論 文

# 고차 대역통과 및 대역저지 타원 필터의 최대 동적구역을 실현하기 위한 새로운 접근법

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# A New Approach to the Maximum Dynamic Range of the High Order Band-Pass and Band-Reject Elliptic Filters

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**요 약** 고차 필터는 각2차 함수를 종속연결함으로써 실현화한다. 본 논문에서는 타원함수의 고차 대역통과와 대역저지 필터 실현화에 있어서 전 동적범위를 향상시키기 위한 방법을 제시하였으며 고역통과 놋치, 저역통과 놋치 그리고 대칭 놋치등 각 2차함수들의 최적배열을 타원함수의 대역통과와 대역저지 필터에 적용하였다.

***ABSTRACT*** High order filters are usually realized by cascading second order stages. In this paper, a simple method of pole-zero pairing in the high order band-pass and band-reject filter realization of the elliptic functions is proposed for the enhancement of overall dynamic range. Furthermore, the optimum sequence of the various biquads of high-pass notch, low-pass notch and symmetrical notch etc., is developed for the elliptic band-pass and band-reject filters.

## I. INTRODUCTION

In the cascade realization of the band-pass and band-reject high order elliptic filters, the given low-pass function of order n must be decomposed into a number of biquads (for n even) or into a first order function in addition to biquads (for n odd) [1].

By using frequency transformation, for n even case, a number of biquads in the low-pass elliptic function change into the high-pass notch (HPN) and the low-pass notch (LPN) biquads in the high order band-pass (B-P) and band-reject (B-R) elliptic filter realizations. For the case of odd n, the first order section transforms into a second-order B-P in the B-P case and a second order symmetric notch in the B-R

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case, respectively [2].

The main advantage of the cascading technique is the relatively simple relation between the poles and zeros of the transfer function and the network elements. Thus, a relatively uncomplicated adjustment of the filter parameters is possible which allows the network elements to have under tolerances [12].

### A. Band-Pass Type Elliptic Functions

As a consequence of the transformation $S \to \frac{S^2+\omega_o^2}{BS}$, where $\omega_0$ is the center frequency and B is the band-width, we have the pole-zero location as shown in Fig. la as illustrated for n=12 band-pass elliptic function. The upper half from the horizontal line intersecting $\omega_0$=1.5 is similar to the pole-zero pattern of the low-pass function; and the lower half is similar to the pole-zero pattern of the high-pass function [2].

Corresponding to each $p_i$ of a low-pass notch function, there exists a pole $p_i'$ of the identical Q for a high-pass notch function.

The pole-zero pairing, therefore, should be conducted as shown below

$$t_i = \frac{(S-Z_i)(S-Z_i^*)}{(S-P_i)(S-P_i^*)} \quad \text{for low-pass notch section of } Q_i \tag{1a}$$

$$t_i' = \frac{(S-Z_i')(S-Z_i^{*\prime})}{(S-P_i')(S-P_i^{*\prime})} \quad \text{for high-pass notch section of identical } Q_i \tag{1b}$$

For optimal sequencing, $t_i'$ should accompany $t_i$ so that a pair of biquads; one of low-pass notch and the other of high-pass notch, forms a subsection in the cascade [1], [2].

### B. Band-Reject Type Elliptic Functions

Using the transformation $S \to \frac{BS}{S^2+\omega_o^2}$, which is the inverse of the band-pass case, we have the pole-zero pattern as shown in Fig. 1b.

By using the analogy, the technique which results in the pairing and sequencing similar to band-pass will be developed but the place of low-pass notch function and high-pass notch function is reversed [2].

The particular cascading sequence that results in the transfer functions, from input to output, having the flattest magnitude in band-pass and the flattest at the bottom in the band-reject will be developed for the elliptic filter realization with reference to the Q of individual biquads.

Making use of the properties pertinent to the elliptic function, a relatively simple method of pole-zero pairing in the band-pass type and the band-reject type will be developed to improve the overall dynamic range of the realized filter [5], [6].

## II. HIGH ORDER BAND-PASS REALIZATION

Let us take the case of n=6 for illustrative purpose. The sixth order elliptic low-pass function may be written as a product of three biquads of different Q's.

It has been proved [2] that the maximum dynamic range the sequence will be ordered as follows where $Q_1 < Q_2 < Q_3$

$$T(S) = K \underset{Q_2}{t_2(S)} \cdot \underset{Q_1}{t_1(S)} \cdot \underset{Q_3}{t_3(S)} \tag{2}$$

The biquad $t_2(s)$ of the moderate $Q_2$ is followed by $t_1(s)$ of low $Q_1$ and $t_3(S)$ of high $Q_3$. The biquad $t_i(s)$ has the pole-zero pairing as shown in Fig. 2a.

In order to further enhance the dynamic range we may distribute the gain K to individual biquads as

$$T(S) = \underset{Q_2}{k_2 t_2(S)} \cdot \underset{Q_1}{k_1 t_1(S)} \cdot \underset{Q_3}{k_3 t_3(S)} \tag{3}$$

Applying the low-pass to band-pass transformation we obtain the band-pass function of n= 12. $t_i(s)$ in (3) yields a biquad couple $t_i'(s)$ and $t_i''(s)$ of an identical $Q_i'$. The component of the biquad couple, viz., $t_i'(s)$ is of the low-pass notch (LPN) type, and $t_i''(s)$ is of the high-pass notch (HPN) type, respectively, and together they exhibit the band-pass characteristics.

$$T_{BP}(S) = k_2 t_2'(S) t_2''(S) \cdot k_1 t_1'(S) t_1''(S) \cdot k_3 t_3'(S) t_3''(S) \quad (4)$$

| LPN | HPN | LPN | HPN | LPN | HPN |
|---|---|---|---|---|---|
| $Q_2'$ | | $Q_1'$ | | $Q_3'$ | |

Each band-pass section is composed of LPN first and HPN second because this will ensure that strong out-of-band signals, especially high frequency ones, will be sufficiently attenuated before reaching the second stage where they might produce overloading or slew-rate limiting. Similarly, it is usually recommended to place a band-pass or high-pass notch section at the end of the cascade [2], [4]. This will help to prevent internally generated low-frequency noise from appearing at the filter output. If the odd function is given, there appears only one second-order band-pass function$\left(\frac{S}{S^2+aS+b}\right)$and it has the moderate band-pass characteristics, therefore, second-order band-pass function is placed at the last stage in the sequence of the coupled biquads band-pass elliptic function.

To obtain the optimum dynamic range in the realization of the elliptic band-pass filter the pole-zero pairing of the coupled biquads must be done as shown in Fig. 2a and then cascading sequence, finally the optimum gain assignment [8], [4] will be carried out. For general development let us now write the function of even order n as a proruct of $\frac{n}{2}$ biquads

$$T(S) = K \prod_{i=1}^{\frac{n}{2}} t_i(S) \quad (5)$$

where the subscript is in the order of increasing Q. The sequence of biquads for the optimum dynamic range in cascade realization has been proved [2] to be

Case (a)

$$T(S) = K\, t_{\frac{n}{4}} \cdot t_{\frac{n}{4}+1} \cdot t_{\frac{n}{4}-1} \cdot t_{\frac{n}{4}+2} \cdot t_{\frac{n}{4}-2} \cdots \quad (6a)$$
$$\text{for } n = 4k,\ k = 1, 2, \cdots$$

and

Case (b)

$$T(S) = K\, t_{\frac{n+2}{4}} \cdot t_{\frac{n+2}{4}-1} \cdot t_{\frac{n+2}{4}+1} \cdot t_{\frac{n+2}{4}-2} \cdot t_{\frac{n+2}{4}+2} \cdots$$
$$\text{for } n = 4k-2,\ k = 1, 2, \cdots \quad (6)$$

When n = 10, for example, we write five biquads in the order of pole $Q's$ to find the cascading sequence as indicated by encircled numbers.

$$T(S) = K\, \underset{④}{t_1(S)} \cdot \underset{②}{t_2(S)} \cdot \underset{①}{t_3(S)} \cdot \underset{③}{t_4(S)} \cdot \underset{⑤}{t_5(S)}$$

cascading sequence

For n = 12, we have

$$T(S) = K\, \underset{⑤}{t_1(S)} \cdot \underset{③}{t_2(S)} \cdot \underset{①}{t_3(S)} \cdot \underset{②}{t_4(S)} \cdot \underset{④}{t_5(S)} \cdot \underset{⑥}{t_6(S)}$$

When n is odd, the first order function $t_0 = \frac{1}{S+\sigma}$ should be properly treated relative to other biquads. For example, if we place $t_0$ in front in the sequence as $t_0 \cdot t_2 \cdot t_1 \cdot t_3$, the minimum of $|t_2|$ occurs at $\omega = 0$ while the minimum of the product $|t_0 \cdot t_2|$ does not occur at $\omega = 0$ but at $\omega = 1$. Thus it contradicts the optimum sequencing method proposed in [2]. On the other hand, if we place $t_0$ as a last entry to the sequence as $t_2 \cdot t_1 \cdot t_3 \cdot t_0$ the procedure coincides exactly as advanced in [2]. This leads to the realization of the first order section as the last stage of the cascade.

The normalized magnitude of the biquads in which $Q_m$ corresponding to midpoint $Q$ exhibits the highest flatness as shown Fig. 3.

For optimal sequencing in the band-pass realization $t_i'$ should accompany $t_i''$ so that a pair of biquads; one of low-pass notch and the other of

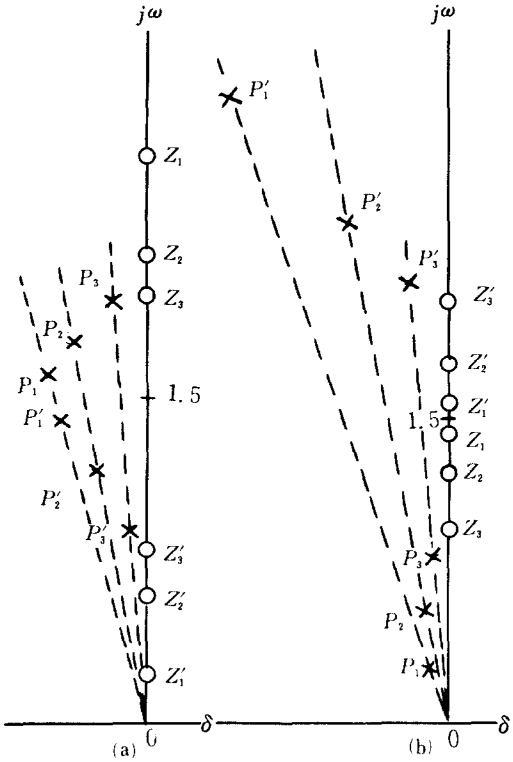



그림 1 극점 및 영점의 위치분포도
Schematic pole-zero location.
(a) 12차의 Elliptic 대역통과함수
Elliptic Band-Pass Function of order 12
(b) 12차의 Elliptic 대역저지함수
Elliptic Band-Reject Function of order 12.

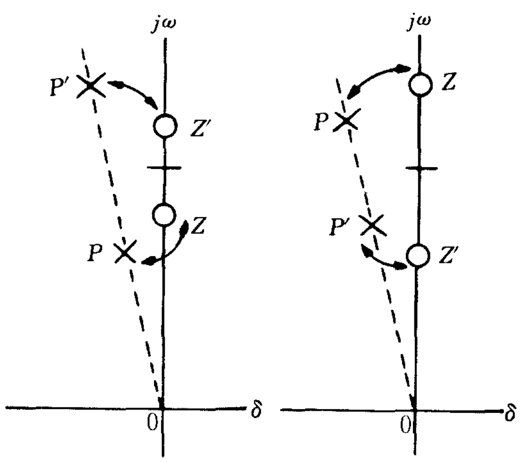



그림 2 Coupled biquads pole-zero pairing
짝지어진 2차함수의 극점-영점 쌍
(a) 대역통과의 경우
Band-Pass Case
(b) 대역저지의 경우
Band-Reject Case.

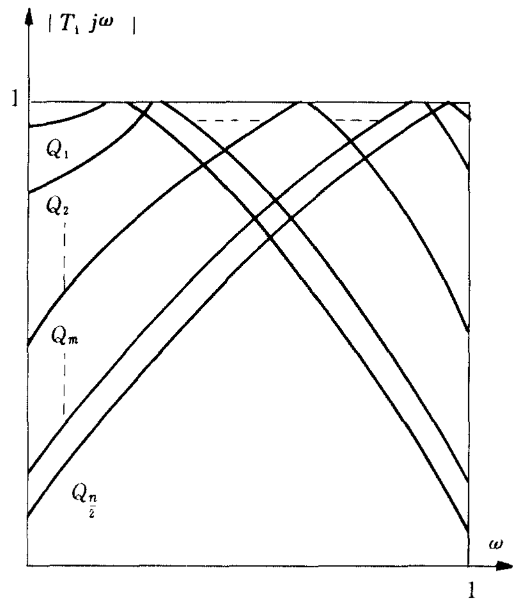



그림 3 2차함수의 정규화된 크기에서 $Q$의 중앙점에 위치하는 $Q_m$이 가장 좋은 평탄도
The normalized magnitude of the biquads in which $Q_m$ corresponding to midpoint $Q$ exhibits the highest flatness.

high-pass noth, forms a subsection in the cascade.

Considering the nature of LP to BP transformation, using the result of (6a) and (6b), we obtain the optimum sequencing as

$$T_{BP}(S)=K[t'_{\frac{n}{4}}\cdot t''_{\frac{n}{4}}][t'_{\frac{n}{4}+1}\cdot t''_{\frac{n}{4}+1}][t'_{\frac{n}{4}-1}\cdot t''_{\frac{n}{4}-1}] \quad (7a)$$

for case $(a)$

$$T_{BP}(S)=K[t'_{\frac{n+2}{4}}\cdot t''_{\frac{n+2}{4}}][t'_{\frac{n+2}{4}-1}\cdot t''_{\frac{n+2}{4}-1}]$$

$$[t'_{\frac{n+2}{4}+1}\cdot t''_{\frac{n+2}{4}+1}] \quad (7b)$$

for case $(b)$

## III. HIGH ORDER BAND REJECT REALIZATION

As a consequence of the transformation $S\rightarrow\frac{BS}{S^2+\omega_o^2}$ which is the inverse of the band-pass, we have the pole-zero pattern as shown in Fig. 1b. By using analogy, we can develop the technique which results in the pairing and sequencing similar to band-pass realization but the place of low-pass notch function and high-pass notch function is reversed.

$$T_{BR}(S) = K[t''_{\frac{n}{4}} \cdot t'_{\frac{n}{4}}]\,[t''_{\frac{n}{4}+1} \cdot t'_{\frac{n}{4}+1}]\,[t''_{\frac{n}{4}-1} \cdot t'_{\frac{n}{4}-1}] \cdots$$

for case (a) (8 a)

$$T_{BR}(S) = K[t''_{\frac{n+2}{4}} \cdot t'_{\frac{n+2}{4}}]\,[t''_{\frac{n+2}{4}-1} \cdot t'_{\frac{n+2}{4}-1}]$$

$$[t''_{\frac{n+2}{4}+1} \cdot t'_{\frac{n+2}{4}+1}] \cdots \quad (8b)$$

for case (b)

## IV. ILLUSTRATIVE EXAMPLES

### Example 1.

Find the elliptic band-pass function for the optimum dynamic range realization under the specipication; passband ripple $K_P$=1dB, stopband attenuation $K_S \geqq 57$dP and stopband frequency $\omega_S$ = 1.3,tutoff frequency $W_C = 1$, bandwidth B=0.3.

From the standard table [3] we find n=6 low-pass elliptic function. For the optimum dynamic range realization [2], we write

$$T(S) = K\,t_2(S) \cdot t_1(S) \cdot t_3(S)$$
$$T(S) = K_2 t_2(S) \cdot K_1 t_1(S) \cdot K_3 t_3(S)$$

where

$$t_2(S) = \frac{(S^2+2.7698611)}{(S^2+0.2909193\,S+0.6785315)}$$

$K_2 = 0.1098799$

$Q_2 = 2.8314733$

$$t_1(S) = \frac{(S^2+1.7658735)}{(S^2+0.5514179S+0.1886488)}$$

$K_1 = 0.0106312$

$Q_1 = 0.7876733$

$$t_3(S) = \frac{(S^2+17.0590037)}{(S^2+0.0781612S+0.9964994)}$$

$K_3 = 1.1642069$

$Q_3 = 12.7716574$

By the frequency transformation we obtain n=12 band-pass transfer function [8],[10],[11]

$$T_{BP}(S) = K_2 t'_2(S) t''_2(S) \cdot K_1 t'_1(S) t''_1(S) \cdot K_3 t'_3(S) t''_3(S)$$
$$= K'_2 t'_2(S) \cdot K''_2 t''_2(S) \cdot K'_1 t'_1(S) \cdot K''_1 t''_1(S) \cdot K'_3 t'_3(S) \cdot K''_3 t''_3(S)$$

Where

$$t'_2(S) = \frac{(S^2+1.3008857)}{(S^2+0.0629874S+1.7184224)}$$

$K_2 = K'_2 K''_2$

$$t''_2(S) = \frac{(S^2+3.8915796)}{(S^2+0.0824723S+2.9460173)}$$

$Q'_2 = 20.8118785 = Q''_2$

$$t'_1(S) = \frac{(S^2+1.4499369)}{(S^2+0.1301475S+2.0117435)}$$

$K_1 = K'_1 K''_1$

$$t''_1(S) = \frac{(S^2+3.4915318)}{(S^2+0.1455614S+2.5164742)}$$

$Q'_1 = 10.8980921 = Q''_1$

$$t'_3(S) = \frac{(S^2+0.6216953)}{(S^2+0.0163357S+1.1659816)}$$

$K_3 = K'_3 K''_3$

$$t''_3(S) = \frac{(S^2+8.1430569)}{(S^2+0.0227449S+3.1327720)}$$

$Q'_3 = 77.8180702 = Q''_3$

Magnitude characteristics of the elliptic band-pass function is shown in Fig.4(a).

### Example 2

Using the method analogous to the band-pass case to find the optimum sequence for the band-reject function, we obtain the band-reject transfer function of n=12 [7],[8],[9].

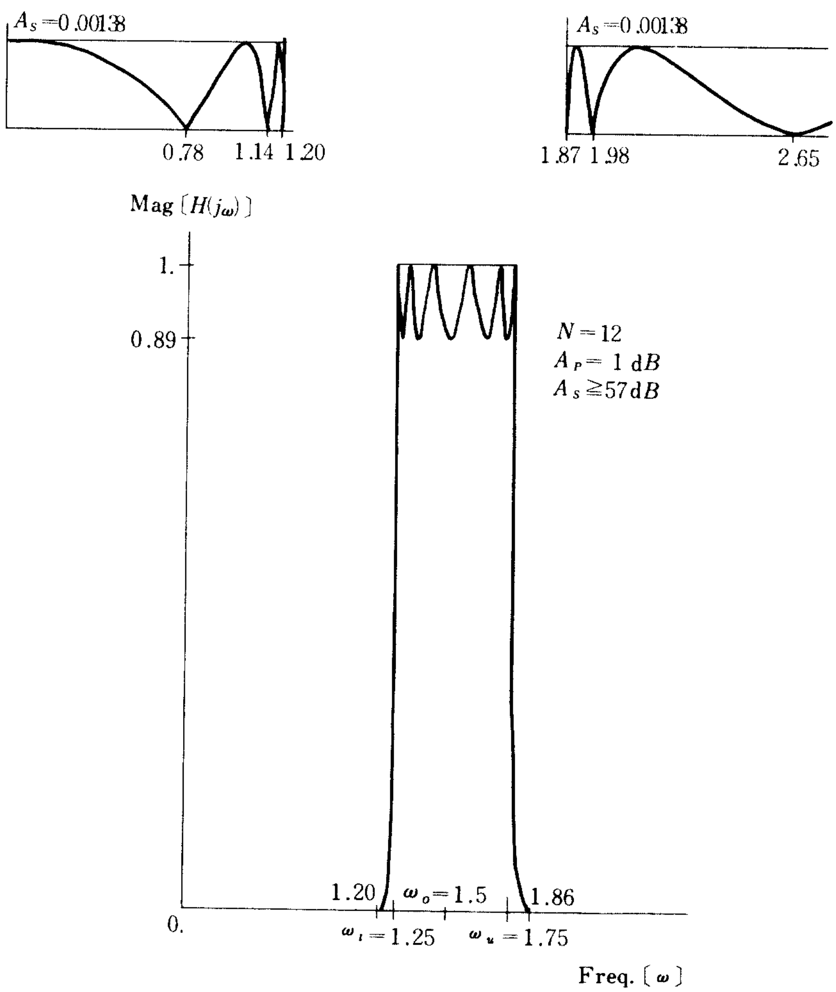
$$T(S) = K_2 t_2''(S)\, t_2'(S) \cdot K_1 t_1''(S)\, t_1'(S) \cdot K_3 t_3''(S)\, t_3'(S)$$
$$= K_2' t_2''(S) \cdot K_2'' t_2'(S) \cdot K_1' t_1''(S) \cdot K_1'' t_1'(S) \cdot K_3' t_3''(S) \cdot K_3'' t_3'(S)$$

where

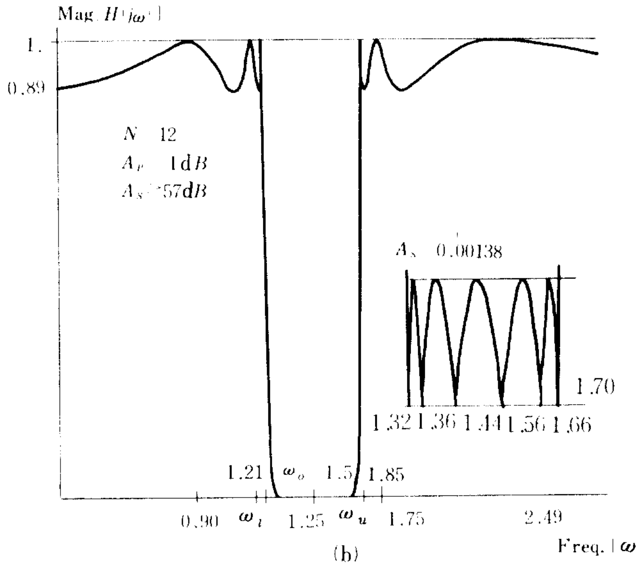
$$t_2''(S) = \frac{(S^2+2.7480273)}{(S^2+0.1281351S+3.3430624)}$$
$$t_2'(S) = \frac{(S^2+1.8422303)}{(S^2+0.9438618S+1.5143306)}$$
$$K_2 = K_2' K_2''$$
$$Q_2' = 14.2693480 = Q_2''$$

$$t_1''(S) = \frac{(S^2+2.8896022)}{(S^2+0.9438618S+4.1026964)}$$
$$t_1'(S) = \frac{(S^2+1.7519715)}{(S^2+0.5176320S+1.2339451)}$$
$$K_1 = K_1' K_1''$$
$$Q_1' = 2.1459830 = Q_1''$$

$$t_3''(S) = \frac{(S^2+2.4390635)}{(S^2+0.0228358S+3.1363850)}$$
$$t_3'(S) = \frac{(S^2+2.0755920)}{(S^2+0.0163821S+1.6141195)}$$
$$K_3 = K_3' K_3''$$
$$Q_3' = 77.5529734 = Q_3''$$




(a)




(b)

그림 4 [illegible]
Magnitude characteristics
(a) Elliptic 대역통과 특성 곡선
Elliptic Band-Pass.
(b) Elliptic 대역저지 특성 곡선
Elliptic Band-Reject.

Magnitude characteristics of the elliptic band-reject function is shown in Fig.4(b).

## V. CONCLUSION

A simple method of pole-zero pairing in the band-pass and band-reject case has been proposed for the cascade realization of elliptic functions which lead to the maximum dynamic range.

It is shown that the sequencing techniques can be developed with reference to the pole Q's in the P-P and B-R elliptic functions.

Two examples are provided to illustrate proposed methods. The first example in case of band-pass realization is the case in which the pole-zero pairing, biquad sequencing, and the gain distribution are all employed to optimize one of the most relevant performance measures. Viz., the dynamic range.

The second example in case of band-reject realization is conducted to enhance the dynamic range as much as possible.

The proposed high order band-pass and band-reject elliptic filter realization may be used not only for RC-active filters, but also for switched capacitor filters.

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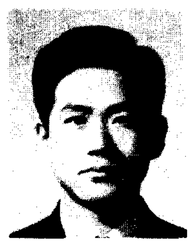

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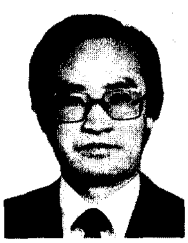

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