

極配置에 의한 同期發電機의 電壓制御器 設計

論 文
34~12~3

Voltage Controller Design of Synchronous Generator by Pole Assignment

任 漢 錫*
(Han-Suck Yim)

요 약

長距離高力率 運轉되는 發電機는 재래의 電壓調整器(AVR)로서는 動態安定度가 低下하여 定格出力 運轉이 不可能한 경우가 있으며 특히 超速應勵磁方式의 AVR에서 이와 같은 現象이 經驗되고 있다. 本研究에서는 이러한 發電機의 動態安定度를 改善하기 위하여 極配置技法에 의하여 電壓 制御器를 設計하고 그 有用性を 보이기 위하여 컴퓨터시뮬레이션한 結果, 動態安定도와 制御性能이 在來式 電壓制御에 비하여 현저히 改善되었음이 確認되었다.

Abstract

A design of robust voltage controller for high speed excitation of synchronous machine was carried out by pole assignment techniques.

An affine map from characteristic polynomial coefficients to feedback parameters is formulated in order to place the system eigenvalues in the desired region.

The feedback parameters determined from linearized model are tested on nonlinear model subjecting it to small disturbances and system faults to show the effectiveness of the controller designed by the proposed technique.

The results obtained indicate that the controller presented improves the dynamic stability and system performances of conventionally controlled synchronous machine significantly.

1. Introduction

Power systems with long transmission lines and fast acting static excitations have made the stability and control problems more difficult than ever.

It is generally observed fact that the conventional feedback control actions of voltage

regulators of synchronous machines have the tendency of contributing to negative damping which can cause undamped modes of dynamic oscillations¹⁾.

Thus synchronous machines being transiently stable on the first swing do not always guarantee that they will return to their steady state operations in a well damped manner. New types of controllers are required to increase the damping of machines during transient periods but not affecting the regular functions of voltage control in steady state operation. Since voltage

*正 會 員 : 建國大 工大 電氣工學科 副教授
接受日字 : 1985年 8月 12日

control can affect the damping of machine oscillations by sensing machine terminal voltage, it seems possible that supplementary signals to the voltage regulator increase damping.

The general method of studying a synchronous generator and its control has been to consider the generator connected through a transmission line to an infinite bus²⁾.

Computer simulation of nonlinear system for a large disturbance model of a synchronous machine is one approach, and that of a linearized small disturbance model is another which is particularly suitable for the determination of proper control settings²⁾.

A great deal of advances has been achieved in this field using excitation control with various stabilizing signals.

Until recently, the design of power system stabilizer (PSS) was based on classical control techniques^{3,4)}.

Modern control theory provides powerful tools for controller design and has been applied to the power system stabilizer design problems.

Since the design problem is one of system stabilization, pole assignment techniques can be used effectively. Pole assignment method aims at placing the closed-loop poles at specified locations or regions, in order to improve the stability of the system.

For system stabilization, specification of unique closed-loop pole locations is not essential and it is enough if a sector region is specified for the closed loop eigenvalues^{5,6)}.

Although this idea is important from the practical point of view, it has received very little attention.

The method based on a technique that assigns the closed loop eigenvalues inside a desired sector in the left half complex plane is simple and may be used for the design of voltage controller with feedback from measurable quantities.

In contrast with proposed technique, optimal control techniques often have the drawback that they require trial and error basis. On the other hand, full state feedback is not always

practically feasible.

In this paper, a state space model of a synchronous machine is developed, subsequently a systematic procedure of the proposed design method is presented.

Finally, several response tests are conducted using digital simulation.

The objective of this study is to investigate the applicability of pole assignment techniques to the practical problem of robust voltage controller design for the improvement of synchronous machine stability.

2. Mathematical Model of the System

2.1 Nonlinear Model

System configuration of synchronous machine with its excitation connected through transmission line to infinite bus is shown in Figure 1.

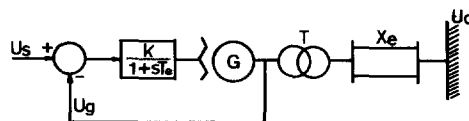


Fig. 1. Single machine-infinite bus system.

The controller is designed for a linearized machine model and subsequently it is tested on nonlinear model.

In order to simplify the analysis, rather simple model of excitation is preferable maintaining the inherent characteristics of it by the time constant and gain^{7,8)}.

Thus the transfer function of excitation is described as first order system by equation¹⁾.

$$G_e(s) = \frac{K}{1+sT_e} \quad (1)$$

For the system stabilization, supplementary signals from rotor angular velocity ω and rotor angle δ are introduced to the exciter, excluding field voltage in the synthesis of voltage control loop because of simplified excitation system. In the modelling, the followings are neglected:

- ohmic resistances

- magnetic saturation
- Speed governor effect

Figure 2 shows vector diagram of a synchronous machine-infinite bus system.

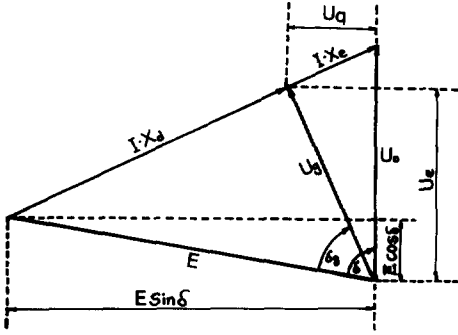


Fig. 2. Vector diagram of the system.

The generator terminal voltage expressed by equation (2) is obtained from this diagram.

$$U_g = \left[\left((E \sin \delta \frac{X_e}{X_d + X_e})^2 + (U_o - (U_o - E \cos \delta) \frac{X_e}{X_d + X_e})^2 \right)^{\frac{1}{2}} \right] \quad (2)$$

For a synchronous machine under constant field excitation, its dynamic system behavior can be described by relating the angular acceleration of the generator rotor to the power imposed on it:

$$\frac{2H}{\omega_0} \cdot \frac{d^2 \theta}{dt^2} = (P_m - P_e - P_d) \quad (3)$$

$$P_d = D \cdot \frac{d\delta}{dt} \quad (4)$$

Rotor angle disturbances oscillating about the operating point at the angular velocity ω_0 are assumed and hence the relationship in angular velocities is given by

$$\dot{\delta} = \omega = \dot{\theta} - \omega_0 \quad (5)$$

The generator output is represented as

$$P_e = \frac{U_o E}{X_d + X_e} \sin \delta \quad (6)$$

Generator field current is equal to machine internal voltage in per unit and formed from the two parts e_{fex} and e_{fina} ⁹⁾.

$$i_f = \frac{1}{1 + sT_d'} (e_{fex} + e_{fina}) \quad (7)$$

where

$$T_d' = \frac{X_d' + X_e}{X_d + X_e} T_d'$$

$$e_{fex} = \frac{1}{1 + sT_e} U_e$$

$$e_{fina} = \frac{X_d - X_d'}{X_d + X_e} U_o T_d' \sin \delta \cdot \omega$$

$$U_e = k_2 (U_s - U_g) - k_3 \omega - k_4 \delta$$

Figure 3 represents equations (1) – (7) in the form of block diagram.

2.2 Linear Model

A linearized generator model of small motion about the operating point is very useful for analyzing oscillatory stability problems. The small perturbation model was suggested already by deMollo, Concordia and others for synchronizing and damping torque analysis ^{3), 10)}. The system is represented on the common basis with nonlinear model and linearized about the initial operating points.

Referring to equations (1)–(7) and vector diagram, Figure 2, the system equations are represented as

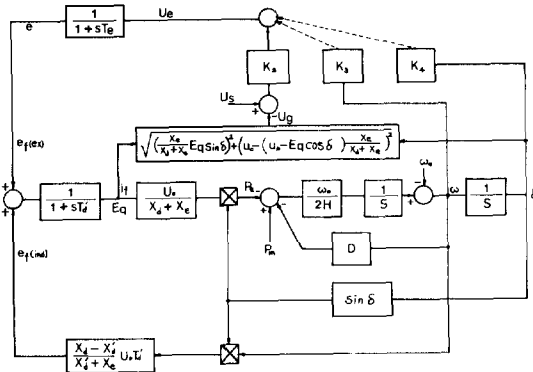


Fig. 3. Nonlinear system block diagram.

$$\frac{2H}{\omega_0} \cdot \frac{d^2 \Delta \theta}{dt^2} = (\Delta P_m - \Delta P_e - \Delta P_v) \quad (8)$$

$$\Delta P_v = D \frac{d \Delta \delta}{dt} \quad (9)$$

$$\Delta P_e = \frac{U_0 \sin \delta_0}{X_d + X_e} \Delta E_q + \frac{U_0 E_{q0} \cos \delta_0}{X_d + X_e} \Delta \delta \quad (10)$$

$$\Delta \dot{\delta} = \Delta \omega = \Delta \dot{\theta} \quad (11)$$

$$\Delta U_g = \frac{X_e \cos \delta_g}{X_d + X_e} \Delta E_q + \frac{X_e E_{q0} \sin \delta_g}{X_d + X_e} \Delta \delta \quad (12)$$

$$\Delta i_r = \frac{1}{1 + s T_d'} (\Delta e_{r,ex} + \Delta e_{r,ind}) \quad (13)$$

where

$$\Delta e_{r,ex} = \frac{1}{1 + s T_e} \Delta U_e$$

$$\Delta e_{r,ind} = \frac{X_d - X_d'}{X_d' + X_e} U_0 T_d' \sin \delta_0 \Delta \omega$$

$$\Delta U_e = k_2 (\Delta U_s - \Delta U_g) - k_3 \Delta \omega - k_4 \Delta \delta$$

$$K_e (\Delta U_s - K_6 \Delta E - K_5 \Delta \delta) \frac{1}{1 + s T_e} = \Delta e \quad (14)$$

$$(\Delta e + K_4 \Delta \omega) \frac{1}{1 + s T_d'} = \Delta E \quad (15)$$

$$(-K_2 \Delta E - D \Delta \omega - K_1 \Delta \delta) K_3 = \Delta \dot{\omega} \quad (16)$$

$$\Delta \omega = \Delta \dot{\delta} \quad (17)$$

where

$$K_1 = \frac{E \cdot U_0}{X_d + X_e} \cos \delta$$

$$K_2 = \frac{U_0}{X_d + X_e} \sin \delta$$

$$K_3 = \frac{\omega_0}{2H}$$

$$K_4 = \frac{X_d - X_d'}{X_d' + X_e} U_0 T_d' \sin \delta$$

$$K_5 = \frac{-X_e}{X_d + X_e} E \sin \delta_g$$

$$K_6 = \frac{X_e}{X_d + X_e} \cos \delta_g$$

The conventional voltage control is pure voltage regulation which derives feedback signal only from machine terminal voltage.

By designating Δe , ΔE , $\Delta \omega$ and $\Delta \delta$ as state variables, the above equations can be arranged as

$$\Delta \dot{e} = \begin{bmatrix} -\frac{1}{T_e} & 0 & 0 & 0 \end{bmatrix} \cdot X \quad (18A)$$

$$\Delta \dot{E} = \begin{bmatrix} \frac{1}{T_d'} & -\frac{1}{T_d'} & \frac{K_4}{T_d'} & 0 \end{bmatrix} \cdot X \quad (18B)$$

$$\Delta \dot{\omega} = \begin{bmatrix} 0 & -K_2 \cdot K_3 & -DK_3 & -K_1 \cdot K_3 \end{bmatrix} \cdot X \quad (18C)$$

$$\Delta \dot{\delta} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \cdot X \quad (18D)$$

$$U = K_e \left[\Delta U_s - [0 \ K_6 \ 0 \ K_5] \cdot X \right] \quad (18E)$$

$$B = \begin{bmatrix} \frac{1}{T_e} & 0 & 0 & 0 \end{bmatrix}^T \quad (18F)$$

and in the desired state space form

$$\dot{X} = AX + BU \quad (19)$$

where $X = \begin{bmatrix} \Delta e & \Delta E & \Delta \omega & \Delta \delta \end{bmatrix}^T$

$$A = \begin{bmatrix} -\frac{1}{T_e} & 0 & 0 & 0 \\ \frac{1}{T_d'} & -\frac{1}{T_d'} & \frac{K_4}{T_d'} & 0 \\ 0 & -K_2 \cdot K_3 & -DK_3 & -K_1 \cdot K_3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Selecting ΔU_g , $\Delta \omega$ and $\Delta \delta$ as output, the output vector Y is denoted by

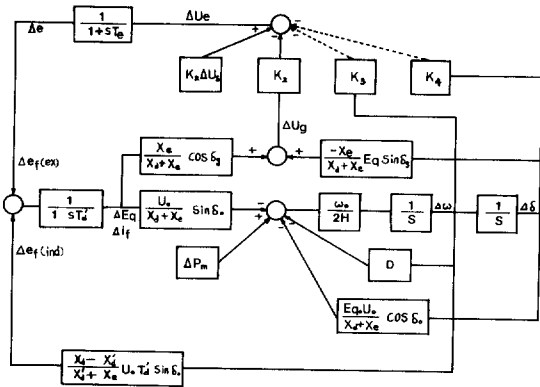


Fig. 4. Linearized system block diagram.

Figure 4 represents equations (8)–(13) in the form of block diagram.

For controller design, two initial operating states (normal and abnormal) and typical system data are selected.

The data given in Appendix 1 are referred to Appendix D, power system control and design by P.M. Anderson and A.A. Foud¹⁾.

2.3 State Space Representation

From the linear model, the system equations for conventional voltage control are described by

$$Y = \begin{bmatrix} \Delta U_g \\ \Delta \omega \\ \Delta \delta \end{bmatrix} = \begin{bmatrix} 0 & K_6 & 0 & K_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot X = C \cdot X \quad (20)$$

The constant matrices A and C are given in Appendix 2 for the study case.

The effect of governor is neglected in the model because of its large time constant.

This is normal approximation for the representation of synchronous machine with fast acting exciter.

If governor effect is to be considered for instance, the rank of control vector becomes higher.

3. Voltage Controller Design

3.1 Pole Placement Formulation

To investigate the effects of exciter gain for both operating conditions on the system stability of conventional voltage control, the selected initial operating states are applied to represent root locus.

Figure 5 and 6 represent the excursion of system roots as exciter gain is increased from 0 to 150.

The system roots of abnormal operation migrate to the unstable side of S-plane already at the low gain of 45 p.u.. Some measures have to be taken to stabilize this system.

Pole placement formulation of an affinity from the space of characteristic polynomial coefficients to the controller parameter space allows determining controller parameter region which places all eigenvalues in the desired region¹¹.

Introducing constant state feedback of the form described by the equation (21) to the state equation (19)

$$U = K \left[U_s - kX \right] \quad (21)$$

where K is control gain, k is feedback vector and U_s is reference input, the closed loop system

equation is given by

$$\dot{X} = [A - KBk]X + KB U_s \quad (22)$$

Some elements of k may be given, e.g., they may be zero for output feedback.

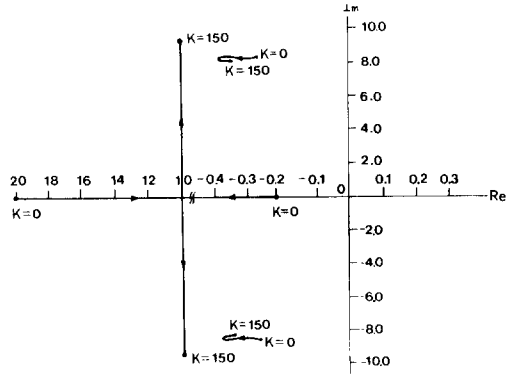


Fig. 5. Root locus of normal operation.

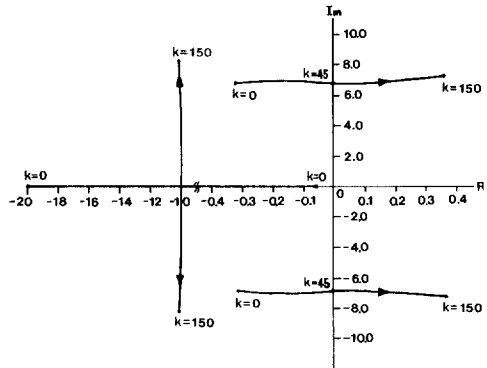


Fig. 6. Root locus of abnormal operation.

The remaining elements constitute the free design parameters. To formulate the relationship between the system eigenvalues and coefficients of characteristic polynomial, the phase variable form of control system is required.

The phase variable representation of the open loop described by the transfer function

$$G(s) = \frac{c_1 + c_2 s + \dots + c_m s^{m-1}}{a_1 + a_2 s + a_3 s^2 + \dots + s^n} \quad (23)$$

takes the form¹²⁾

$$\dot{\bar{X}} = \bar{A} \bar{X} + \bar{B} U \tag{24}$$

$$Y = \bar{C} \bar{X} \tag{25}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ -a_1 & -a_2 & -a_3 & \dots & \dots & -a_n \end{bmatrix}$$

$$\bar{B} = [0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 1]^T$$

$$\bar{C} = [c_1 \quad c_2 \quad \dots \quad c_m \quad 0 \quad \dots \quad 0]$$

Thus the system transfer function can be determined from the phase variable representation by simple inspection of above relations.

In terms of the closed loop, the task may be similarly accomplished. If the closed loop expression for the control is given by

$$U = K [U_s - \bar{k} \bar{X}] \tag{26}$$

where \bar{k} is the feedback vector in terms of phase variables, then the system representation becomes

$$\dot{\bar{X}} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 1 \\ -(a_1 + K\bar{k}_1) & -(a_2 + K\bar{k}_2) & \dots & \dots & -(a_n + K\bar{k}_n) \end{bmatrix} \bar{X} + K \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} U_s \tag{27}$$

$$Y = [c_1 \quad c_2 \quad \dots \quad c_m \quad 0 \quad \dots \quad 0] \bar{X} \tag{28}$$

where $\bar{k}_i (i=1,2,\dots,n)$ is the element of \bar{k} .

Thus the closed loop transfer function is given by

$$\frac{Y(s)}{U_s(s)} = \frac{K(c_1 + c_2 s + \dots + c_m s^{m-1})}{(a_1 + K\bar{k}_1) + (a_2 + K\bar{k}_2)S + \dots + (a_n + K\bar{k}_n) s^{n-1} + s^n} \tag{29}$$

The denominator is the characteristic polynomial of the closed loop system and its coefficients are represented by the simple equation

$$a k_i = a_i + K \bar{k}_i \quad i = 1, 2, \dots, n \tag{30}$$

The characteristic polynomial of the closed loop system can be related to preassigned eigen-

values S_i 's:

$$P(s) = a_{k_1} + a_{k_2} s + a_{k_3} s^2 + \dots + a_{k_n} s^{n-1} + s^n = \prod_{i=1}^n (s - s_i) \tag{31}$$

If a_i 's and a_{k_i} 's are known, then \bar{k}_i for preassigned poles can be determined from equations (30) and (31).

The elements of \bar{k}_i are the coordinates of a parameter space in which the design is performed.

3.2 Phase Variable Representation

It is necessary to transform the system to phase variable form since most systems are not naturally described in phase variables. The feedback parameters determined in phase variables have to be transformed back to the original state variable representation for practical use.

Transforming the system to phase variables is possible if the system is controllable.

Controllability may be determined by the test, which states that the system expressed in equation (32) is controllable if and only if the n by n controllability matrix

$$M_c = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B] \tag{32}$$

is nonsingular, i.e. $\text{Det} [M_c] \neq 0^{18}$.

The transformation matrix is related to phase variable form as described in equations (33A)–(33E).

$$\bar{X} = T^{-1} X \tag{33A}$$

$$\bar{A} = T^{-1} A T \tag{33B}$$

$$\bar{B} = T^{-1} B \tag{33C}$$

$$\bar{C} = C T \tag{33D}$$

$$\bar{k} = k T \tag{33E}$$

The transformation matrix is determined using controllability matrix:

$$T^{-1} = d \begin{bmatrix} M_c^{-1} I \\ M_c^{-1} A \\ M_c^{-1} A^2 \\ \vdots \\ M_c^{-1} A^{n-1} \end{bmatrix} \tag{34A}$$

$$T=[T^{-1}]^{-1} \tag{34B}$$

where $d=[0 \ 0 \ \dots \ 0 \ 1]$,

I is identity matrix.

For fourth order system, the following equations are obtained from equations (30) and (31).

$$a_{k4}=a_4+\bar{k}_4=-(s_1+s_2+s_3+s_4) \tag{35A}$$

$$a_{k3}=a_3+\bar{k}_3=(s_1s_2+s_3s_4)+(s_1+s_2)(s_3+s_4) \tag{35B}$$

$$a_{k2}=a_2+\bar{k}_2=-((s_1+s_2)s_3s_4+(s_3+s_4)(s_1s_2)) \tag{35C}$$

$$a_{k1}=a_1+\bar{k}_1=(s_1s_2s_3s_4) \tag{35D}$$

The exciter gain K is selected as unity and it does not lose generality. Referring to equation (20) and Figure 4, the feedback vector for output feedback is represented by

$$k=[0 \ K_6k_2 \ k_3 \ K_5k_2+k_4] \tag{36}$$

The transformation matrices for the system are given in Appendix 3 and have the general form:

$$T=\begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & 0 \\ 0 & T_{32} & 0 & 0 \\ T_{41} & 0 & 0 & 0 \end{pmatrix} \tag{37}$$

From equations (33E) and (35A)–(37), the elements of the transformed feedback vector are given by

$$k_1=T_{21}K_6k_2+T_{41}K_5k_2+T_{41}k_4=a_{k1}-a_1 \tag{38A}$$

$$\bar{k}_2=T_{22}K_6k_2+T_{32}k_3=a_{k2}-a_2 \tag{38B}$$

$$\bar{k}_3=T_{23}K_6k_2=a_{k3}-a_3 \tag{38C}$$

$$\bar{k}_4=0=a_{k4}-a_4 \tag{38D}$$

Transformed matrices and vectors are given in Appendix 3.

3.3 Feedback Parameter Determination

The design objective is to find feedback parameters which shift the closed loop eigenvalues into a sector defined by sloping line S

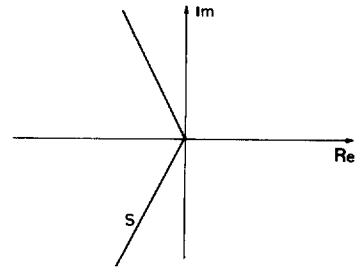


Fig. 7. Sector region for system eigenvalue.

as shown in Figure 7.

Since the control system is not full state feedback, preassigning all eigenvalues is not possible.

If some of eigenvalues are assigned, then the rest of them has to be checked if it lies inside the sector.

Let a conjugate complex pair of eigenvalues be assigned, then the rest may be either complex pair or two real eigenvalues.

Assuming the case of two conjugate complex pairs,

$$s_1=\alpha_1+j\beta_1 \tag{39A}$$

$$s_2=\alpha_2+j\beta_2 \tag{39B}$$

$$s_3=\alpha_3+j\beta_3 \tag{39C}$$

$$s_4=\alpha_4+j\beta_4 \tag{39D}$$

where $\alpha_1=\alpha_2, \alpha_3=\alpha_4, \beta_1=-\beta_2, \beta_3=-\beta_4$, the following relations are obtained by substituting the sector boundary condition $S=\left|\frac{\beta_i}{\alpha_i}\right|$ in equations (35A)–(35D).

$$a_{k1}=\alpha_1^2(1+s^2)\alpha_3^2(1+s^2)=d_1k_2+d_5k_4+a_1 \tag{40A}$$

$$a_{k2}=-\left(2\alpha_1\alpha_3^2(1+s^2)+2\alpha_3\alpha_1^2(1+s^2)\right) \\ =d_2k_2+d_3k_3+a_2 \tag{40B}$$

$$a_{k3}=\alpha_1^2(1+s^2)+\alpha_3^2(1+s^2)+(2\alpha_1)(2\alpha_3) \\ =d_1k_2+a_3 \tag{40C}$$

$$a_{k4}=-\left(2\alpha_1+2\alpha_3\right)=a_4 \tag{40D}$$

where

$$d_1=T_{23}K_6$$

$$d_2=T_{22}K_6$$

$$d_3=T_{32}$$

$$d_4=T_{21}K_6+T_{41}K_5$$

$$d_5=T_{41}$$

From equations (40A)–(40D), the feedback parameters k_2 and k_3 are obtained as functions of α_1 and S .

$$k_2 = \frac{\alpha_1(1+s^2) + (d_5 k_4 + a_1)/\alpha_1^2(1+s^2)}{d_1 - d_4/\alpha_1^2(1+s^2)} - \frac{2\alpha_1(2\alpha_1 + a_4) - a_3}{\alpha_1^2(1+s^2)} \quad (41)$$

$$k_3 = \frac{-2\alpha_1(d_4 k_2 + d_5 k_4 + a_1)/\alpha_1^2(1+s^2)}{d_3} + \frac{(2\alpha_1 + a_4)\alpha_1^2(1+s^2) - d_2 k_2 - a_2}{\alpha_1^2(1+s^2)} \quad (42)$$

The pair of real eigenvalue remainders ($S_3 = \alpha_3, S_4 = \alpha_4$) results in the same expressions for k_2 and k_3 as equations (41) and (42).

It is worth noting that the sum of eigenvalues is constant and equal to a_4 as shown in equation (40 D).

Thus the variation of α_1 from zero to $-a_4/2$ is sufficient for pole assignment.

In the procedure of determining k_2 and k_3 regions, k_4 is kept constant so that the eigenvalue locations may not be influenced from it.

The result for the slope $s=3$ is shown in Figure 8. Two dominant regions are available: One of them is the combination of small value feedback parameters, the other is that of large values.

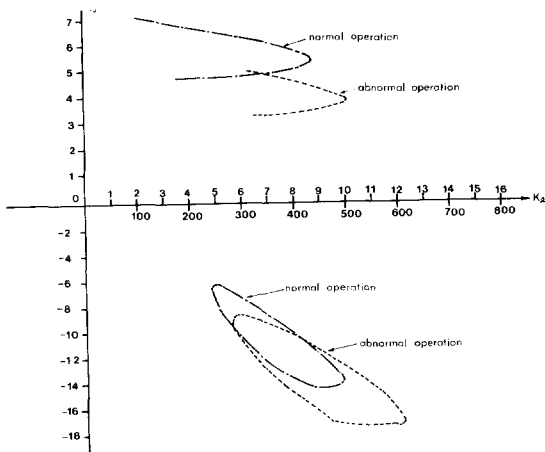


Fig. 8. Feedback parameter regions for $S=3$.

4. System Performance

4.1 Feedback Parameter Selection

The major function of a voltage regulator is to maintain machine terminal voltage at a pre-set value, regardless of changing requirements of excitation due to changes in load and other system conditions.

Secondly fast response is desirable in compromise with system performances.

To meet these requirements, a high value controller gain is preferable.

The gain values used in actual systems are typically 200-400 per unit¹⁴.

The region of high value feedback parameters agrees with these typical values of controller gain.

Since for low value combination the common region of both initial operating conditions is very small as seen in Figure 8, little flexibility is allowed in choosing parameters.

For performance tests the feedback parameters are chosen from the both regions as

$$k_2 = 6.5, k_3 = 5 \text{ for low value region}$$

$$k_2 = 400, k_3 = -12 \text{ for high value region}$$

4.2 Steady State Value

It is usually desired to determine the steady state values of system responses.

Assuming the controlled system is stable, the final values can be determined from the relations:

$$\dot{\bar{X}}_{ss} = \bar{X}_{ss} = 0$$

$$\bar{X}_{ss} = \bar{A} \bar{X}_{ss} + \bar{B} r = 0$$

$$\bar{X}_{ss} = -\bar{A}^{-1} \bar{B} r$$

$$= \begin{bmatrix} 1 \\ a_{k1} \\ 0 \\ 0 \\ 0 \end{bmatrix} r$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -a_{k1} & -a_{k2} & -a_{k3} & -a_{k4} \end{bmatrix}$$

$$\begin{aligned}
 B^T &= [0 \quad 0 \quad 0 \quad 1] \\
 Y_\infty &= \bar{C} \bar{X}_\infty = -\bar{C} \bar{A}^{-1} \bar{B} r \\
 &= \begin{bmatrix} \Delta u_{K^*} \\ \Delta \omega_\infty \\ \Delta \delta_\infty \end{bmatrix} = \begin{bmatrix} \frac{c_{11}}{a_{k1}} \\ 0 \\ \frac{c_{31}}{a_{k1}} \end{bmatrix} r = \begin{bmatrix} \frac{c_{11} \cdot k_2 \cdot \Delta U_s}{a_1 + d_4 k_2 + d_5 k_4} \\ 0 \\ \frac{c_{31} \cdot k_2 \cdot \Delta U_s}{a_1 + d_4 k_2 + d_5 k_4} \end{bmatrix} \quad (43)
 \end{aligned}$$

where

$$\hat{C} = CT = \begin{bmatrix} K_6 T_{21} + K_5 T_{41} & K_6 T_{22} & K_6 T_{23} & 0 \\ 0 & T_{32} & 0 & 0 \\ -T_{41} & 0 & 0 & 0 \end{bmatrix}$$

Since the relations $\bar{C}_{11} = K_6 T_{21} + K_5 T_{41}$ and $Cu = d_4$ hold, upon examination of equation (43), one notes that the control system can provide zero steady state terminal voltage error by simply adjusting k_4 .

However maintaining zero terminal voltage error is not so important that small value of k_4 is satisfactory in reducing the error.

The calculated steady state values of the system for 0.1 p.u control input are:

$$Y_\infty = \begin{bmatrix} 0.1437 \\ 0 \\ -0.329 \end{bmatrix} \begin{matrix} \text{normal operation} \\ k_2 = 6.5, k_3 = 5 \end{matrix}$$

$$Y_\infty = \begin{bmatrix} 0.1005 \\ 0 \\ -0.23 \end{bmatrix} \begin{matrix} \text{normal operation} \\ k_2 = 400, k_3 = -12 \end{matrix}$$

$$Y_\infty = \begin{bmatrix} 3.058 \\ 0 \\ -8.69 \end{bmatrix} \begin{matrix} \text{abnormal operation} \\ k_2 = 6.5, k_3 = 5 \end{matrix}$$

$$Y_\infty = \begin{bmatrix} 0.1016 \\ 0 \\ -0.289 \end{bmatrix} \begin{matrix} \text{abnormal operation} \\ k_2 = 400, k_3 = -12 \end{matrix}$$

From these results, it is clear that the small value combination of feedback parameters is not applicable.

4.3 Performance Tests by Simulation

To investigate the effectiveness of designed controller, simulations are carried out for both sets of feedback parameters by subjecting the nonlinear system model to distinct type of disturbances:

- disturbance A: 0.1 Us step disturbance

- disturbance B: 0.1 Pm step disturbance of 0.1 sec. duration
- disturbance C: interruption of one of double circuits ($X_e = 0.35 \rightarrow 0.55$)
- disturbance D: three phase fault of 0.2 sec duration applied on the infinite bus

The parameter k_4 is kept constant as 2.5 for both parameter combination.

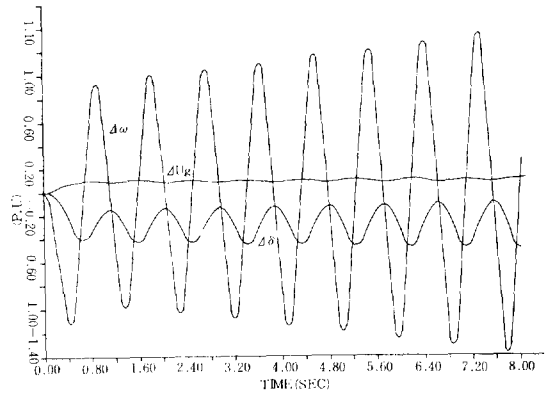


Fig. 9. System response to $\Delta U_s = 0.1$ p.u (Abnormal operation, $k_e = 50$)

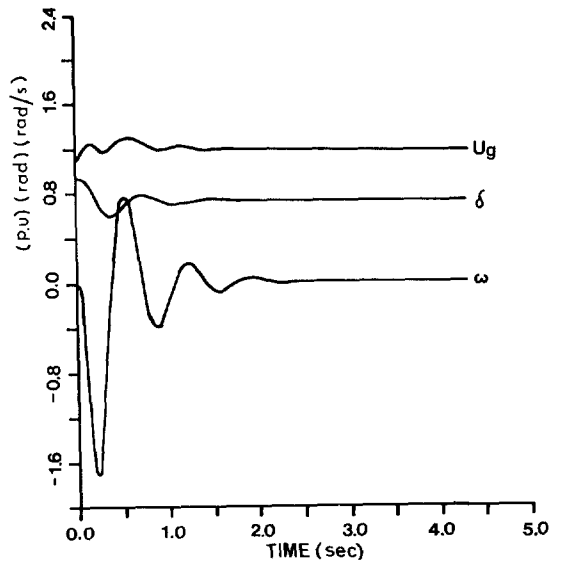


Fig. 10. Disturbance A (normal operation $k_2 = 400, k_3 = -12$)

The initial machine variables of nonlinear model are determined from the following four equations.

$$(U_s - U_g) k_2 - \delta k_4 = e_f = E$$

$$P_e = P_m = \frac{E \cdot U_0}{X_d + X_e} \sin \delta$$

$$U_g = \left[\left(E \sin \delta \frac{X_e}{X_d + X_e} \right)^2 + \left(U_0 - (U_0 - E \cos \delta) \frac{X_e}{X_d + X_e} \right)^2 \right]^{1/2}$$

$$Q = \frac{U_0 (E \cos \delta - U_0)}{X_d + X_e}$$

The test results are summarized in Figures 9 through 15. The system was oscillatory or unstable without stabilizing signals for both initial operating conditions as shown previously. The low value feedback does not bring the system completely stable for the disturbances.

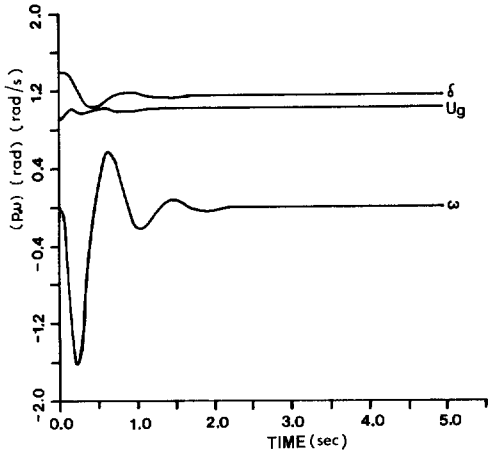


Fig. 11. Disturbance A (abnormal operation $k_2=400, k_3=-12$)

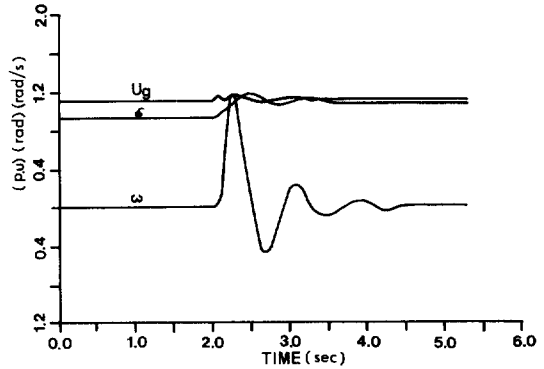


Fig. 13. Disturbance C (normal operation $k_2=400, k_3=-12$)

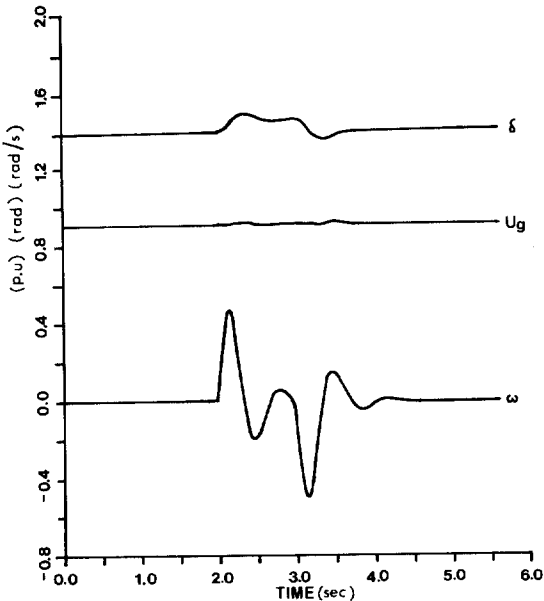


Fig. 12. Disturbance B (abnormal operation $k_2=400, k_3=-12$)

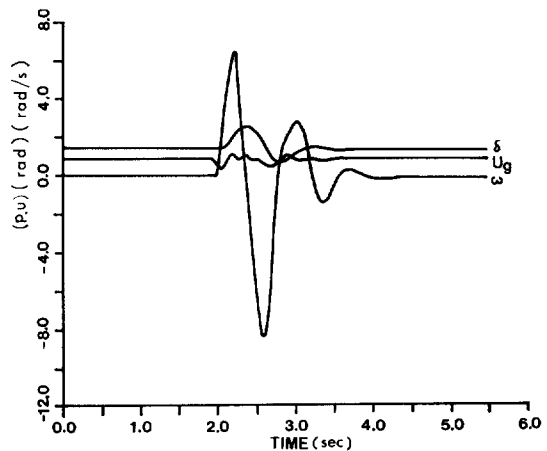


Fig. 14. Disturbance D (abnormal operation $k_2=400, k_3=-12$)

In particular, this control has no ability to overcome the interruption of one of double circuits and three phase fault as exemplified by Figure 15.

With the high value feedback, the disturbances are damped out very fast and the system is stable with respect to all types of disturbances.

Accordingly the high value feedback parameter combination is the better one of the feedback laws considered.

5. Conclusion

A design of robust voltage controller has been formulated in a procedure of pole assignment techniques using distinct system initial states to provide stabilizing control signals for a fast excitation system of a synchronous machine. Two regions of feedback parameters which shift the system eigenvalues into a specified sector was determined by pole assignment.

The test by simulation proved that stabilization signals are very effective for improving system stability.

The high value set of feedback parameters has shown predominantly better performances than the low value set.

In case with high value feedback, power angle scarcely affects system performances.

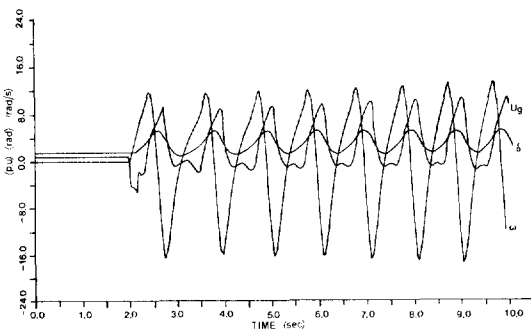


Fig. 15. Disturbance D (abnormal operation $k_2=6.5, k_3=5$)

Thus the design can be free from the difficulty in access to power angle and feedback from the rotor velocity deviation alone is adequate in stabilizing the system.

Whereas the classical PSS design requires generally a trial and error process or practically not feasible full state feedback, the output feedback design performed in this study provides a definite solution and demonstrated the suitability of the proposed techniques.

Acknowledgment

This work was supported by Korean Ministry of Education during my stay at the University of Dortmund, Germany, for a year from Dec. 1983 to Dec. 1984. I wish to express my appreciation to Prof. Dr. Handschin for his support and advice in carrying out this study.

Nomenclature

General

- A System matrix
- B control vector
- C output matrix
- U, r control input
- U_s reference input
- X state vector
- Y output vector
- \circ subscript denoting initial condition
- S laplace operator
- S_i i th system eigenvalue
- Δ prefix denoting linearized variables.

System parameters (p.u. except as indicated)

- X_e external reactance
- X_d synchronous reactance
- X_d' transient reactance
- T_d' open circuit field time constant, sec.
- T_d' effective transient time constant, sec.
- K_e, K excitation system gain
- T_e excitation system time constant, sec.
- D damping coefficient
- H machine inertia constant, sec.
- k feedback parameter

System variables (p.u. except as indicated)	
$e, e_f(ex)$	field voltage
$e_f(ind)$	equivalent field voltage induced by speed change
U_e	control signal to exciter
E, E_q	voltage behind synchronous reactance.
U_g	generator terminal voltage
U_o	infinite bus voltage
P_m	mechanical power input
P_e	electric power output
θ	rotor angle, rad
δ	rotor angle with respect to infinite bus
δ_g	rotor angle with respect to generator terminal voltage

$$B = [20.0 \quad 0 \quad 0 \quad 0]^T$$

$$C = \begin{bmatrix} 0 & 0.1278 & 0 & -0.2277 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Abnormal operation

$$A = \begin{bmatrix} -20.0 & 0 & 0 & 0 \\ 0.4045 & -0.4045 & 1.8032 & 0 \\ 0 & -22.566 & -0.2857 & -6.49 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = [20.0 \quad 0 \quad 0 \quad 0]^T$$

$$C = \begin{bmatrix} 0 & 0.1317 & 0 & -0.314 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Appendix I

Initial states and system data

Normal operation	Abnormal operation
$P_e = 0.85$ p.u.	$P_e = 0.68$ p.u.
$Q_e = 0.41$ p.u.	$Q_e = 0.0$ p.u.
$p.f = 0.9$	$p.f = 1.0$
$X_e = 0.35$ p.u.	$X_e = 0.55$ p.u.
$T_e = 0.05$ sec.	$T_e = 0.05$ sec.
$H = 3.5$ sec.	$H = 3.5$ sec.
$T_d'o = 7.0$ sec.	$T_d'o = 7.0$ sec.
$T_d' = 2.0512$ sec.	$T_d' = 2.4723$ sec.
$\delta_o = 0.9436$ rad.	$\delta_o = 1.3954$ rad.
$\delta_g = 0.6682$ rad.	$\delta_g = 0.9730$ rad.
$U_o = 1.0$ p.u.	$U_o = 1.0$ p.u.
$E_{qo} = 2.2571$ p.u.	$E_{qo} = 1.6229$ p.u.
$X_d = 1.8$ p.u.	$X_d = 1.8$ p.u.
$X_d' = 0.28$ p.u.	$X_d' = 0.28$ p.u.
$D = 2.0$ p.u.	$D = 2.0$ p.u.

Appendix 2.

Linearized system matrices

Normal operation

$$A = \begin{bmatrix} -20.0 & 0 & 0 & 0 \\ 0.4875 & -0.4875 & 1.9535 & 0 \\ 0 & -20.2818 & -0.2857 & -33.1812 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Normal operation

$$T = \begin{bmatrix} 323.52 & 1458.82 & 15.46 & 20.0 \\ 323.52 & 2.7856 & 9.75 & 0.0 \\ 0.0 & -197.75 & 0.0 & 0.0 \\ -197.75 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ -323.52 & -1475.0 & -88.4 & -20.77 \end{bmatrix}$$

$$\bar{B} = [0.0 \quad 0.0 \quad 0.0 \quad 1.0]^T$$

$$\bar{C} = \begin{bmatrix} 86.37 & 0.3560 & 1.2460 & 0.0 \\ 0.0 & -197.75 & 0.0 & 0.0 \\ -197.75 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Abnormal operation

$$T = \begin{bmatrix} 52.50 & 945.1 & 13.80 & 20.0 \\ 52.50 & 2.31 & 8.09 & 0.0 \\ 0.0 & -182.56 & 0.0 & 0.0 \\ -182.56 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ -52.5 & -948.6 & -61.1 & -20.7 \end{bmatrix}$$

$$\bar{B} = [0.0 \quad 0.0 \quad 0.0 \quad 1.0]^T$$

$$\bar{C} = \begin{bmatrix} 62.24 & 0.3044 & 1.0655 & 0.0 \\ 0.0 & -182.56 & 0.0 & 0.0 \\ -182.56 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

References

- 1) P.M. Anderson and A.A. Fouad, "power system control and stability", vol. 1, The Iowa state university press, 1977, pp. 321-322.
- 2) W.K. Marshal and W.J. Smolinski, "Dynamic stability determination by synchronizing and damping torque analysis", IEEE Trans. vol. PAS-92, pp. 1239-1246, 1973.
- 3) F.P. DeMello and C. Concordia, "Concept of synchronous machine stability as affected by excitation control", IEEE Trans. vol. PAS-88, pp. 316-329, 1969.
- 4) K. Bollinger and A. Laha, "power system stabilizer design using root locus method", IEEE Trans. vol. PAS-94, pp. 1484-1488, 1975.
- 5) R. Luus and R. Mutharasam, "stabilization of linear system behavior by pole shifting", Int. J. Control vol. 20, pp. 395-405, 1974.
- 6) K.R. Padigar, S.S. prabhu, M.A. pai and K. Gomathi, "Design of stabilizers by pole assignment with output feedback", Electrical world & Energy System vol. 2, No. 3, pp. 140-146, 1980.
- 7) V.H. Quintana, M.A. Zohdy and J.H. Anderson, "On the design of output feedback excitation controllers of synchronous machines", IEEE Trans. vol. PAS-95, pp. 954-961, 1976.
- 8) Robert T.H. Alden and Adel A. Shaltout, "Analysis of damping and synchronizing torques part I", IEEE Trans. vol. PAS-98, pp. 1696-1700, 1979.
- 9) Andreas Kubbe, "Dynamisches Spannungsverhalten in Kurzzeitbereich", Dortmund University, EV-8425, 1984, pp. 47-49.
- 10) W.G. Heffron and R.A. Phillips, "Effect of a modern-amplidyne voltage regulator on underexcited operation of large turbine generators", AIEE Trans. vol. PAS-71, pp. 692-697, 1952.
- 11) J. Ackermann, "Parameter space design of robust control systems", IEEE Trans. vol. AC-25, pp. 1058-1072, 1980.
- 12) D.G. Schultz and J.L. Molsa, "State functions and linear control systems", McGraw Hill, 1967, pp. 35-43.
- 13) D. Burghes and A. Graham, "Introduction to control theory including optimal control", John Wiley & Sons Inc., 1980, pp. 149-154.
- 14) J. Hurley, "Power system stabilization via excitation control", IEEE 81 EHO 175-0 PWR, 1980, pp. 2-11.