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— Research Paper —

A Model of Quality and Capacity Variation

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Abstract

The problems of product quality selection and pricing are considered in congested and uncongested markets. In a congested market, such as a computer service market, product quality (the level of congestion) is partly a function of the amount of usage, which in turn depends on user choice. In an uncongested market, product qualities are set solely by providers. A model of quality and capacity variation is developed using a state equation description to represent user optimizing behavior. The model is used to study the problem of scarce resources among competing user demands through quality-dependent pricing.

1. Introduction

Selection of the products that are produced and sold is variously referred to as product differentiation, product selection, and monopolistic competition [21]. The classical theory of product differentiation has its origin in Chamberlin's work on monopolistic competition [6]. This theory treats different quality levels of a product as if they were different products. The problem with this approach is that there is no metric to determine the closeness of different products.

Houthakker [10] studies the quality problem by introducing qualities as separate variables directly into the consumer's utility function. In view of the importance of producer's product quality choices, relatively little analysis of this type has appeared in the literature until recently. Sweeney [23] formalizes the concept of housing quality in the context of consumer choice, while a survey by Schmalensee [20] examines the use of durability as a scalar quality measure. The "characteristics" framework developed by Lancaster [13] assumes that products themselves do not directly provide utility, but rather provide basic characteristics which consumers value.

Mussa and Rosen [16] consider a class of monopoly pricing problems involving a quality-differentiated spectrum of products of the same generic type. They show that consumers distribute themselves along the quality spectrum by a process of self-selection. Their analysis is limited to the case where each individual consumes one unit of the good. Our continuous demand case extends their work.

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Quality differentiation by a monopolist has been studied in several other contexts. A quality-dependent pricing model with positive demand externalities is developed and analyzed by Oren, Smith, and Wilson [18]. Salop [19] studies the monopolist's pricing problem in the presence of imperfect information. Imperfect discrimination thorugh user self-selection is also discussed in the literature on nonuniform pricing [9, 14, 15, 22].

Donaldson and Eaton [8], and Chiang and Spatt [5] study the problem of quality-dependent pricing in a framework with discrete user groups and discrete quality levels. Their work assumes that quality is not costly to the producer.

The theory of congested systems has primarily focused on the control of congestion through the pricing of services in nonmarket situations, such as highways or airports. An extensive literature review appears in Agnew [2]. Diamond [7] examines the problem of pricing congested facilities in the presence of consumption externalities in a general context. Agnew [2] studies the stability and transient behavior of a congestion-prone system and analyzes policies for the control of congestion using an optimal control model.

The model presented here allows user heterogeneity with respect to both value of time and reservation price. The model is used to propose a quality-dependent pricing scheme which may enhance the utilization of system capacity in the presence of congestion.

Two important assumptions are made throughout this paper. First, system capacity is assumed to be added in small increments at a constant unit capacity cost. In a market with congestion, system capacity (e. g., the number of servers) is measured by throughput capability. The level of congestion is an increasing function of the total amount of services provided and a decreasing function of the allocated system capacity. In the absence of congestition, system capacity can be regarded as a single input of the resource mix. The congestion function can thus be seen as the production function in uncongested markets. System capacity is assumed to be fixed in the short run. Second, users are distributed continuously over the range of user time values. This is a natural assumption with a large number of users and is simple to work with.

The system state equation is a consequence of optimizing behavior by users. The provider knows the distribution of user types over the range, but he does not know the identity of any particular user, so that each user faces the same price-quality schedule. Product choice is made through a process of user self-selection, given the price-quality schedule set by the provider.

2. Formulation of the Optimal Control Problem

The derivation of the system state equation from the user self-selection process follows an approach used by Mussa and Rosen [16] and extended by Agnew [1]. The individual user is assumed to be characterized by his time value and reservation price. Users are assumed to be continuously distributed with respect to value of time and reservation price. Assuming that congestion is a function of both the amount of serivce provided and the amount of capacity allocated, a capacity constraint is explicitly considered.

2. 1. User Optimizing Behavior as System Constraints

We assume that a user population is distributed continuously according to both their time values and reservation prices. Users with the same value of time may have different reservation prices. In general there is no correlation between the value of time and reservation price.

Each individual user with time value ω is assumed to maximize his surplus (reservation price minus user cost), given the price-quality schedule $p(t)$ set by the service provider:

$$\max_t \, [r - (p(t) + \omega t)] \tag{1}$$

where r represents the user's reservation price. It is important to note that price depends only on quality level, because a service provider cannot identify or discriminate among users either by their time values or their reservation prices. Differentiating Eq. (1) with respect to t yields the first-order necessary condition for surplus maximization (see Figure 1):

$$p'(t) = -\omega \tag{2}$$

where the prime denotes differentiation with respect to t. Equation (2) states that the optimal price-quality combination is the point where the marginal price of service quality equals the negative of the user's valuation of time.

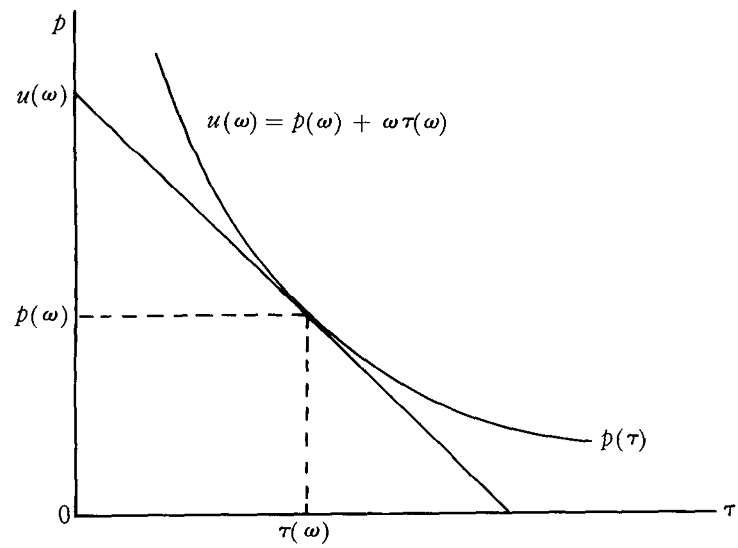

Fig. 1: The price-quality selection by users with time value ω, given a price-quality schedule $p(\tau)$.

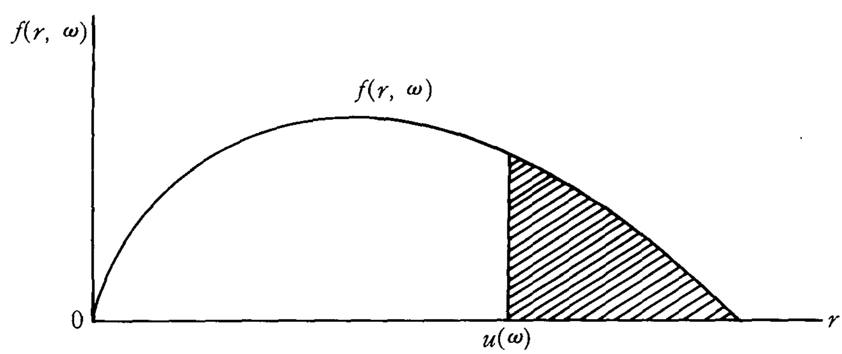

Fig. 2: The demand by users with time value ω, given a user cost $u(\omega)$.

Users with the same value of time choose the same price-quality combination and incur the same user cost, unless their reservation prices are lower than the minimum user cost for them. Let $f(r, \omega)$ represent the joint density of the number of users at reservation price r and user time value ω. Given user cost u, the demand density by users with time value ω, $q(u, \omega)$, can be obtained by integrating $f(r, \omega)$ over the range of reservation prices u, (see Figure 2):

$$q(u, \omega) = \int_{u}^{\infty} f(r, \omega)\, dr \tag{3}$$

Equation (3) states that the demand by users with time value ω consists of users whose reservation price is higher than the given minimum user cost. Note that $q(0, \omega)$ is the total number of users with time value ω. The size of the user population is $\int_{\omega_0}^{\omega_1} q(0, \omega)$ where we assume $\omega \in [\omega_0, \omega_1]$.

Differentiating Eq. (3) with respect to u yields

$$q_u(u, \omega) = -f(u, \omega) < 0 \tag{4}$$

which states that the number of users (i. e., the demand) increases as user cost decreases.

Having considered the user's optimizing behavior, we now examine the system constraints resulting from this behavior. Following Agnew [1], it is assumed that both price and user time required per unit of consumption have a one-to-one relationship with time value ω. Based on this assumption, we have :

$$u(\omega) = p(t(\omega)) + \omega \cdot t(\omega) \tag{5}$$

Differentiating both terms of Eq. (5) with respect to ω gives

$$u'(\omega) = p'(t) \cdot t'(\omega) + \omega \cdot t'(\omega) + t(\omega) \tag{6}$$

From the user's optimizing condition (2), we substitute $p'(t)$ for $-\omega$ in Eq. (6):

$$u'(\omega) = t(\omega) \tag{7}$$

which will be used as the system equation in the provider's optimization problem. Equation (7) states that the marginal increase in user cost with respect to user time value equals the user time incurred by users at equilibrium. Since $t(\omega)$ is nonnegative over the range of user time values, user cost increases with the value of time.

Figure 3 illustrates the typical curves of $p(t)$, $p(\omega)$, and $t(\omega)$ in $p-t-\omega$ space, and provides a geometrical interpretation of the users' choice processes.

2. 2. System Capacity Constraints

In the presence of congestion, the user time required to consume one unit of service has been assumed to be an increasing function of the amount of service provided. Capacity is assumed to be varied among users with different time values. Let $k(\omega)$ denote the capacity al-

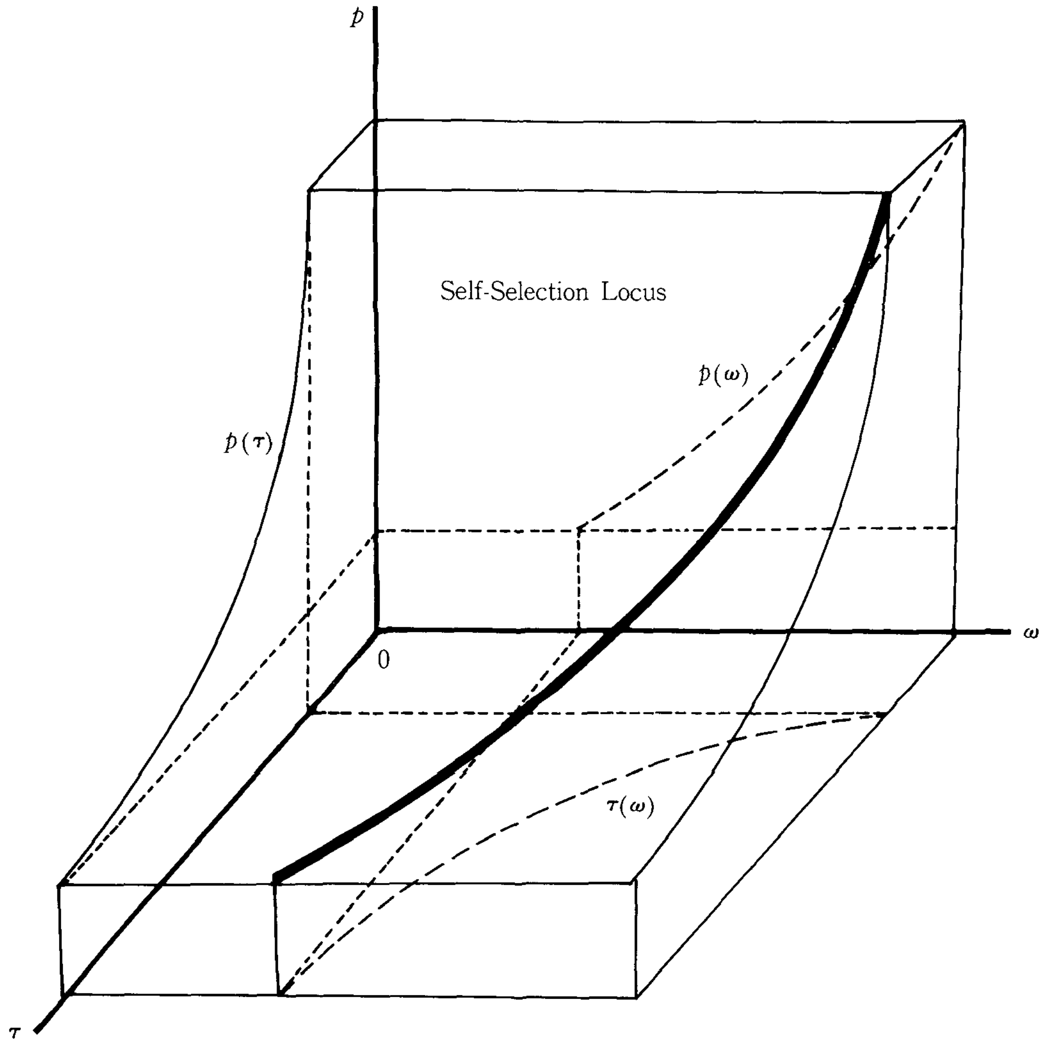

Fig. 3: Typical curves of $p(\tau)$, $p(\omega)$, and $\tau(\omega)$, and $\tau(\omega)$ in $p-\tau-\omega$ space.

located to users with time value ω. Define the utilization factor $\rho(\omega)$ by

$$\rho(\omega)=\frac{q(u(\omega),\ \omega)}{k(\omega)} \tag{8}$$

where k measures service throughput capability and has the same units as q. As in Agnew's work [1], the user time required to consume one unit of service is assumed to be only a function of the utilization factor, so that

$$\begin{aligned} t &= t(\rho) \\ t'(\rho) &> 0 \\ t''(\rho) &> 0 \end{aligned} \tag{9}$$

The strict convexity of $t(\rho)$ states that the user time required per unit of consumption increases with increased utilization factors, at an increasing rate. An example of a particular functional form of $t(\rho)$ is $t(\rho) = t_0/(1-\rho)$.

Assuming that Eq. (9) can be inverted:

$$\rho = \rho(t) \tag{10}$$

From Eq. (8), we substitute ρ for q/k in Eq. (10) and express the result in terms of k:

$$k(\omega) = \frac{1}{\rho(t(\omega))} \cdot q(u(\omega),\ \omega) \tag{11}$$

Defining $d(t)$ as $1/\rho(t)$, Eq. (11) can be rewritten as

$$k(\omega) = k(q,\ t) = d(t) \cdot q(u,\ \omega) \tag{12}$$

where d is the inverse of the utilization factor. It can be shown that $d(t)$ is a strictly decreasing convex function with respect to t, i. e. $d'(t) < 0$, $d''(t) > 0$, from the strict increasing concavity of $t(\rho)$.

Given a limited capacity K^o, the capacity allocated to all users must not be greater than the available capacity K^o:

$$\int_{\omega_0}^{\omega_1} k(\omega)\ d\omega \leqq K^o \tag{13}$$

which is called a system capacity constraint. Subsituting $k(\omega)$ from Eq. (12) into (13) yields

$$\int_{\omega_0}^{\omega_1} k(q(u(\omega),\ \omega),\ t(\omega))\ d\omega \leqq K^o \tag{14}$$

where $k(q,\ t) = d(t)\ q(u,\ \omega)$ in this analysis.

Since system capacity is fixed in the short run, provider behavior under limited capacity may be interpreted as short-run provider behavior. In reality, system capacity may also be constrained in the long run due to a physical constraint. For example, the space available for widening the highway may be limited. Thus formulation with a capacity constraint may be used to examine a provider behavior either in the short-run case or in the case where long-run capacity is constrained. The capacity-allocating behavior of a provider in the long run when capacity can be varied without limit can be examined simply by ignoring the capacity constraint.

2. 3. Production Costs

The total production costs of providing q units to users with time value ω depends on both the amount of services provided, q, and the amount of capacity allocated, k, i. e., $C(q, k)$. For analytic purposes, $C(q, k)$ can be represented as

$$C(q,\ k) = m \cdot q + n \cdot k \tag{15}$$

where m denotes the constant unit cost of use and n denotes the constant unit cost of allocated capacity. The capacity cost may include the expenditures for rent, depreciation, or maintenance of the system. We substitute k from Eq. (12) into Eq. (15):

$$C(q,\ k) = [m + n \cdot d(t)]\ q(u,\ \omega) \tag{16}$$

For notational convenience, define $c(t)$ by

$$c(t) = m + n \cdot d(t) \tag{17}$$

where $d(t)$ is again the inverse of the utilization factor. Since $d(t)$ is a strictly decreasing convex function of t, $c(t)$ is also a strictly decreasing convex function of t.

3. Profit Maximization

A producer is assumed to maximize his profit. His problem is to choose the price-quality schedule that maximizes his profits subject to the system constraints imposed by user self-selection process and capacity constraint.

Given the price-quality schedule $p(t)$ set by the service provider, and user's optimizing behavior discussed in Section 2.1, the revenue earned from users with time value ω can be expressed as

$$p(\omega) \int_{u(\omega)}^{\infty} f(r,\ \omega)\ dr = p(\omega)\ q(u(\omega),\ \omega) \tag{18}$$

where $u(\omega) = p(\omega) + \omega(\omega)$, $f(r,\ \omega)$ represents the joint density of users at reservation price r and time value ω, and $q(u(\omega),\ \omega)$ represents the demand for service quality $t(\omega)$ We recall that the total costs of producing q units with quality t was assumed to be the sum of variable cost and capacity cost.

$$C(q,\ k) = c(\mathrm{t}) \cdot q(u,\ \omega) \tag{19}$$

where $c(t) = m + n \cdot d(t)$, and m and n denote the constant unit cost of use and capacity, respectively. Since profit is defined to be the difference between revenue and production cost, the profit $\pi(\omega)$ from users with time value ω can be expressed as

$$\pi(\omega) = p(\omega)\ q(u(\omega),\ \omega) - c(t(\omega))\ q(u(\omega),\ \omega) \tag{20}$$

There, the total profit over all users can be obtained by integrating Eq. (20) in the interval $[\omega_0,\ \omega_1]$:

$$\pi[u(\omega),\ t(\omega),\ \omega_0,\ \omega_1]$$

$$= \int_{\omega_0}^{\omega_1} [p(\omega)\ q(u(\omega),\ (\omega\ ) - c(t(\omega))\ q(u(\omega),\ \omega)]\ d\omega \tag{21}$$

where $p(\omega) = u(\omega) - \omega t(\omega)$. Using the system constraints obtained in Section 5.2, the optimization problem faced by profit-maximizing service provider can be stated as

$$\max_{t(\omega)} \int_{\omega_0}^{\omega_1} [p(\omega)\ q(u(\omega),\ \omega) - c(t(\omega)\ q(u(\omega),\ \omega)]\ d\omega \tag{22}$$

subject to

$$u'(\omega) = t(\omega) \tag{23}$$

$$\int_{\omega_0}^{\omega_1} k(q(u(\omega), \omega), t(\omega))\, d\omega \leqq K^o \tag{24}$$

$$p(\omega) = u(\omega) - \omega\ t(\omega) \tag{25}$$

$$t'(\omega) \leqq 0 \tag{26}$$

$$t(\omega) \geqq 0,\ p(\omega) \geqq 0 \tag{27}$$

where $k(q, t) = d(t) \cdot q(u, \omega)$ and K^o is the amount of the available system capacity. Equation (23) describes the system state equation derived from the user behavior in Section 2.1. Constraint (24) is the system capacity constraint; a provider can use capacity only up to the available capacity K^0. Constraint (26) states that the service quality assigned to users increases with the value of user time. Finally, constraints (27) describes the usual nonnegativity constraints of price and service quality. Assuming an interior solution, the constraints (26) and (27) are satisfied at equilibrium. In his control problem, $u(\omega)$ represents a state variable and $t(\omega)$ represents a control variable.

We first consider the case in which the system capacity constraint (24) is not binding. This case corresponds to the case where an available capacity is unlimited with a constant unit capacity cost n. To apply Pontryagin's maximum principle, the Hamiltonian is formed by taking the integrand of the profit function and adjoining the state equation multiplied by the costate variable:

$$H(u, t, \lambda, \omega) = (u - \omega t - c(t))q(u, \omega) + \lambda t \tag{28}$$

where λ denotes a costate variable.

Let an optimal path be a pair of functions $[u(\omega), t(\omega)]$ defined for $\omega_0 \leqq \omega \leqq \omega_1$ which yield the maximum profit. The first necessary condition for an optimal path is for t to maximize the value of the Hamiltonian [3]. Differentiating Eq. (28) with respect to t gives the first necessary condition for an optimum:

$$\frac{\partial H}{\partial t} = (-\omega - c'(t))\ q(u, \omega) + \lambda = 0$$

or

$$\lambda = (\omega + c'(t))\ q(u, \omega) \tag{29}$$

where the subscript t indicates differentiation with respect to t. Taking the second derivative of Eq. (28) with respect to t gives

$$\frac{\partial^2 H}{\partial t^2} = -c''(t)\ q(u, \omega) < 0$$

because of the concavity of $c(t)$, so that the condition (29) is sufficient for the maximization of the Hamiltonian (28).

It can be shown that the costate variable $\lambda(\omega)$ represents the marginal valution of the state variable $u(\omega)$ [3, 12]:

$$\lambda(\omega) = \frac{\partial}{\partial u}\left[\int_{\omega}^{\omega_1} (u - vt - c(t))\ q(u, v)\ dv\right]. \tag{30}$$

In this case, $\lambda(\omega)$ represents the effect on profits of a small increase in the user cost $u(\omega)$, charged to the users with time value ω. The economic reasoning behind this representation is intuitive. Raising $u(\omega)$ by one unit implies that the user cost charged to all users with time values higher than ω increases by one unit. Thus Eq. (30) represents the intergral of the marginal profit due to a small increase in the user cost charged to all users with time values higher than ω. From a social welfare point of view, it is shown that the marginal cost of increasing quality should be equal to the negative of user time value i, e., $c'(t) = -\omega(\lambda(\omega) = 0)$ for all ω over the range of user time values [17]. In the profit maximization case, however, $\lambda(\omega)$ may be positive, negative, or even zero, depending on the distribution of user time values and the user demand characteristics, such as demand elasticity.

Let H^o denote the optimal value of the Hamiltonian, then the maximum principle states that along an optimal path H^o satisfies two differential equations:

$$u' = \frac{\partial H^o}{\partial \lambda}$$
$$\lambda' = -\frac{\partial H^o}{\partial u}$$

or

$$u' = t \tag{31}$$

$$\lambda' = -[q(u, \omega) + (u - \omega t - c(t))\, q_u(u, \omega)]. \tag{32}$$

Equation (31) is again a state equation which states that the rate of change in user cost with respect to user time value must equal the service quality assigned to the users with time value ω, at equilibrium. Equation (32) is the analogue of the equality of marginal revenue and marginal cost. In this case, $\lambda'(\omega)$ need not to be zero.

Finally, with the assumption that $u(\omega)$ is positive over the range of user time values, we have the following transversality conditions:

$$\lambda(\omega_0) = (\omega_0 + c'(t_0))\, q(u(\omega_0), \omega_0) = 0 \tag{33}$$
$$\lambda(\omega_1) = (\omega_1 + c'(t_1))\, p(u(\omega_1), \omega_1) = 0 \tag{34}$$

Since $u(\omega_0)$ and $u(\omega_1)$ are not specified, marginal profits at both ω_0 and ω_1 should be zero. Otherwise the profits can be improved by changing user costs, since the change in the optimal profit with respect to the state variable at each end point is the corresponding costate variable.

Differentiating Eq. (29) with respect to ω yields

$$\lambda' = (1 + c'' \cdot t')p + (\omega + c')\,(q_u \cdot u' + q_\omega). \tag{35}$$

Combining Eq. (32) and Eq. (35) yields the Euler equation [11]:

$$(1 + c'' \cdot t')q + (\omega + c')\,(q_u \cdot u' + q_\omega) + q + (u - \omega t - c)q_u = 0. \tag{36}$$

Define e_u as the demand elasticity with respect to user cost:

$$e_u = -\frac{u}{q} q_u$$

Rearranging Eq. (36) and using the above equation for demand elasticity, Eq. (36) can be expressed as

$$\frac{1}{e_u} = \frac{p - c(t)}{p + \omega t} + [(\frac{t}{u} + \frac{q_\omega}{q_u \cdot u})(\omega + c') - \frac{q}{q_u \cdot u}(1 + c'' \cdot u'')] \tag{37}$$

Before interpreting this equation, we consider the case in which users are homogeneous with respect to their values of time. If users are homogenous with respect to their values of time, we have the following relationship among price, quality, and demand elasticity:

$$\frac{p - c(t)}{p + \omega t} = \frac{1}{e_u} \tag{38}$$

or

$$\frac{c(t) + \omega t}{p + \omega t} = 1 - \frac{1}{e_u} \tag{39}$$

where $c(t)$ is the unit cost of producing of quality t products, ω is is the value of user time, and e_u is the demand elasticity with respect to user cost. The left term in Eq. (39) is seen as the ratio of social cost to user is inversely proportional to demand elasticity with respect to user cost. Equation (38) states that the ratio of markup price to user cost increases with the decrease of demand elasticity. This interpretation is the analogue of the classical monopoly pricing result which states that the ratio of markup price to price is inversely proportional to the demand elasticity with respect to price.

However, in (37) the problem is complicated by the fact that users with different time values interact and a profit-maximizing monopolist exploits the user self-selection process. The second term of the right side in Eq. (37) is seen as a result of user interactions. This term is shown to be zero when $c'(t)$ is equal to the negative of the user's time value ω. This implies the possibility of a case in which no user interactions occur. Depending on the distribution of user time values and reservation prices, the sign of this term may be negative, zero, or positive.

In many practical situations, there may not be unlimited capacity available due to physical limitations. For example, the spectrum allocated to land mobile radio service may be limited, because of the natural limitation of the total spectrum. Another important case is the case where system capacity is fixed in the short run. In order to obtain the necessary conditions when the capacity constraint is binding, the modified Hamiltonian is formed by adding the inner product of the Lagrange multiplier and the negative integrand of the system capacity constraint to the Hamiltonian defined in Eq. (28):

$$H(u, t, \lambda, \mu) = (u - \omega t - c(t))\, q(u, \omega) + \lambda t - \mu k(q, t) \tag{40}$$

where λ is a costate variable, and μ is the Lagrange multiplier associated with the capacity constraint (14). In the interest of simplicity, we assumed that the following relationship among capacity, quality, and the amount of service provided:

$$k(\omega) = d(t(\omega))\, q(u(\omega), \omega) \tag{41}$$

where $c(t(\omega)) = m + n \cdot d(t(\omega))$, and n is the constant unit capacity cost.

Substituting $k(\omega)$ from Eq. (41) into Eq. (40) and rearranging the result yields

$$H(u, t, \lambda, \mu) = [u - \omega t - (c(t) + \mu \cdot d(t))]\, q(u, \omega) + \lambda t. \tag{42}$$

Since the analysis is similar to the previous case, we suppress the details here. The resulting necessary conditions can be summarized as follows:

$$\frac{\partial H}{\partial t} = [-\omega - (c'(t) + \mu \cdot d'(t))]\ q(u, \omega) + \lambda = 0 \tag{43}$$

$$\lambda' = -\frac{\partial H}{\partial u} = -[q(u, \omega) + (u - \omega t - (c(t) + \mu \cdot d'(t)))\ q_u(u, \omega)] \tag{44}$$

$$\lambda(\omega_0) = [\omega_0 + (c'(t_0) + \mu \cdot d'(t_0))]\ q(u(\omega_0), \omega_0) = 0 \tag{45}$$

$$\lambda(\omega_1) = [\omega_1 + (c'(t_1) + \mu \cdot d'(t_1))]\ q(u(\omega_1), \omega_1) = 0 \tag{46}$$

$$\int_{\omega_0}^{\omega_1} d(t)\ q(u, \omega)\ d\omega = K^o \tag{47}$$

It can be shown that Lagrange multipliter μ measures the marginal profit associated with an increase in the capacity [4]:

$$\mu = \frac{\partial \pi}{\partial K^o} \tag{48}$$

In an economic sense, μ can be interpreted as the shadow price of the capacity. Differentating Eq. (43) with respect to ω and substituting λ' in Eq. (44) into this results yields the Euler equation. The Euler equation is the second-order differential equation with the unknown constant Lagrange multiplier μ, which can be solved by using the transversality conditions (45) and (46). The Lagrange multiplier μ can be determined from the capacity constraint (47). The detailed analysis will be presented with a specific example next.

4. An Example

To illustrate the results obtained in this paper we assume simple functional forms for user demand, unit congesting cost, and production cost as follows. Suppose that user demand is of the constant elasticity form

$$q(u, \omega) = a\,\omega^{\beta-\alpha} u \qquad \omega \in [\omega_0, \omega_1] \tag{49}$$

where a, α, and β are parameters. The parameter α measures the demand elasticity with respect to user cost and is assumed to be greater than one, while β represents the shape of the user distribution. The unit congestion function is assumed to be

$$t(q, k) = \left(\frac{q}{k}\right)^{1/\gamma} \tag{50}$$

where γ is a parameter which represents the degree of congestion. Finally, production cost is assumed to be only capacity cost:

$$C(q, k) = n \cdot k \tag{51}$$

where n is a constant unit capacity cost. Solving Eq. (50) for k gives

$$k(t, q) = t^{-\gamma}\, q \tag{52}$$

Substituting k from Eq. (52) into Eq. (51) yields

$$C(q, k) = n \cdot t^{-\gamma} q \tag{53}$$

which implies $c(t) = n \cdot t^{-\gamma} q$ in this example.

Form Eq. (52), the total capacity K can be expressed in terms of t and q:

$$K = \int_{\omega_0}^{\omega_1} t^{-\gamma} q \, d\omega \tag{54}$$

Given these specific but simple functional forms, we can obtain the profit maximizing values such as user cost, price, quality, total capacity allocated, and the price-quality schedule, in explicit forms [17].

Since the form of the profit maximizing user cost is complex and so little can be said in general, we consider a specific numerical example. The values of α, β, γ and n are assumed to be 2, 0, 1, and 1/4 respectively.

Now suppose that a user population is continuously distributed over the range of user time values [1, 2]. The user cost incurred by users with time value ω is then

$$u(\omega) = 1.9364(\omega + 0.4667)^{1/3}, \quad \omega \in [1, 2].$$

Differentiating $u(\omega)$ with respect to ω yields the quality assigned to users with time value:

$$t(\omega) = 0.6455(\omega + 0.4667)^{-2/3}, \quad \omega \in [1, 2].$$

In order to examine the effects of user time value distribution on the quality choice by a profit maximizer, we consider the case where a user population is distributed over the range of time values [1, 3]. In this case, we have the following user cost and the quality assignment:

$$u(\omega) = 2.0203(\omega + 0.5631)^{1/3}, \quad \omega \in [1, 3].$$
$$t(\omega) = 0.6735(\omega + 0.5631)^{-2/3}, \quad \omega \in [1, 3].$$

The profit maximizer offers a lower quality service to users with time values less than 2.

5. Conclusions

A theory of quality selection and optimal capacity allocation in both congested and uncongested markets has been developed. Resource utilization has been achieved through quality-dependent pricing schedules, which lead to users' self-selection process. It has been shown that capacity cost should be differently charged, depending on the relative utilization levels. Therefore, cost for capacity is recovered through all users in the market. In the case where capacity is limited, this model has been used to price capacity as shadow pricing.

In a monopolistic market, profit maximization has been considered. The problem of system capacity utilization through quality-dependent pricing has been formulated as an optimal control model and solved explicitly for a special case. As a whole, in the presence of congestion, market efficiency can be restored through nonlinear quality-dependent pricing.

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