectively, and crosses, filled and open circles denote mean wavelengths λ_1, λ_2, and λ_3, respectively, in Table 2 and Table 3.

emerging from the present anaysis is the fact that four filters (named as 34, 38, 42, 46 in Figure 3) are needed to accommodate the various optimal mean wavelengths from different conditions. Among those filters, 34 and 46 filters may work better if bandwidths were broaden by more than 200Å.

IV. COMPARISON WITH OTHER FILTER SYSTEMS

Among many filter systems, the Strömgren *uvby* and DDO systems are widely used at present estimating surface gravity, metallicity and effective temperature of stars. The former system consists of four filters whose central wavelengths are at 3460Å (*u*), 4120Å (*v*), 4700Å (*b*) and 5500Å (*y*), respectively and the basic photometric parameters are $(b\text{-}y)$, $m_1=[(v-b)-(b-y)]$ and $c_1=[(u-v)-(v-b))]$. On the other hand, the DDO system consists of six intermediate band filters.

Utilizing the characteristics of the above filters (Lee 1985), we have computed the weighted differential volumes and differential lengths for these systems. The results are given in Table 4, where the DDO system is divided into two sets of filters. The set of 35, 38, 42, 48 filters show higher resolubility than the other set of the DDO filters. The reason seems to be partly due to the fact that the filters of 41, 42, 45, 48 have all relatively narrow passbands so that the differential volume is reduced because of the weight given to equation (5).

A more explicit comparison between our optimal system and the *uvby* system is shown in the color plane of Figure 4, where Figure 4-a refers to our optimal system and Figure 4-b, to the *uvby*

Table 4. $(DVW)^{1/3}$ of Two Photometric Systems.

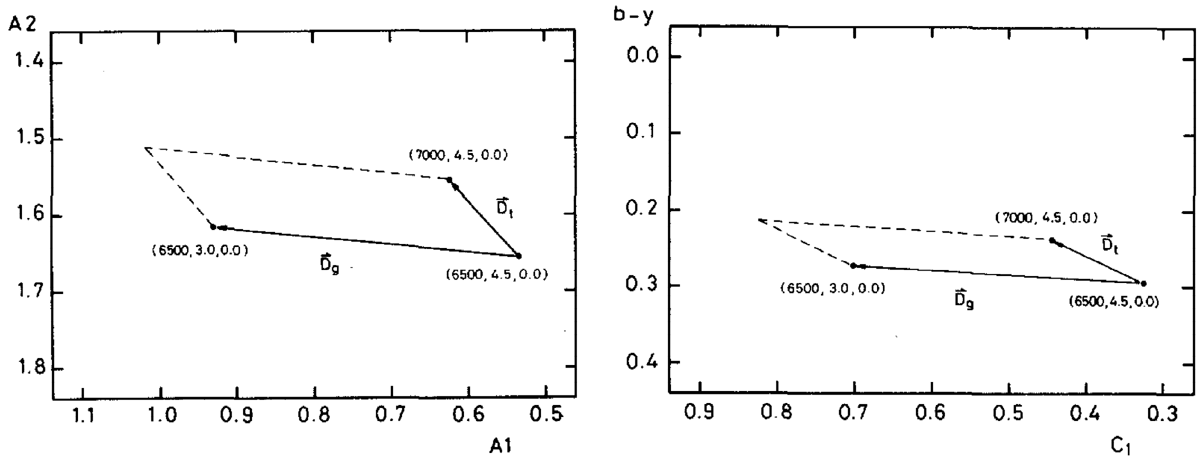
T_{eff} (°K)	uvby	DDO(41, 42, 45, 48)	DDO(35, 38, 42, 48)
5500	1.5	.18	.99
6000	2.3	.27	1.9
6500	3.3	.39	3.4
7000	4.3	.13	5.0
7500	5.2	.75	6.3

system. The area of parallelogram made by the two differential vectors is certainly greater in the case of our optimal system than in the *uvby* system. It may be said that the accuracy of T_{eff} and log g deduced from the colors of our system is higher than that of the *uvby* system and the DDO system.

V. CONCLUSION

In the present study, we demonstrated that the resolubility of physical quantities for a given photometric filter system can be evaluated quantitatively by computing the weighted differential volume and lengths. In this process, we weighted each component of differential vectors in the color space by 1/(color error)^2, noting that the integration time is inversely proportional to the squre of error in magnitudes.

Making use of Kurucz(1978)'s model atmospheres, most optimal filter systems were examined. It is found that for F and G dwarfs, the wavelengths of optimal filter set resides in the near UV region (2400< λ <4200Å). When the atmospheric extinction is taken into account, the derived optimal filter systems varies slightly, depending on the conditions for resolubility parameters (see Table 2 and Table 3). Finally, we presented a most optimal filter system for F, G dwarfs which is made of



(a) (b)

Fig. 4. Area of parallelogram made by two differential vectors D_t and D_g in the color-color diagrams is shown for the cases of our optimal system in Fig. 4-a and the *uvby* system in Fig. 4-b. A1 and A2 are colors of our optimal system.

Table 5. The Optimal Filter System for F, G Dwarfs.

Filters	Mean Wavelength(Å)	Half Bandwidth(Å)	Similar Filters
34	3400	200	35 (DDO), *u (uvby)*
38	3850	200	38 (DDO)
42	4190	200	42 (DDO), *v (vuby)*
46	4600	200	

our 34, 38, 42, 46 filter combination (see Table 5). The 34 and 38 filters are found to be very sensitive to all of the three physical parameters, while 46 filter, to metallicity and gravity, and 42 filter, only to effective temperature. The set of our 34, 38 and 42 filters turns out to be quite similar to those of the DDO filters as indicated in Table 5.

For simplicity, we have fixed the bandwidth of all filters in the present study. However, we may take the bandwidth as a free parameter to optimize the photometric filter combinations. In view of the fact that the UV region ($\lambda < 4000$Å) of stellar spectra of F, G dwarfs shows a steep descent in energy distribution, it appears that a better filter system can be found if we are allowed to vary the bandwidths of the filters. This case will be taken up further in the near future.

The authors express thier thanks to Prof. Yun, Hong Sik for his careful reading the manuscript and to Mr. Park, Byung Gon for his help during computer works.

REFERENCES

Conte, S. D. and Boor, C. D. 1980, *Elementary Numerical Analysis* (New York : McGraw-Hill), p.209.

Golay, M. 1974, *Introduction to Astronomical Photometry* (Boston : Reidel), p.302.

Kurucz, R. L. 1978, Center for Astrophysics Preprint Series, No. 1050.

Lamla, E. 1982, in *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology,* group VI, Vol 2, subvol. b, ed. K. Schaifers and H. H. Voigt (New York : Springer-Verlag), p.46.

Lee, See-Woo 1985, *Astronomical Observations and Analyses* (Seoul : Min-Eum-Sa), p.251 and p.275.

Massa, D. and Lillie, C. F. 1978, Ap. J., **221,** 833.

McClure, R. D. 1976, A. J., **81,** 182.

Steinlin, U. W. and Buser, R. 1974, in *Multicolor Photometry and the Theoretical HR Diagram,* ed. A. G. D. Philip and D. S. Hayes (Dudley Obs. Rept. No. 9), p.349.