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Thermal Aspects of Nuclear Waste Disposal

Wook Han*

Abstract: This study evaluates temperature profiles with distances from the center of nuclear wastes. The mathematical model to understand disposal plan is applied. The evaluation for thermal aspects includes mathematical derivation, numerical solution and some limitations. The numerical results and procedures of this work predict temperature variation in the radioactve wastes.

INTRODUCTION

Geophysicists can be hired by the government or company to evaluate the thermal aspects of radioactive waste disposal for the nuclear power plant complex being considered in some places on the globe. The radioactive waste in the subsurface is a possible solution of ultimate disposal. Deep salt formation guarantees a complete sealing against atmosphere and groundwater, and is favorable at present time for the permanent disposal plan of high radioactive waste.

The disposal plan is to bury the waste in cylindrical containers down large drill holes. If high radioactive wastes are deposited underground high temperature may occur. The temperature increase influences the physical behavior of the waste and the surroundings and should be kept under control. Each container would be 10 feet long, 1 foot in diameter and would generate 5 kilowatts of heat. The plan is to put 30 such containers in each drill hole and space the holes one per acre.

The cost for the waste depends on the volume of the storage cavities and the maximum concentration of the waste. Mathematical model has been developed to evaluate the influence of some parameters on the temperature. The model can predict the temperature distribution within and around cavities. The thermal conductivity and diffusivity should be known.

* Dept. of Env. Sci., Korea Military Academy

MATHEMATICAL MODEL

Model

The evaluation might include the following:

(a) Model to mathematically describe the problem.

(b) Temperature as a function of space and time for the model.

Let us further assume that

(a) neglect the diameter of the radioactive (material) wastes cylinders and regard them as infinite continuous line sources arranged as shown in Figure 1.

(b) neglect the tangential heat flow and consider the temperature profile on the plane A-B.

Using assumptions (a) and (b), model the problem as follows: continuous line source, generating Q(cal/hr·unit length) for $t>0$, parallel to the Z axis through the point $(r', 0)$ in the cylinder $r=R_0$. The cylinder is at zero temperature at $t=0$, and no heat flow occurs across the boundary (insulated boundary).

For the calculations, the following physical properties of the rock are introduced:

$k=1,800$ cal/m·hr·°C
thermal conductivity,

$\rho=2.67\times10^6$ g/m^3
density of the rock,

$C_p=0.065$ cal/g·°C
specific heat at constant pressure,
$\alpha=k/\rho C=0.0104$ m²/hr
thermal diffusivity,
$Q=1.411\times10^6$ cal/hr·m
surface heat flow.

Solution

The differential equations (after transformation) are

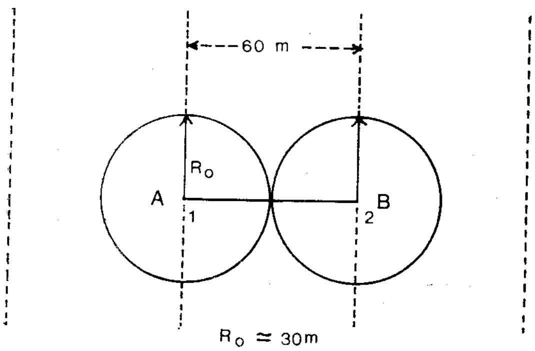

Fig. 1 Continuous line source.

$$\frac{\partial^2 \bar{T}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{T}}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \bar{T}}{\partial \theta^2}-q^2\bar{T}=0,\quad 0\leq r<R_0 \tag{1}$$

$$\frac{\partial T}{\partial r}=0 \text{ at } r=R_0 \tag{2}$$

$$\lim_{R\to 0} R\frac{\partial T}{\partial R}=-\frac{Q}{2\pi k\rho} \tag{3}$$

$$(R=(r^2+r'^2-2rr'\cos\theta)^{\frac{1}{2}} \tag{4}$$

where T is temperature increase r radius of cylinder The symbols except earlier explanations are constants for calculations.

From equation (1) and (3),

$$T=\frac{Q}{2\pi k\rho}k_0(qR)$$

By the addition theorem,

$$T=\frac{Q}{2\pi k\rho}\sum_{n=0}^{\infty}\epsilon_n I_n(qr)K_n(qr')\cos n\,\theta,\quad \text{when } r<r' \tag{5}$$

Where $\epsilon_n=1$ for $n=0$, and
$\epsilon_n=2$ for $n=1, 2, \cdots$

For $r>r'$, interchange r and r' in equation (5). Thus the solutions of equations (1)~(4) are

$$\left.\begin{aligned}T&=\frac{Q}{2\pi k\rho}\sum_{n=0}^{\infty}\epsilon_n\{I_n(qr)K_n(qr')\\&\quad+a_nI_n(qr)\}\times\cos n\theta,\quad 0<r\leq r'\\T&=\frac{Q}{2\pi k\rho}\sum_{n=0}^{\infty}\epsilon_n\{I_n(qr')K_n(qr)\\&\quad+a_nI_n(qr)\}\times\cos n\theta,\quad a>r>r'\end{aligned}\right\} \tag{6}$$

From the boundary condition (2) and equation (6),

$$a_n=-\frac{I_n(qr')K_n'(qR_0)}{I_n'(qa)}$$

Substituting a_n into (6) and inverting

$$T=\frac{Q}{4\pi^2 ik}\sum_{n=0}^{\infty}\epsilon_n\cos n\,\theta\int_{r-i\infty}^{r+i\infty}\frac{I_n(\mu r)\{K_n(\mu r')I_n'(\mu R_0)-I_n(\mu r')K_n'(\mu R_0)\}e^{\lambda t}d\lambda}{\lambda I_n'(\mu R_0)} \tag{7}$$

Complete solution can be obtained by performing the integration of equation

$$\begin{aligned}T=\frac{Q}{2\pi k}&\left\{\frac{2\alpha t}{R_0^2}+\frac{r^2+r'^2}{2R_0^2}-\frac{3}{4}-\frac{1}{2}\ln\left(1-\frac{2rr'}{R_0^2}\cos\theta+\frac{r^2r'^2}{R_0^4}-\frac{1}{2}\right)\ln\left(\frac{r'^2}{R_0^2}-\frac{2rr'}{R^2_0}\cos\theta+\frac{r^2}{R_0^2}\right)\right.\\&-\frac{Q}{\pi k}\sum_{n=0}^{\infty}\epsilon_n\cos_n\theta\sum_{m=1}^{\infty}e^{-\alpha\cdot\alpha^2_{n,m}t}\frac{J_n(r,\alpha_{n,m})J_n(r',\alpha_{n,m})}{(R_0^2\alpha^2_{n,m}-n^2)J_n^2(\alpha,\alpha_{n,m})}\end{aligned} \tag{8}$$

Where $\alpha_{n,m}$ are the roots of $J_n'(\alpha\ \alpha_n)=0$

If $r'=0$ and the heat flow is axisymmetric, equation (8) reduces to

$$\frac{T(r,t)}{(Q/\pi k)}=\frac{1}{2}\left\{\frac{2\alpha t}{R_0^2}+\frac{r^2}{2R_0^2}-\frac{3}{4}-\ln\frac{r}{R_0}\right\}-\sum_{m=1}^{\infty}e^{-\alpha\cdot\alpha^2_{0,m}t}\frac{J_0(r\ \alpha_{0,m})}{(R_0^2\ \alpha^2_{0,m})J_0^2(R_0\ \alpha_{0,m})} \tag{9}$$

Where $J_0'(R_0\ \alpha_{0,m})=0$

Table 1 can be used for the further calculations.

Table 1 Numerical values.

m	$R_0\alpha_{0,m}$	$\alpha_{0,m}$	$J_0(R_0d_{0,m})$	$\alpha_2{}^{0,m}$	$\frac{(R_0d_{0,m})^2}{J_0^2(R_0\alpha_{0\ m})}$
1	3.8317	0.128	−0.40	0.0164	2.349
2	7.0156	0.234	0.30	0.0548	4.43
3	10.1735	0.339	−0.25	0.115	6.47
4	13.3237	0.444	0.22	0.197	8.591
5	16.4706	0.549	−0.196	0.301	10.31

At $r=0.17\text{m}(\simeq 0)$

$$\frac{T(0.17,t)}{(Q/\pi k)} = \frac{1}{2}\left\{\frac{2\alpha t}{900}+0.00002-0.75-\ln(0.0056)\right\} - \left\{\frac{J_0(0.022)}{2.349}e^{-0.0164\alpha t}+\frac{J_0(0.04)}{4.43}e^{-0.0548\alpha t} + \frac{J_0(0.058)}{6.47}e^{-0.115\alpha t}+\frac{J_0(0.0755)}{8.591}e^{-0.197\alpha t} + \frac{J_0(0.0933)}{10.31}e^{-0.301\alpha t}\right\} \quad (10)$$

At $r=10\text{m}$

$$\frac{T(15,t)}{(Q/\pi k)} = \frac{1}{2}\left\{\frac{2\alpha t}{900}+\frac{1}{18}-\frac{3}{4}-\ln\left(\frac{10}{30}\right)\right\} - \left\{\frac{J_0(1.28)}{(2.349)}e^{-0.0164\alpha t}+\frac{J_0(234)}{4.43}e^{-0.0548\alpha t} + \frac{J_0(3.39)}{6.47}e^{-0.115\alpha t}+\frac{J_0(4.44)}{8.591}e^{-0.197\alpha t} + \frac{J_0(5.49)}{10.31}e^{-0.301\alpha t}\right\} \quad (11)$$

At $r=30\text{m}$ (center)

$$\frac{T(30,t)}{(Q/\pi k)} = \frac{1}{2}\left\{\frac{2\alpha t}{900}+\frac{1}{2}-\frac{3}{4}\right\} - \left\{\frac{J_0(3.83)}{2.349}e^{-0.0164\alpha t}+\frac{J_0(7.02)}{4.43}e^{-0.0548\alpha t} + \frac{J_0(10.17)}{6.47}e^{-0.115\alpha t}+\frac{J_0(13.32)}{8.591}e^{-0.197\alpha t} + \frac{J_0(16.47)}{10.31}e^{-0.301\alpha t}\right\} \quad (12)$$

The calculated temperature profile at three locations are shown in Figure 2 for two different elapsed time.

The above equations were evaluated with the help of on electronic computer.

DISCUSSION/LIMITATIONS

The sloution predicts temperatures that are probably too great because temperature will reach the melting point of the granite(chapman, 1978).

After 40 years, the temperature will be stable as shown in Figure 3. For the geologic environments suitable for above disposal, granite is not recommended. We need a small container or the plan to bury the waste in cylindrical containers down to deep drill hole. The salt cavity can be suitable for the waste disposal. And the porosity, groundwater leaching and the melting point of the surrounding rocks should be considered. As a result the rocks of the surrounding the container will be affected by the disposal container. This might include the effect of different rock types on the temperature regime.

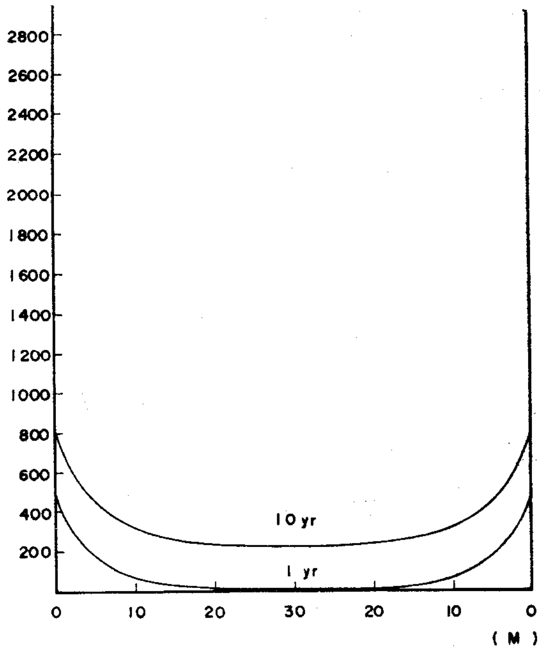

Fig. 2 Temperature profile.

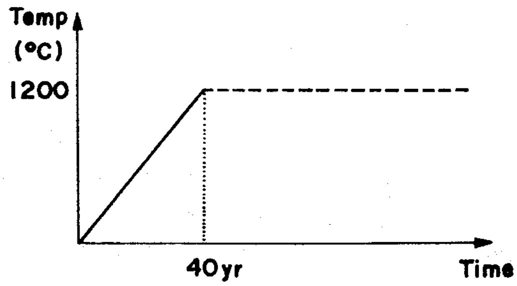

Fig. 3 Temperature vs. time.

Different rock types have various thermal conductivity and so the rocks with high thermal

Table 2 Radioactive composition of waste after Bruce(1960).

Isotope	$t_{\frac{1}{2}}$(yrs)	$\lambda(10^{-8}s^{-1})$	A_0(cal cm^{-3}s^{-1})	T(°C)
$90S_r$	28.0	0.0785	—	—
$137C_s$	30.0	0.0735	5.22×10^{-5}	242.8
$106R_u$	1.0	2.20	3.98×10^{-5}	185.2
$147P_m$	2.6	0.846	5.92×10^{-6}	27.5
$144C_e$	0.78	2.76	2.44×10^{-4}	1137.0

conductivity are suitable for this disposal plan.

The composition and heat production of the radioactive waste to be considered is shown in Table 2. The data in Table 2 include the energy released by daughter products.

$t_{\frac{1}{2}}$=half-life

λ=decay constant

A_0=heat production

T=temperature

The temperature rise which is caused by a mixture of radioactive isotopes in a spherical undergound cavity is given by Kappelmeyer and Haenel(1974).

With the numerical values given in Table 2 the maximum temperatures attained at the center of the cavity are of the order of 500°C as shown in Figure 4. Since salt melts at about 770°C, a cavity larger than 2 m radius is not recommendable. The significant feature of the temperature rise due to the radioactive waste is the radial distribution of the temperature disturbance. Figure 4 shows the temperatures at different distances from the center for 60 years. At 20m from the center of the heat source the maximum temperature disturbance is ~10°C and occurs after three years. The temperature rise from the radioactive waste is limited to small zones around the cavities and does not influence a great geologic complex.

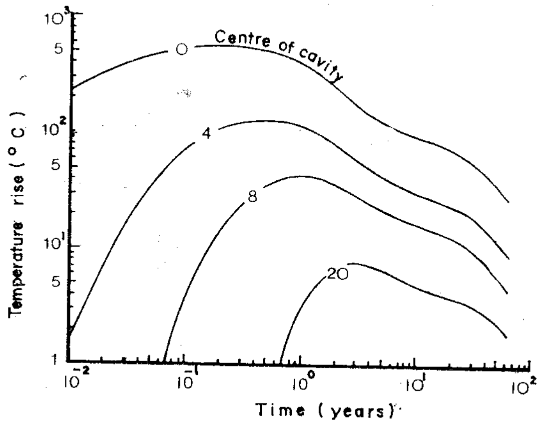

Fig. 4 Temperature with distance from the center.

More complex models for cylinders and for various thermal properties are treated by Schmidt (1969). Temperature rise due to the disposal of radioactive wastes in the subsurface may be an important issue in the near future. Considerations about the waste disposal showed that a mathematical treatment is not satisfactory unless limitations of analysis and a half life are included.

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핵 폐기물 처리에 따른 地熱問題

韓 郁*

요약 : 본 연구는 핵 폐기물의 중심부에서 거리가 변화함에 따른 온도 곡선을 포함하여 폐기물 처리 계획을 이해하기 위한 수학적 모델이 적용되었다. 지열 문제를 위하여 수식의유도, 수치결과 및 제한점들을 평가하였으며 본 연구 결과와 절차로부터 방사성 물질로 인한 온도의 변화를 예측할 수 있다.

* 陸士 環境學科