論 文

영상신호 2차원 코사인 변환계수의 분포근사화

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Distribution Approximation of the Two Dimensional Discrete Cosine Transform Coefficients of Image

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요 약 영상신호의 이차원 코사인 변환부호화에 있어서 변환계수의 분포근사화는 매우 중요하다. 그 이유는 분포양자화 시 분포함수를 잘못 가정하면 양자화잡음이 매우 커지기 때문이다. 본 논문에서는 일반화된 가우시안 분포함수를 이용하여 test를 행한 결과 AC변환계수들은 shape parameter가 0.6인 일반화된 가우시안 분포로 잘 근사화된다는 결과를 얻었다. 이차원코사인 변환부호화의 컴퓨터 시뮬레이션을 통해 Laplacian이나 Gaussian분포로의 근사화와 비교한 결과 shape parameter가 0.6인 일반화된 가우시안 분포로 근사화하는 경우 실험치와 이론치가 거의 일치하며 출력 신호대 잡음비도 가장 크게 나타났다.

ABSTRACT In two-dimensional discrete cosine transform(DCT) coding, the measurements of the distributions of the transform coefficients are important because a better approximation yields a smaller mean square distortion. This paper presents the results of distribution tests which indicate that the statistics of the AC coefficients are well approximated to a generalized Gaussian distribution whose shape parameter is 0.6. Furthermore, from a simulation of the DCT coding, it was shown that the above approximation yields a higher experimental SNR and a better agreement between theory and simulation than the Gaussian or Laplacian assumptions.

1. Introduction

In image coding system two dimensional discrete cosine transform(DCT) has been widely used because of its resemblance to the Karhunen-Loeve transform of image which are usually modelled as highly correlated first order Markov process. The unitary DCT are defined as follows.[1]

$$v_{kl} = \frac{2}{N} c_k c_l \sum_{i,j=0}^{N-1} u_{ij} \cos\frac{(2i+1)k\pi}{2N} \cos\frac{(2j+1)l\pi}{2N}, \quad (1)$$

$$\text{Where } c_u = \begin{cases} \frac{1}{\sqrt{2}}, & u=0 \\ 1, & \text{otherwise.} \end{cases} \quad (2)$$

Here $\{u_{ij},\ i,\ j=0,1,\cdots,(N-1)\}$ and $\{v_{kl},\ k,\ l=0,1,\cdots,(N-1)\}$ denote the input and transformed image arrays, respectively.

There have been several different assumptions

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on the distributions of the transform coefficients. An intuitive assumption is that the DC coefficient has a Rayleigh distribution since it is the sum of positive values, and that, based on the central limit theorem the other coefficients Gaussian distributions. But it is well known that the non-DC coefficients are close to Laplacian rather than to Gaussian from the previous results.(2)(3)

Recently, Reininger(4) performed a Kolmogorov-Smirnov test on the DCT coefficients in order to identify the distribution that best approximates the statistics of the DCT coefficients. In the tests, the Gaussian, Laplacian, Gamma, and Rayleigh distributions were considered. He reported that for many images the DC coefficient is best approximated by a Gaussian distribution and non-DC coefficients are best approximated by Laplacian distributions, and that by using Laplacian quantizers for the non-DC coefficients, the SNR of the reconstructed image can be improved as compared with the case of Gaussian quantizers.

In this paper, some tests are made on the non-DC coefficients to more precisely approximate the distributions with the generalized Gaussian distributions(6) varying the shape parameter. The distribution of DC coefficient is not considered because its effects on the overall performance of DCT coding is negligible. The results show that non-DC coefficients are well approximated by the generalized Gaussian distribution with shape parameter 0.6 rather than with shape parameter 1.0(Laplacian), and that the corresponding SNR is also improved as compared with the Laplacian assumption.

This paper is organized as follows. Section II describes the generalized Gaussian distribution. Section III describes the distribution test methods and how they were done, and section IV describes the results of the tests. In section V, comparisons between the theoretical and experimental block quantization errors for a two dimensional DCT coding are made for different assumptions on the distributions of the coefficients.

2. Generalized Gaussian Distribution

The generalized Gaussian distribution is a useful model describing a great variety of pdf's. It is defined by

$$p(\xi;\sigma,c)=\frac{c\,\eta(\sigma,c)}{2\Gamma(1/c)}\exp\{-[\eta(\sigma,c)\,|\xi|]^{c}\},\qquad(3a)$$

with

$$\eta(\sigma,c)=\frac{1}{\sigma}\left\{\frac{\Gamma(3/c)}{\Gamma(1/c)}\right\}^{1/2}\qquad(3b)$$

The shape of this pdf can be varied by means of the shape parameter c without affecting the variance σ^2. Here, c=1 yields the Laplacian and c=2 the Gaussian distribution. For c ∞ the pdf tends to the uniform distribution, whereas for c<1 there is a sharp peak at ξ=0. They are demonstrated in Fig. 1 for some values of c.

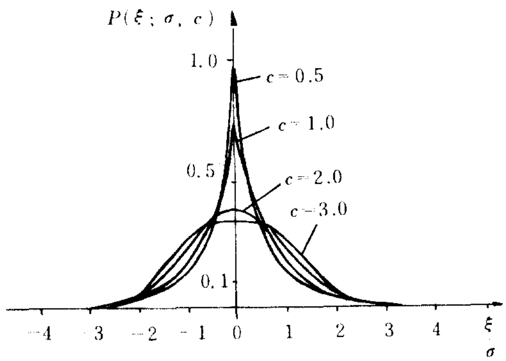

Fig. 1. Generalized Gaussian probability density function.

3. Goodness-of-Fit Tests for Non-DC Coefficients

A well known test for goodness of fit of distributions is the Kolmogorov-Smirnov(KS) test. For a given set of data $X=(x_1, x_2,\cdots, x_M)$, the KS test compares the sample distribution function

$F_X(\cdot)$ to a given distribution function $F(\cdot)$ which is an indefinite integral of Eq.(3). The sample distribution function is defined by

$$F_X(Z) = \begin{cases} 0, & z < x_{(1)} \\ \frac{n}{M}, & x_{(n)} \leqq z < x_{(n+1)}, \quad n = 1, 2, \cdots, M-1 \\ 1, & z \geqq x_{(M)} \end{cases} \tag{4}$$

where $x_{(n)}$, n = 1,2,..., M are the order statistics of the data X. The KS test statistic t_1 is then defined by

$$t_1 = \max_{i=1,2,\cdots,M} |F_X(x_i) - F(x_i)| \tag{5}$$

Another test used in this paper is as follows. The test statistic is defined by the integral of the absolute difference between the pdf of a given distribution p(x) and the normalized histogram of the data $p_X(x)$. Test statistic t_2 is written as

$$t_2 = \int_{-\infty}^{\infty} |p_X(x) - p(x)| \, dx \tag{6}$$

When testing the data against several distributions, the distribution that yields the smallest statistic is the best fit for the data.

The tests were performed to approximate the distributions of the 2-D DCT coefficients computed for the two image GIRL and COUPLE. These images have size 256x256 pels, with 8 bit grey levels, which were digitized in USC (University of Sourthern California). The transform block size was 16x16. The tests were performed on the ten high energy coefficients v_{01}, v_{02}, v_{03}, v_{10}, v_{11}, v_{12}, v_{13}, v_{20}, v_{21}, and v_{22}. The data for a given coefficient "ij" consisted of pel points $v_{ij}(k)$, k = 1,2,...,M, where the index k represents the position of the block in the image. For each transform coefficient the sample variance σ_{ij}^2 was calculated according to

$$\sigma_{ij}^2 = \frac{1}{M} \sum_{k=1}^{M} v_{ij}(k)^2 \tag{7}$$

It was assumed that the mean of the non-DC coefficients were zero, since, in image transform coding, the transmission of the mean of each coefficient is actually impossible and the zero mean assumption may be reasonable in concept. Furthermore the symmetry of the probability density was also assumed so that the tests were made only with the absolute values of the coefficients v_{ij}. The data were then tested against c=0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 2.0 generalized Gaussian distributions which have variances equal to the sample variances.

4. Test Results

Partial results of the tests for c=0.6, c=1.0 (Laplacian), and c=2.0(Gaussian) are shown in Table 1 and Table 2 for GIRL and COUPLE, respectively. From the tables it can be seen, for most coefficients, the statistics t_1 and t_2 are the smallests when c=0.6. For some coefficients such as v_{01} or v_{02}, the Laplacian distribution yields the smaller values. By the way, in image transform coding, a single distribu-

Table 1 Test statistics for GIRL image.

	t_1			t_2		
	c=0.6	c=1.0	c=2.0	c=0.6	c=1.0	c=2.0
v_{01}	0.162	0.086	0.208	0.518	0.418	0.537
v_{02}	0.125	0.052	0.163	0.449	0.400	0.525
v_{03}	0.104	0.077	0.183	0.412	0.434	0.561
v_{10}	0.066	0.189	0.275	0.311	0.477	0.636
v_{11}	0.054	0.166	0.260	0.402	0.463	0.632
v_{12}	0.075	0.154	0.225	0.405	0.436	0.554
v_{13}	0.040	0.163	0.257	0.292	0.431	0.598
v_{20}	0.058	0.190	0.283	0.439	0.552	0.726
v_{21}	0.072	0.161	0.262	0.393	0.499	0.599
v_{22}	0.066	0.149	0.233	0.433	0.542	0.659

Table 2 Test statistics for COUPLE image.

	t_1			t_2		
	c=0.6	c=1.0	c=2.0	c=0.6	c=1.0	c=2.0
v_{01}	0.047	0.118	0.241	0.382	0.456	0.638
v_{02}	0.081	0.134	0.225	0.425	0.527	0.683
v_{03}	0.053	0.149	0.272	0.349	0.475	0.674
v_{10}	0.085	0.182	0.264	0.458	0.558	0.680
v_{11}	0.056	0.185	0.308	0.301	0.481	0.694
v_{12}	0.062	0.177	0.298	0.459	0.500	0.691
v_{13}	0.120	0.249	0.372	0.383	0.559	0.775
v_{20}	0.125	0.198	0.267	0.525	0.570	0.659
v_{21}	0.092	0.229	0.342	0.381	0.538	0.747
v_{22}	0.083	0.212	0.335	0.402	0.518	0.730

tion has to be assumed for the simplicity of encoding and decoding. Moreover it is known that the quantizer mismatch effects for the shape parameter are smaller when the smaller shape parameter is chosen.[7] Therefore it can be said that the DCT non-DC coefficients have approximately c=0.6 generalized Gaussian distribution rather than the Laplacian distribution, and that c=0.6 assumption will yield larger output signal-to-noise ratio.

5. Transform Coding Simulation

To confirm the results of section IV, the block quantization of the transform coefficients was performed based on three different distribution assumptions for average coding rate of 1bit/pel. Here the variances of the coefficients were assumed to be known to the receiving part. For each assumption the optimum symmetric uniform and nonuniform quantizers were designed and used for all non-DC coefficients. DC coefficient was quantized uniformly over its range for simplicity. The quantizers for the non-DC coefficients were scaled to the sample variances of the coefficients for the image, and the bit allocation was determined by Pratt scheme[8] which is optimum when the variances of the coefficients are known.

The performances of the transform coding based on different distribution assumptions were measured theoretically and experimentally in terms of signal-to-noise ratio. They are shown in Table 3 and Table 4. In the tables the theoretical SNR is computed, for nonuniform quantization, from

$$\text{theoretical SNR} = \frac{\sum_{i,j=1}^{16} \sigma_{ij}^2 K(b_{ij}) \exp(-2 l_n 2\, b_{ij})}{\sum_{i,j=1}^{16} \sigma_{ij}^2} \qquad (8)$$

where $K(b_{ij})\exp(-2\ln 2\ b_{ij})$ is the distortion rate function for unit vaiance input. For uniform quantization similar expressions can be obtained.[9] The experimental SNR is given by

$$\text{experimental SNR} = \frac{\sum_{i,j=1}^{16} (v_{ij} - \hat{v}_{ij})^2}{\sum_{i,j=1}^{16} \sigma_{ij}^2} \qquad (9)$$

where $\hat{v}_{ij}$ denotes the reconstructed value or v_{ij}.

Notice that when it is assumed that the non-DC coefficients are Gaussian or Laplacian, the theoretical SNR's are about 2.5dB or 0.6 dB higher than the experimental SNR's, repectively. But when it is assumed that they are c=0.6 generalized Gaussian, the differences between the theoretical and experimental values are within 0.1dB. Furthermore, using c=0.6 generalized Gaussian quantization, the highest experimental SNR's have been resulted. The above statements are equally true for both uniform and nonuniform quantization.

Table 3 Theoretical and experimental performance(SNR) for different symmetric nonuniform quantizers (DCT, 1 bit/pel, block size=16).

image		c=0.6	c=1.0	c=2.0
GIRL	theory	31.52	31.97	32.70
	experiment	31.60	31.46	30.29
COUPLE	theory	33.30	33.80	34.62
	experiment	33.30	33.07	31.78

Table 4 Theoretical and experimental performance(SNR) for different symmetric uniform quantizers (DCT, 1 bit/pel, block size=16).

image		c=0.6	c=1.0	c=2.0
GIRL	theory	30.70	31.61	32.62
	experiment	30.89	30.76	29.35
COUPLE	theory	32.50	33.40	34.52
	experiment	32.43	32.25	30.97

6. Conclusions

The results shown in this paper indicate that the non-DC transform coefficients are better approximated by a generalized Gaussian distribution with a shape parameter 0.6 rather than the previous Laplacian assumption. Assuming that the non-DC coefficients are c=0.6 generalized Gaussian, a better agreement between theory and experiment and higher coding performance have been obtained, in both cases of uniform and non-uniform quantization.

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